20249

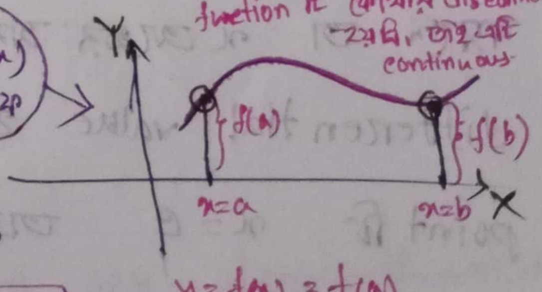
Rolle's Theorem

Theorem: Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=0$ and $f(b)=0$ $[f(a)=f(b)]$ এটা নিয়মানুসারেও হবে, then there is ~~only~~ at least one point c in the open interval (a,b) such that $f'(c)=0$

বাংলায়: যদি $f(x)$ একটি বাস্তব function হয় এবং function টি $a\le x\le b$ ~~ব্যবধিতে~~ বা $[a,b]$ (closed interval) ব্যবধিতে এমনভাবে অবস্থিত হয় যেন,

(i) $a\le x\le b$ (closed interval $[a,b]$) ব্যবধিতে function টি continuous.

$y=f(x)$ এর মান



function টি কোথায় disconnect হয়নি, তাই এটি continuous

$y=f(x)=f(a)$

$y=f(x)=f(b)$

(ii) $a<x<b$ (open interval (a,b)) ব্যবধিতে function টি differentiable.

$\boxed{y=\frac{d}{dx}x \;/\; y=f'(x) \;/\; y'}$

$x=a$ এর পর থেকে $x=b$ এর আগ পর্যন্ত প্রতিটা point differentiable বা প্রতিটি point এ

স্পর্শকের ঢাল অংকন করা যায়।

(iii) $f(a) = f(b)$ অর্থাৎ $x=a$ point এবং $x=b$ point এ অংকিত লম্ব $f(a)$ এবং $f(b)$ সমান হতে হবে। [Important]

তবে,
a and b এর মধ্যে function টির ~~কমপক্ষে~~ x এর কমপক্ষে এমন একটি মান থাকতে হবে যাতে $f'(c)=0$ হয়।
(অসংখ্য থাকতে পারে তবে must একটি থাকতে হবে।)

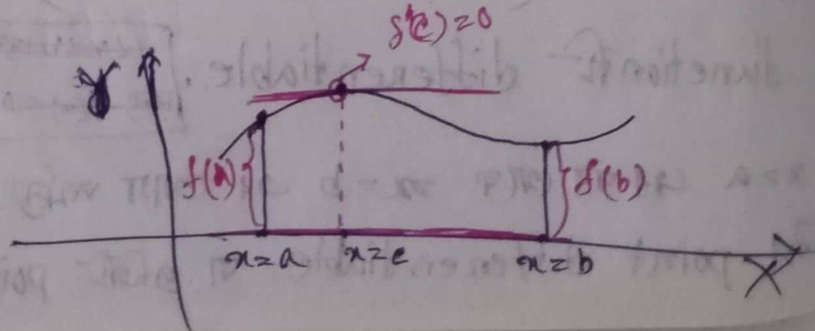
অর্থাৎ $x=a$ এবং $x=b$ এর মাঝে অন্তত একটি point এর differentiate করলে তার মান হবে 0। এবং ধরি ঐ point টা হবে c।

* $f'(c)=0$ means slope of tangent to the curve at point c is equal to zero. That means the tangent at c is parrallal to "x" axis.

আমরা জানি,
differentiate = The slope of tangent
অতএব = স্পর্শকের ঢাল।

$x=a$ এবং $x=b$ point এর মধ্যে যে point এ স্পর্শ অংকন করলে তা x অক্ষের সমান্তরাল হবে ঐ point এর differential value হলো $=0$, ~~অর্থাৎ~~ ধরি, point টি $x=c$ তাহলে $y = \frac{d(x)}{dx}$ বা $\frac{d(c)}{dx}$ বা $y = f'(c)$।

$\therefore f'(c) = 0$.



Example:1 $f(x) = x^2 + 2x - 8$ এ function এ $-4 \le x \le 2$ ব্যবধিতে রোলের উপপাদ্য যাচাই কর?

