DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
UNIVERSITY OF BARISHAL

Second Year First Semester Final Examination, Session: 2020-2021

Course Title: Complex Variable and Matrices, Course Code: MATH-2109

Full Marks: 60, Credits: 3, Time: 3 Hours

[N.B.: Answer any *FIVE* questions. Numbers given on the right side indicate full marks]

1. (a) Define functions of a complex variable with examples. Find the three cube roots of $z = i$. **[4]**

   (b) Define limits and continuity of a complex function at a particular point. Using the rules of differentiation differentiate the following functions: **[8]**

   $$(i)\, f(z) = 3z^4 - 5z^3 + 2z \quad (ii)\, f(z) = \frac{z^2}{4z+1} \quad (iii)\, f(z) = (2z - 1/z)^6$$

2. (a) State and proof Cauchy-Riemann Equations. **[4]**

   (b) What is analytic function. Using the Cauchy-Riemann equations show that the function $f(z) = (2x^2 + y) + i(y^2 - x)$ is not analytic at any point. **[6]**

   (c) Write the polar form of Cauchy-Riemann Equations. **[2]**

3. (a) Define harmonic function and harmonic conjugate functions with examples. **[3]**

   (b) Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane. Hence find the harmonic conjugate function of u. **[6]**

   (c) Evaluate $\oint_C \bar{z}dz$, where C is given by $x = 3t, y = t^2, -1 \le t \le 4$. **[3]**

4. (a) State Cauchy's integral formula and its derivative formula. **[4]**

   (b) Evaluate the following integral $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$. **[6]**

   (c) State Liouville's theorem. **[2]**

5. (a) Expand $f(z) = \frac{1}{1-z}$ in a Taylor series with center $z_0 = 2i$. **[4]**

   (b) Find the Laurent expansion of the functions $f(z) = \frac{1}{z(z^2 - 3z + 2)}$ valid for each of the regions: $(i) 0 < |z| < 1 (ii) 1 < |z| < 2 (iii) |z| > 2$. **[5]**

   (c) Classify the isolated singular points. **[3]**

6. (a) Define an invertible matrix. If A is an invertible matrix, then AT is also invertible and [6]

(i) $(A^T)^{-1} = (A^{-1})^T$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

(b) Define Orthogonal, Involutory and Idempotent matrix. Prove that the matrix [6]

$A = \frac{1}{6}\begin{pmatrix} 3 & 3 & 3 & 3 \\ 3 & -5 & 1 & 1 \\ 3 & 1 & 1 & -5 \\ 3 & 1 & -5 & 1 \end{pmatrix}$ is orthogonal.

7. (a) Determine the value of a such that the following system of equations in x, y and z has: [6]

(i) a unique solution, (ii) no solution, (iii) more than one solution

$$x - 3z = -3$$
$$2x + ay - z = -2$$
$$x + 2y + az = 1$$

(b) Find the solution of the following system of linear equations with the help of matrices: [6]

$$2x - 3y + 4z = 1,$$
$$3x + 4y - 5z = 10,$$
$$5x - 7y + 2z = 3.$$

8. (a) Define eigenvectors and eigenvalues of an $n \times n$ matrix. Determine the eigenvalues and eigenvectors of the matrix: [6]

$$A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

(b) What is Cayley-Hamilton theorem? Verify this theorem for the matrix: [6]

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

Good Luck!!!