University of Barisal

Department of Computer Sciences & Engineering

2nd Year 2nd Semester Final Examinations-2024, Session: 2022-23

Course Code: MATH-2211, Course Title: Vectors, Fourier analysis, and Laplace transforms

Full Marks: 60 Time: 3 Hours

Answer any **five (05)** from the following questions. Right side of the question shows the maximum marks.

1.(a) Find the cross product of the vectors $(1,\ 2,\ 3)$ and $(3, -4,\ 2)$ and also find the sin angle between them. 4

(b) Find the angle which the vector $3\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$ makes with the coordinate axis. 4

(c) Find the volume of the parallelepiped whose edges are represented by the $\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k},\ \ \boldsymbol{b} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{c} = 3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$. 4

2.(a) If $\boldsymbol{u} = x^2zi + yz^2j - xyk$ and $\boldsymbol{v} = xy^2i - yzj + zxk$ then find 6
(i) $(\nabla.\boldsymbol{u})\boldsymbol{v}$. (ii) $(\nabla \times \boldsymbol{u}) \times \boldsymbol{v}$, (iii) $\boldsymbol{v}.\nabla \times \boldsymbol{u}$.

(b) Prove that: $curl\ (\boldsymbol{r} \times \boldsymbol{s}) = \boldsymbol{r}\ div\ \boldsymbol{s} - \boldsymbol{s}\ div\ \boldsymbol{r} + \boldsymbol{s}.\nabla\boldsymbol{r} - \boldsymbol{r}.\nabla\boldsymbol{s}$. 6

3.(a) If $\overline{F} = x^3z\hat{\imath} - 2xy^2\hat{\jmath} + 2yz^3\hat{k}$ then find the gradient, divergence and curl of F. 6

(b) Find the line integral $\oint y^2dx - x^2dy$ about the circle $x^2 + (y-1)^2 = 1$. 6
If $x = cost$, then $y = 1 + sint$ which represent the circle.

4.(a) State and prove the Green's theorem for vector calculus. 6

(b) Verify Green's theorem in the plane for $\oint(2x - y^3)dx - xydy$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. 6

5.(a) Define Fourier series. Expand $f(x) = \sin x\ , 0 < x < \pi$ in a fourier cosine series. 6

(b) Define half range fourier sine and cosine series. Show that, an even function can have no sine terms in its fourier expansion. 6

6.(a) State and proved the Parseval's Identity for fourier expansion. 6

(b) Expand $f(x) = x,\ 0 < x < 2,$ in a half-range fourier cosine series. Also, write down the Parseval's Identity corresponding for the fourier expansion. 6

7.(a) Find the Laplace transforms of the following functions: 6
(i) $F(t) = t\ sinat$ (ii) $F(t) = t\ cosat$.

(b) If $\mathcal{L}\{F(t)\} = f(s)$ then prove that $\mathcal{L}\{t^nF(t)\} = (-1)^n \frac{d^n}{ds^n} f(s) = (-1)^n f^n(s)$ where $n = 1, 2, 3, \ldots\ldots$ 6

8.(a) Define inverse Laplace transformation. Prove that: (i) $\mathcal{L}^{-1}\left\{\frac{1}{s^{n+1}}\right\} = \frac{t^n}{\Gamma(n+1)}; n > 1$ 6
(ii) $\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2+b^2}\right\} = \frac{1}{b}e^{at}sinbt$.

(b) If $\mathcal{L}\{F(t)\} = f(s)$ then prove that : (i) $\mathcal{L}\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$ 6
(ii) $\mathcal{L}\{F'(t)\} = sf(s) - F(0)$.