Solve:

(i) $f(x) = x^2 + 2x - 8$ function টি $-4 \le x \le 2$ বা $[-4, 2]$ closed interval এ continuous.

(ii) $f(x) = x^2 + 2x - 8$

$f'(x) = \frac{d}{dx}(x^2 + 2x - 8) = 2x + 2$

$f'(x) = 2x + 2$ function টি $-4 < x < 2$ বা $(-4, 2)$ open interval এ differentiable.

(iii) $f(a) = f(b)$ অর্থাৎ -4 ও 2 এর জন্য $f(x) = 0$ হবে।

$f(a)$ বা $f(-4) = (-4)^2 + 2.(-4) - 8 = 16 - 8 - 8 = 0$

$f(b)$ বা $f(2) = 2^2 + 2.2 - 8 = 4 + 4 - 8 = 0$

$\therefore f(-4) = f(2)$.

$f'(x) = 2x + 2$

বা, $f'(c) = 2c + 2$

$\therefore f'(c) = 0$

$\Rightarrow 2c + 2 = 0$

$c = -1$

i.e. $-4 \le -1 \le 2$, so that we can say $f(x)$ function এর $-4 \le x \le 2$ ~~এর~~ interval এ ~~ব্যবধিতে~~ রোলের উপপাদ্য প্রযোজ্য.

Example 2:
দেখাও যে $f(x) = \frac{x^2+2}{x-1}$ function টি $-4 \le x \le 2$

ব্যবধিতে রোলের উপপাদ্য প্রযোজ্য নয়।

solve: Given that

$$f(x) = \frac{x^2+2}{x-1} \quad \Big| \quad x \ne 1$$

এখানে, $x = 1$ point এ function টি অসংজ্ঞায়িত।

① $f(x)$ ~~[illegible]~~ function টি $-4 \le x \le 2$ interval এ continuous নয়।

সুতরাং $f(x) = \frac{x^2+2}{x-1}$ function টি রোলের উপপাদ্য প্রযোজ্য নয়।

Semester final Q.

Example: 3 Find the two x-intercepts of the function $f(x) = x^2 - 5x + 4$ and confirm that $f'(c) = 0$ at some point c between those intercepts.

Sol^n: Given that,

$$f(x) = \cancel{x^2+} \; x^2 - 5x + 4$$

[a, b] এর মান না থাকলে function টিকে বিশ্লেষণ করে x এর মান বের করবো।

$\therefore f(x) = 0$ $\qquad \therefore x = [1, 4]$

$x^2 - 5x + 4 = 0$

$x^2 - 4x - x + 4 = 0$ $\qquad$ ~~[illegible]~~ this function is continuous on

$(x-4)(x-1) = 0$ $\qquad$ the closed interval $[1, 4]$.

$f(x) = x^2 - 5x + 4$

$\boxed{f'(x) = 2x - 5}$

This function is differentiable on the open interval $(1,4)$ and $f(a)$ or $f(1)$ and $f(b)$ or $f(4)$ is equal. So this function is satisfied Rolle's theorem.

$f'(x) = 2\cancel{x}-5$

Now, $f'(c) = 0$

$f'(x) = 2x-5$

$f'(c) = 2c-5$

$\therefore f'(c) = 0$

$2c-5=0$

or $c = \frac{5}{2}$ is ~~the~~ a point in the interval $(1,4)$ at which

$f'(c) = 0$

Mid term

Example 4: If $f(x) = \frac{1}{4}x^2+1$, show that f satisfies the hypothesis of the mean value theorem on the interval $[-1,4]$ and find a number c in the open interval $[-1,4)$ that satisfies the conclusion of the theorem. Illustrate the result graphylly.

