



DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
UNIVERSITY OF BARISHAL
FINAL EXAMINATION

Course Title: Mathematical Analysis for Computer Science
 Course Code: CSE-3201
 3rd Year 2nd Semester
 Session: 2019-20(Admission)
 (Answer Any Five Questions)

Time: 3 Hours

Marks: 60

1. a) Write down the reason of studying mathematical analysis in Computer Science and Engineering. [3]
 b) Derive the recursive formula for the minimum number of moves required to solve the Tower of Hanoi problem with n disks. [3]
 c) How many intersection points are there when n distinct lines are drawn in the plane, assuming no two lines are parallel and no three lines meet at the same point? [3]
 d) Derive the recursive formula for the position of the survivor in the Josephus problem, given that every k -th person is eliminated from a circle of n people. [3]
2. a) Define Box and Whisker Plot. Set Q contains 20 positive integer values. The smallest value in Set Q is a single digit value and the largest value in Set Q is a three digit value. What are possible values for the range of Set Q ? Find out the minimum and maximum range for set Q . [Range in a set means, $Range_Q = Q_{max} - Q_{min}$] [3]
 b) Solve the recurrence relation $T(n)=T(n-1)+n$ with $T(1)=1$. What is the closed-form solution for $T(n)$? [3]
Hint: This is the sum of the first n natural numbers. Use the formula for the sum of an arithmetic series: $S(n) = \frac{n(n+1)}{2}$
 c) To prove the statement: $m = \lfloor \log_2(n) \rfloor + 1$ [4]
 where n is an m -bit integer, we need to relate the number of bits required to represent n in binary to the logarithm of n .
 d) If 724 is the greatest common divisor of positive integers A and B , and 726 is the least common multiple of A and B , then $AB=?$ [2]
3. a) A number in decimal notation is divisible by 3 if and only if the sum of its digits is divisible by 3. Prove this well-known rule, and generalize it. [2]
 b) Define relative primality. The formula for the n -th Catalan number is: $C_n = \frac{1}{n+1} \binom{2n}{n}$ [4]
 calculate C_3 ?
 c) Define bayes theorem. What do you mean by priori and posterior probability. A bag I contains 4 white and 6 black balls while another Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from Bag I. [3]
 d) What do you mean by complex numbers? Prove that $\sqrt{2}$ is irrational number. [3]
4. a) Draw the transition diagram for each case. Is it irreducible? For both yes or no, each cases justify your answer with proper explanation. [4]

$$(a) P = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \quad (b) P = \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix} \quad (c) P = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & 1/5 & 4/5 \end{pmatrix}$$

$$(d) P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (e) P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

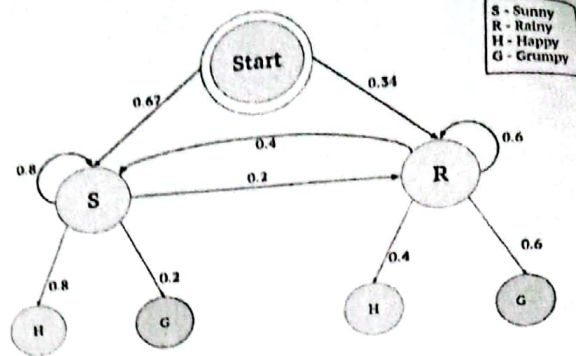
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b) Write down the following

i. Define hidden states and observed variable

ii. Find the stationary distribution for the figure along with the transition matrix.

iii. Find out the probability where observed mood sequences are HHG with SSR sequence. Write down the program to find the most likely weather sequence for the observed mood sequence.



c) Find out the transition matrix for the following scenario

- The President of the United States tells person A his or her intention to run or not to run in the next election. Then A relays the news to B, who in turn relays the message to C, and so forth, always to some new person. We assume that there is a probability that a person will change the answer from yes to no when transmitting it to the next person and a probability b that he or she will change it from no to yes. We choose as states the message, either yes or no.
- In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale.

5. a) Explain the concepts of divergence and convergence, and their applications.

b) Cribbage players have long been aware that $15 = 7 + 8 = 4 + 5 + 6 = 1 + 2 + 3 + 4 + 5$. Find the number of ways to represent 1050 as a sum of consecutive positive integers.

c) Define stochastic process. Determine whether the following series converges or diverges:

i. $\sum_{n=0}^{\infty} (-1)^n \left(\frac{5}{n}\right)$	ii. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$	iii. $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$	iv. $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$	v. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{n^3 + 1}$
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d) What are p-values? Is the harmonic series divergent or convergent? Justify your answer.

6. a) Write down the importance of Game Theory in computer science and engineering.

b) Write down the meaning of competitive situation, strategy, two person zero sum game, maximin-minimax principle.

c) Define Saddle point. Solve the following pay-off matrix to find the following

- Saddle point
- Best strategy for player A and B

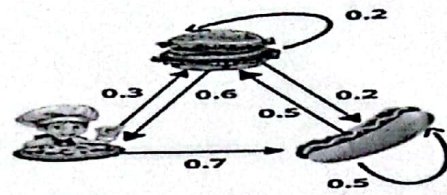
Player A	Player B			
	Strategies	I	II	III
I		6	8	6
II		4	12	2

d) Using Kendall's notation, write down the type of queue for the following scenarios:

- Exponentially distributed inter-arrival times and service times, with m servers.
- Exponentially distributed inter-arrival times and general service times with infinite servers.
- A system with exponential inter-arrival and service times, m servers, a maximum of K customers in the system, and N potential customers.

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- a) From the following diagram find out the transition matrix. Suppose we are in the pizza day, find the 5th day probability for the overall scenario.



- b) Write short notes on transient, recurrent and classes in Markov chain with examples.

c)

Truth										
Prediction	Dog	Dog	Dog	No Dog	Dog	No Dog	Dog	No Dog	Dog	Dog
	✓	✗	✓		✓		✗		✓	✗

i. Find Precision, Recall, Accuracy and F1- Measure for predicting the dog.

ii. Fill the ?? with correct True Positive(TP), True Negative(TN), False Positive(FP) and False Negative(FN).

		Actual Class	
		Positive Class	Negative Class
Predicted Class	Positive Class	??	??
	Negative Class	??	??

iii. Fill up the confusion matrix

		PREDICTION	
		DOG	NOT A DOG
TRUTH	DOG	??	??
	NOT A DOG	??	??

8. a) Define Queuing network. Write short notes on different customer behavior in queuing model. [3]
- b) Define pure birth process. Briefly describe the M/M/1: infinity ∞ /FCFS model or birth and death model. [4]
- c) Basic M/M/1 Queue: A service station has a single server where customers arrive according to a Poisson process with an average arrival rate of $\lambda=6$ customers per hour. The service times are exponentially distributed with an average service rate of $\mu=8$ customers per hour. [3]
1. Calculate the traffic intensity ρ
 2. Find the average number of customers in the system L .
 3. Determine the average time a customer spends in the system W .
- d) **Problem Statement:** A bank has a single teller who serves customers at an average rate of 10 customers per hour. Customers arrive at the bank according to a Poisson process with an average arrival rate of 12 customers per hour. [2]
1. What is the average number of customers in the queue L_q ?
 2. What is the average time a customer spends waiting in the queue W_q ?