University of Barisal

Department of Computer Science & Engineering

2nd year 1st Semester Final Examination Session: 2021-22

Course Code: MATH-2109 Course Title: Complex Variables and Matrices

Full Marks: 60 Time: 3 Hours

Answer Any **FIVE (5)** from the following questions. All parts of each question must be answered consecutively. The figures in the right margin indicate full marks.

1. (a) Define matrix, idempotent and involutory matrix. Show that, 6

$$A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix} \text{ is an idempotent and } B=\begin{bmatrix}4 & 3\\ -5 & -4\end{bmatrix} \text{ is an involutory matrix.}$$

(b) Is each matrix has inverse? Give reason. Find the inverse matrix of 6

$$A=\begin{bmatrix}-1 & 2 & -3\\ 2 & 1 & 0\\ 4 & -2 & 5\end{bmatrix}$$

2. (a) What do you mean by echelon and canonical form of a matrix? Reduce the following matrix to the echelon form and hence obtain its rank. 7

$$A=\begin{bmatrix}1 & 3 & 5 & 6\\ 4 & 1 & -2 & 4\\ -2 & 0 & 3 & 1\end{bmatrix}$$

(b) Define orthogonal matrix. Verify whether the matrix $A=\frac{1}{3}\begin{bmatrix}2 & 2 & 1\\ 2 & -1 & 2\\ -1 & 2 & 2\end{bmatrix}$ is orthogonal or not. 5

3. (a) Determine for what values of λ and μ so that the following linear system has (i) a unique solution (ii) more than one solution (iii) no solution. 6

$$x+y-z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu.$$

(b) Reduce the following system of linear equations 6

$$\begin{cases}x_1 - x_2 + x_3 + x_4 - 2x_5 = 0\\ 2x_1 + x_2 - x_3 - x_4 + x_5 = 1\\ 3x_1 + 3x_2 - 3x_3 - 3x_4 + 4x_5 = 2\\ 4x_1 + 5x_2 - 5x_3 - 5x_4 + 7x_5 = 3\end{cases}$$

into echelon form and hence solve it.

4. (a) Define linearly independent and dependent vectors. Test whether the vectors $(1,-2,1), (2,1,-1)$ and $(7,-4,1)$ are linearly independent or dependent. 6

(b) Define basis and dimension. Show that the vectors $\{(0,0,1,1), (0,0,0,1), (1,1,1,1), (0,1,1,1),\}$ are basis of $\mathbb{R}^4$. 6

5. (a) Express the vector $\boldsymbol{v}=(1,2,6)$ as a linear combination of the following vectors $\boldsymbol{v_1}=(2,1,0), \boldsymbol{v_2}=(1,-1,2), \boldsymbol{v_3}=(0,3,-4)$. 6

(b) Define vector space and subspace. Show that, $W=\{(a,b,c)\in\mathbb{R}^2 : a+b=0\}$ is a subspace of $\mathbb{R}^3$. 6

6. (a) Define eigenvalue and eigenvector of a square matrix. Find all the eigenvalues and the corresponding eigenvectors of the following matrix: 8

$$\begin{bmatrix}1 & 2 & -1\\ 0 & -2 & 0\\ 0 & -5 & 2\end{bmatrix}.$$

(b) Test whether the transformation defined as follow is linear or not. 4

$$T: \mathbb{R}^3 \to \mathbb{R}^2 : T(x, y, z) = (z, x + y)$$

Also find their kernel.

7. (a) Define complex number with example.
Show that $|z|\sqrt{2} \geq |Re(z)| + |Im(z)|$, where z is any complex number. (1+3)

(b) (i) Find all the roots of $(1+i)^{1/5}$ and locate them graphically. (2+2)

(ii) Solve the equation $z^2 + (2i - 3)z + 5 - i = 0$.

(c) What are Cauchy-Riemann equations? Verify that the Cauchy-Riemann equations are satisfied for the function $f(z) = z^2$. (4)

8. (a) Define limit of a complex function. Prove that $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exist. (1+2)

(b) If $f(z) = \bar{z}$, then show that $f(z)$ is continuous everywhere but not differentiable. (4)

(c) Define analytic function with example. If $f(z) = u + iv$ is an analytic function and $u - v = e^x(cosy - siny)$, then find $f'(z)$. (1+4)