DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING
UNIVERSITY OF BARISHAL
FINAL EXAMINATION
Course Title: Design and Analysis of Algorithms
Course Code: CSE-2201
2nd Year 2nd Semester
Admission Session: 2021-22

Time: 3 Hours | Marks: 60

(Answer any FIVE questions)

1. a) Illustrate the operation of merge sort on the array A<41, 52, 26, 38, 57, 9, 49 >. Explain Worst case, Best case, and Average Case complexity of Insertion sort. [3]

   b) Use Strassen's algorithm to compute the matrix product $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 4 & 2 \end{pmatrix}$ Show your work. [3]

   c) Given the following points: [6]

   - (0, 0)
   - (1, 1)
   - (2, 2)
   - (3, 1)
   - (2, -1)
   - (1, -1)

   Perform Graham Scan step by step and list the final convex hull vertices in order.

2. a) Show how to implement a first-in, first-out queue with a priority queue. Show how to implement a stack with a priority queue. [6]

   b) Find Longest Common Subsequence using Dynamic Programming Technique with illustration [6]
   X={A, B, C, B,D, A,B} and Y={B,D,C,A,B,A}.

3. a) Consider a hash table of size 7 with hash function $h(k) = k \mod 7$. Draw the table that results after inserting in the given order, the values, 15, 25, 12, 43, 17, 21, 79 for each of the scenarios below: [6]

   i) When collisions are handled by separate chaining.

   ii) When collisions are handled by linear probing.

   iii) When collisions are handled by double hashing. Using a second hash function $h(k) = 5 - (k \mod 5)$.

   b) Define a Linked List. Write a function to insert an element in the middle of a Linked List. Let consider the following linked list with 12 as the head. Now write an algorithm to insert a new node (15) in the middle of node 18 and 7. [6]

4. a) Write down the number of hits does the following string matching algorithms encounter in the text ***T= abahabaaaba*** when looking for the pattern ***P= ababa*** using Finite state automata algorithm. [5]

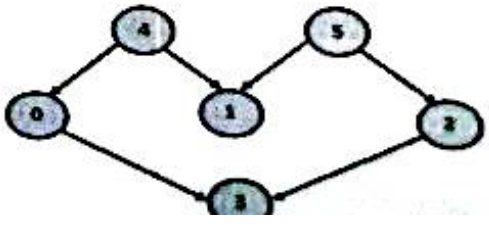
   b) Define divide and conquer. Write down the pseudo code for finding the result of 7^{54} % 8 using divide and conquer. [4]

   c) Represent the *tower of hanoi* problem using master theorem of recurrence. [3]

5. a) Topological sort algorithm for Directed Acyclic Graph (DAG) is a linear arrangement of vertices such that for every directed edge x-y, vertex x comes before y in the arrangement. Topological sorting is only applicable on graphs that are DAG. Now your task is to prove that from the above graph that, "There can exist more than one topological sorting for a given graph." Using [6]

   i. DFS

   ii. Kahn's Algorithm



b) Define disjoint set and answer the following questions. [6]

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CONNECTED-COMPONENTS(G)
1  for each vertex v ∈ G.V
2      MAKE-SET(v)
3  for each edge (u, v) ∈ G.E
4      if FIND-SET(u) ≠ FIND-SET(v)
5          UNION(u, v)
```

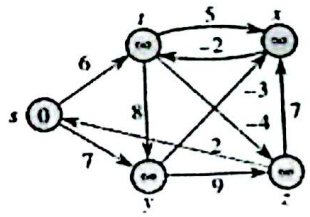
i. Show that after all edges are processed by CONNECTED-COMPONENTS, two vertices are in the same connected component if and only if they are in the same set.

ii. During the execution of CONNECTED-COMPONENTS on an undirected graph G(V, E) with k connected components, how many times is FIND-SET called? How many times is UNION called? Express your answers in terms of |V|, |E|, and k.

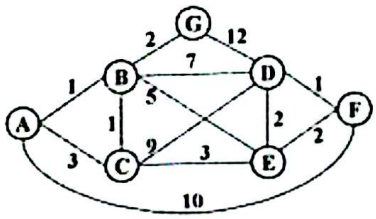
6. a) Consider the following adjacent list for directed graph. [6]
adj(y) = [x], adj(x) = [z], adj(z) = [y,w], adj(w) = [x], adj(s) = [z,w], adj(v) = [s,w], adj(t) = [u,v], adj(u) = [v].

i. Draw a graph from the above information
ii. Rewrite the procedure DFS, using a stack to eliminate recursion.
iii. Identify the tree edge, back edge, cross edge and forward edge for the graph.

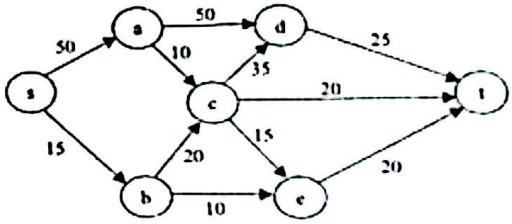
b) Define Relaxation. Run the Bellman-Ford algorithm on the directed graph of the figure, using vertex "s" as the source. In each pass, relax edges in the same order as in the figure, and show the d and Π values after each pass. Now, change the weight of edge .(z,x) to 4 and run the algorithm again, using s as the source. [6]



7. a) Consider the folling weighted and undirected graph to find the path from A to every other vertexs using Dijkstra's algorithm. [4]

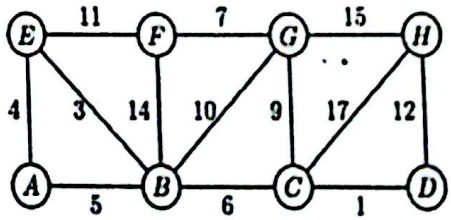


b) What do you mean by flow network and residual network? Use the Ford-Fulkerson Algorithm to find a flow function that maximizes the flow in the below network having the capacities shown and calculate the total flow that results. List the augmenting paths that you use to get your solution in order to get full credit. [4]



c) Define bipartile graph. Write down the algorithm of Edmond-Karp and when we use the algorithm with appropiate example. [4]

8. a) Consider the given weighted graph . [6]
i. Run Prim's algorithm starting from vertex A. Write the edges in the order which they are added to the minimum spanning tree.
ii. Run Kruskal's algorithm starting from vertex A. Write the edges in the order which they are added to the minimum spanning tree.



b) Run the following All-Pair-Shortest-Paths algorithms on the weighted and directed graph. [6]
i. EXTEND-SHORTEST-PATHS algorithm
ii. Floyd-Warshall algorithm