DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

UNIVERSITY OF BARISAL

Final Examination 2024

Course Title: Vectors, Fourier Analysis and Laplace Transforms

Course Code: MATH-2211

2nd year 2nd Semester

Session: 2022-23 (Admission: 2021-22)

Time: 3 hour **Marks:** 60

Answer any five Questions from the followings.

1. (a) Define Fourier series. Determine the process finding of Fourier coefficients in the interval $(-\pi, \pi)$. [6]

   (b) Find the Fourier series expansion of the function $f(x) = \begin{cases} 0, & -\pi < x \le 0 \\ x, & 0 < x \le \pi \end{cases}$ and hence evaluate the sum $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. [6]

2. (a) Show that average of the square of the function $f(x)$ over $(-\pi, \pi)$ is [6]

$$a_0{}^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n{}^2 + b_n{}^2).$$

   (b) Define periodic function. Express $f(x) = x$ as a half range sine series in interval $0 < x < 2$. [6]

3. (a) Define Laplace transform. If $L\{F(t)\} = f(s)$ then show that $L\{F^n(t)\} = S^n f(s) - \sum_{k=0}^{n-1} F^{(k)}(0)s^{n-k-1}$. [7]

   (b) Prove that, [5]

$$L\left\{\int_0^t \frac{\sin u}{u}\right\} = \frac{1}{s}\tan^{-1}\frac{1}{s}$$

4. (a) Evaluate each of the following using Laplace Transforms: [9]

   $(i)\ L\{\operatorname{erf}\sqrt{t}\}$ $\quad (ii)\ L\{t^2 \cos at\}$ $\quad (iii)\ \int_0^{\infty} t\, e^{-2t} \cos t\, dt$

   (b) If $L\{F(t)\} = f(s)$, then show that $L\left\{\int_0^t F(u)du\right\} = \frac{f(s)}{s}$. [3]

5. (a) Define Scalars and Vectors. Find the angles which the vector $\vec{A} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ makes with the positive coordinate axes. [6]

   (b) Determine a unit vector perpendicular to the plane $\vec{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{B} = 4\hat{i} + 3\hat{j} - \hat{k}$. [3]

   (c) Prove that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$ [3]

6. (a) Find the directional derivative of $f(x,y,z) = x^3 z - yz^2 + z^2$ at the point (2,-1,1) in the direction of the vector $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$. [5]

   (b) If $\varphi = 3xyz - x^2y^2z^2$ then find $\vec{\nabla}\varphi$ and $|\vec{\nabla}\varphi|$ at the point (1,1,1). [4]

   (c) If $\vec{A}(x,y,z) = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$, find curl$\vec{A}$ at the point (1,-1,1). [3]

7. (a) Using Stoke's theorem, show that $\oint(yzdx + zxdy + xydz) = 0$, where c is the curve $x^2 + y^2 = 1, z = y^2$. [4]

   (b) State Green's Theorem. Verify Green's theorem in the plane for $\oint\{(xy + y^2)dx + x^2dy\}$ where c is the boundary of the region bounded by $y = x$ and $y = x^2$. [5]

   (c) If the vector $\vec{F}(x,y,z) = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational then find the values of a, b, c. [3]

8. (a) Define subspace. Show that $T = \{(a,b,c,d) \in R^4 : 2a - 3b + 5c - d = 0\}$ is a subspace of R^4. [5]

   (b) What do you mean by basis of a vector space? Show that the vectors $\{(1, 2, 0), (0, 5, 7), (-1, 1, 3)\}$ form a basis for R^3. [7]