University of Barishal
Department of Computer Science and Engineering
Course Title: Discrete Mathematics
Course Code: CSE-1203
1st Year 2nd Semester (B.Sc.) Final Examination-2023
Admission Session: 2023-2024

Time: 03 Hours | Marks: 60

N.B.: Answer any **Five** questions out of the followings. All parts of each question must be answered consecutively. Right side of the question shows the maximum marks.

1. a) Define exclusive disjunction, tautology, contradiction, and contingency with example. — 4

   b) Explain the conditional statement: 'If you get 90% marks on the exam, you will be awarded a trophy.' Describe each case of implication. — 4

   c) Define logically equivalence. Show that the compound propositions $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent. — 4

2. a) Define proof techniques with example. Proof the following statement with contradiction and direct proof technique. — 5

   "The product of any two even numbers is always even."

   b) Write short note on existential quantification. Negate the statement: $\forall x \, (P(x) \rightarrow Q(x))$ — 4

   c) What is the truth value of $\forall x \, P(x)$, where P(x) is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4? — 3

3. a) Let $A = \{1, 2, 3\}$ and the relation $R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$. Determine whether R is **reflexive**, **symmetric**, and **antisymmetric**. — 4

   b) Let $A = \{ x \mid x$ is an even integer between 1 and 15 $\}$. Write out the elements of set A. — 3

   c) Define function and relation. Consider the function $f: A \rightarrow B$ where $A = \{1, 2, 3, 4\}$, $B = \{w, x, y, z\}$ and $f = \{(1, w), (2, x), (3, y), (4, x)\}$. Is f **injective**? Is it **surjective**? — 5

4. a) Write down the differences between Big-O and Big-θ notation with example? — 3

   b) A teacher wants to sort the scores of 1,000 students in ascending order. He has two algorithms available: **Algorithm A:** Bubble Sort ($O(n^2)$), **Algorithm B:** Merge Sort ($O(n \log n)$) Which algorithm is better in this scenario? Explain why based on time complexity. — 4

   c) Define mathematical induction and give its applications. Show that if n is an integer greater than 1, then n can be written as the product of primes. — 5

5. a) Define sum and product rule. Using mathematical induction, show that for all $n \geq 1$: — 5
   $$1 + 3 + 5 + \cdots + (2n - 1) = n^2.$$

   b) The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently? — 3

   c) Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there? — 4

6. a) Show that the number of permutations of 2 objects from n objects is twice the number of combinations of 2 objects from n objects. — 4

   Required:
   i) Write down the formulas for permutation and combination.
   ii) Substitute $r = 2$ in both formulas.
   iii) Simplify and show the relationship.
   iv) Explain your answer in words.

b) Explain the difference between **independent events** and **dependent events** in the context of conditional probability. Provide an example of each.   4

c) A company receives parts from three suppliers: X, Y, and Z.   4
  - 50% of the parts come from X, 30% from Y, and 20% from Z.
 - 1% of parts from X are defective, 2% from Y, and 3% from Z.

If a part is defective, what is the probability it came from **supplier Z**?

7.a) Explain the difference between a **Hamiltonian circuit** and an **Eulerian circuit**. Provide one real-life example where each can be applied.   5

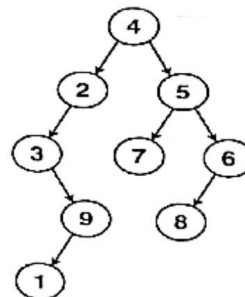**b)** Determine whether the graphs G and H shown in the figure are **isomorphic**.   3

G H
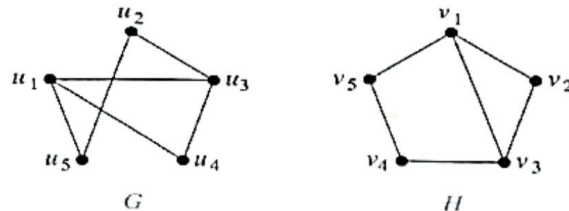
**c)** What is planner graph? Draw the planner graph of the given graph.   4
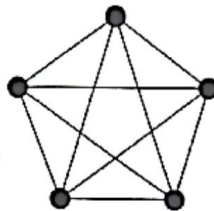
8.a) Consider the following binary tree: 6

i) Write the **in-order traversal** of the tree.

ii) Write the **post-order traversal** of the tree.

iii) Briefly explain the difference between in-order and post-order traversal.

b) Define Handshaking Theorem/Lemma. Find the in-degree and out-degree of each vertex in the following graph with directed edges.   3
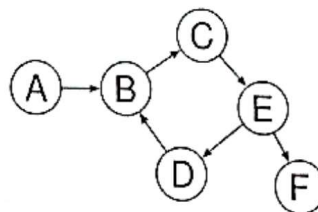
c) Define graph coloring and chromatic number of a graph and find the chromatic number of   3
i) $K_{3,3}$ ii) cycle with even number of vertices