DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

UNIVERSITY OF BARISAL

B.Sc. (Hons) Final Examination 2023

Course Title: Mathematical Analysis for Computer Science

Course Code: CSE 3201

3rd year 2nd Semester

Session: 2022-23 (Admission: 2020-21)

Time: 3 hours **Marks:** 60

Answer any FIVE questions. Marks are indicated at the end of the question.

1. a) Define random variable with example. What are the different types of random variables? Explain with appropriate example. [4]

   b) Write down the properties of random variable. [4]

   c) State and prove the unbiasedness property of the sample variance. [4]

2. a) What do you mean by stochastic process? Briefly describe different types of stochastic process with respect to time and state space. Give example in each case. [4]

   b) Define index set, state space and process with independent increment. [4]

   c) What is second order process? Write down the properties of covariance function. [4]

3. a) Define stationary process and evolutionary process. Suppose $[X(t), t \in T]$ be a stochastic process where $\Pr[X(t)=n]=\frac{e^{-at}(at)^n}{n!}$; $n=0,1,2, \ldots, a>0$. Is this process stationary? [6]

   b) Define martingales process. Let $\{Z_i\}$; $i=1, 2, \cdots$ be a sequence of *i.i.d.* random variables with mean 0 and let $X_n=\sum_{i=1}^{n} Z_i$, then show that $\{X_n\}_{n=1}^{\infty}$ is a martingale. [6]

4. a) What is Markov chain? Briefly discuss different types of Markov chain with example. [5]

   b) State and prove the first entrance decomposition formula. [3]

   c) Define transition probability. Find the probability distribution of stochastic process. [4]

5. a) Define transition probability. State and derive the Chapman Kolmogorov equation for determining the higher order transition probabilities of a Markov Chain. [5]

   b) Define Communicate state, class of state, periodicity and absorbing state. [3]

   c) If state i is recurrent and $i \leftrightarrow j$ then state j is recurrent. [4]

6. a) Define the following terms: (i) continuous time markov chain (ii) Pure Birth Process (iii) A Birth Process with Linear Birth Rate [3]

   b) Determine the Probability Function for the Yule Process. [5]

c) State and prove Chapman Kolmogorov's Backward differential equations. [4]

7. a) Under usual notations, prove that [4]

$$\lim_{h\to 0} \frac{1-P_{ii}(h)}{h} = v_i$$

$$\lim_{h\to 0} \frac{P_{ij}(h)}{h} = q_{ij} \qquad \text{for } i \neq j$$

b) Find the relation between μ *and* λ. [3]

c) Suppose that customers arrive (at a super market) at a Poisson rate of 1 per every 12 minutes and that the service time is exponential at a rate of 1 per every 8 minutes. Then find [5]

1) Find the proportion of time that there is no customer in the super market
2) Find the proportion of time the super market is busy
3) Average number of customer in the super market
4) Average amount of time a customer spent in that queue to get into the super market.

8. a) Briefly discuss the concept of queuing process with example. Also write down assumptions and the elements of queuing process. [5]

b) What is M / M /1 Queueing Model? Describe the derivation of M / M /1 Queueing Model. [7]

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