University of Barishal

Department of Computer Science and Engineering

Course Title: Discrete Mathematics
Course Code: CSE-1203
1st Year 2nd Semester Final Examination
Admission Session: 2022-2023

Time: 03 Hours **Marks:** 60

N.B.: Answer any **Five** questions out of the followings. All parts of each question must be answered consecutively. Right side of the question shows the maximum marks.

1.a) Verify that validating of the following inference. If one person is more successful than another, then he has worked harder to deserve success. X has not worked harder than Y. Therefore, X is not more successful than Y. **4**

b) Define logically Equivalences of compound proposition. Show that this implication is a tautology by using truth table. **4**

$$[(p \rightarrow q) \cap (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

c) What are the differences between 'one to one' and 'onto' function? Provide examples. **4**

2.a) Define predicates and quantifiers with examples. **3**

b) How can this English sentence be translated into logical expression? **4**
"You can't ride the roller coaster if you're under 4 feet tall, unless you're over 16."

c) Let Q(x) denotes the statement "x = x + 1". What is the truth value of the qualification ∃xQ(x), where domain consists of all real numbers? **5**

3.a) What do you know about algorithm complexity? Analyze the time complexity of the following algorithm: **5**

```
void test_algorithm(int arr[], int n)
{
    for (int i = 0; i < n; i++) {
        for (int j = i; j < n; j++) {
            printf("(%d, %d)\n", i, j);
        }
    }
}
```

```
int main()
{
  int arr[] = {1, 2, 3, 4, 5};
  int n = sizeof(arr) / sizeof(arr[0]);
  test_algorithm(arr, n);
  return 0;
}
```

b) Show that ∀x (P(x) ∧ Q(x)) and ∀x P(x) ∧ ∀x Q(x) are logically equivalent. **4**

c) For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive with explanation. **3**
i) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
ii) {(2, 4), (4, 2)}

4.a) Suppose $q(n) = b_k n^k + b_{k+1} n^{k+1} + \ldots + b_r n^r$, where r > k and the degree of q(n) is r. Prove that $q(n) = O(n^r)$. **4**

b) Represent with Venn diagram the relationship **4**
i) $A \cup B = \{x | x \in A \vee x \in B\}$ ii) $A \cap B = \{x | x \in A \wedge x \in B\}$

c) Let A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, and C = {2, 4, 6, 8} **4**

Prove or disprove: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

5.a) A university offers the following courses: 38 courses in Mathematics and 42 courses in Computer Science. **3**

If a student can choose either a Mathematics course or a Computer Science course, how many choices does the student have in total? Explain your reasoning using the Sum Rule.

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b) A library has two sections: i) Fiction Section: 8 books ii) Non-fiction Section: 10 books 4
If a person wants to borrow one book from either section and one magazine (there are 6 magazines available), how many total choices does the person have?

c) Explain how the generalized pigeonhole principle can be used to show that among any 91 integers, there are at least ten that end with the same digit. 5

6.a) *"A patient goes to see a doctor. The doctor performs a test with 99 percent reliability--that is, 99 percent of people who are sick test positive and 99 percent of the healthy people test negative. The doctor knows that only 1 percent of the people in the country are sick. Now the question is:* ***if the patient tests positive, what are the chances the patient is sick?"*** 4

Hints (Bayes's theorem):	$P(A\|B) = \frac{P(B\|A)P(A)}{P(B)}$
P(A) is the probability of event A P(B) is the probability of event B	P(A|B) is the probability of observing event A if B is true P(B|A) is the probability of observing event B if A is true.

Table I: Illustrates the scenario in a hypothetic population of 10,000 people

Test Status	Diseased	Not Diseased	Population
Test +	99	99	198
Test -	1	9801	9802
Total:	100	9900	10,000

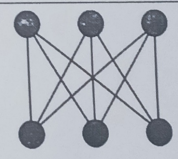
b) In how many ways can we select three students from a group of five students to stand in line for a picture? In how many ways can we arrange all five of these students in a line for a picture? 4

c) A committee of 4 members is to be formed from a group of 10 people. 4
i) How many different committees can be formed if there are no restrictions?
ii) Suppose the group consists of 6 men and 4 women. How many committees can be formed if the committee must include exactly 2 men and 2 women?

7.a) Consider the following undirected graph G: 6
It has 6 vertices, with degrees 2, 3, 3, 2, 4, and 4, respectively.
i) Determine whether the graph G has an **Eulerian circuit**. Justify your answer using the **necessary and sufficient condition** for Eulerian circuits.
ii) If G does not have an Eulerian circuit, explain whether it has an **Eulerian path**, and if so, identify the starting and ending vertices.

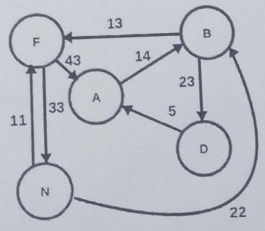
b) Let G be a 4-regular connected planar graph having 16 edges. Find the number of regions of G. 2

c) What is planner graph? Draw the planner graph of the given graph. 4



8.a) Illustrate how Kruskal's algorithm and Prim's algorithm are used to find a minimum spanning tree, using a weighted graph with at least eight vertices and 15 edges. 5

b) Define Handshaking Theorem/Lemma. Find the in-degree and out-degree of each vertex in the following graph with directed edges. 4



c) Define Isomorphic graph and Bipartite graph with example. 3