Soln:

① $f(x) = \frac{1}{4}x^2+1 = \frac{x^2+4}{4}$; ~~this function is continuous~~

this function is continuous on the closed interval $[-1,4]$ and it is also differentiable on " open " $(-1,4)$

Now, $f(-1) = \frac{1}{4}(-1)^2+1 = \frac{1}{4}+1 = \frac{5}{4}$

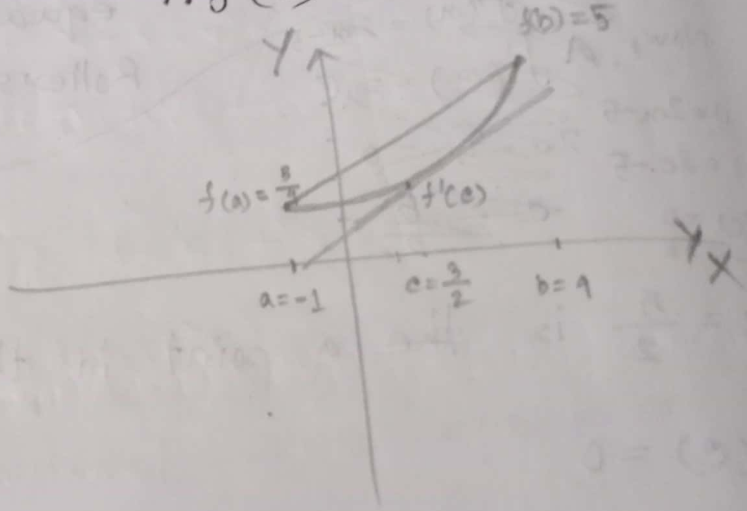
$f(4) = \frac{1}{4}(4)^2+1 = \frac{16}{4}+1 = 5$

$\therefore f(-1) \neq f(4)$ so,

Here, $f(x) = \frac{1}{4}x^2 + 1$

$$f'(x) = \frac{1}{4} \cdot 2x = \frac{x}{2}$$

$$\therefore f'(c) = \frac{c}{2}$$



$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\therefore \frac{c}{2} = \frac{5 - \frac{5}{4}}{4 - (-1)}$$

$$\Rightarrow \frac{c}{2} = \frac{\frac{20-5}{4}}{5}$$

$$\Rightarrow \frac{c}{2} = \frac{15}{4 \times 5}$$

$$\Rightarrow c = \frac{6}{4}$$

$$\therefore c = \frac{3}{2} \quad \text{Ans}$$

Example: 5 verify Rolle's theorem for $f(x) = (x+1)(x-1)(x-4)$ in $[-1, 4]$. Also write down the geometrical interpretation of Rolle

(i) $f(x) = (x+1)(x-1)(x-4)$. According to Rolles theorem,

$f(x) = 0$. So,

$$(x+1)(x-1)(x-4) = 0 \quad \text{——— (1)}$$

$$\therefore x = -1, 1, 4$$

$$\text{or } x = \{-1 \leq x \leq 4$$

$$\text{or } x = [-1, 4]$$

The function $f(x) = (x+1)(x-1)(x-4)$ is continuous on the closed interval $[-1, 4]$.

(ii) $f(x) = (x+1)(x-1)(x-4)$. According to Rolle's theorem

$f'(x) = 0$. So, $\cancel{f(x)} = \frac{d}{dx}$ $f(x) = (x-1)(x+1)(x-4)$

$$= (x^2-1)(x-4)$$

$$= x^3 - 4x^2 - x + 4.$$

Now, $f'(x) = \frac{d}{dx}(x^3 - 4x^2 - x + 4) = 0$

$$\Rightarrow 3x^2 - 8x - 1 = 0 \quad \text{——— (2)}$$

$f'(x) = 3x^2 - 8x - 1$ function is differentiable on the open interval $(-1, 4)$.

(iii) If $f(a) = f(b)$, then the function satisfies Rolle's theorem.

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$f(x) = (x+1)(x-1)(x-4)$ in $[-1,4]$

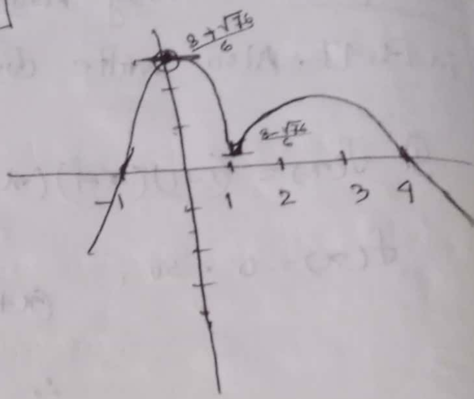
$f(-1) = (-1+1)(-1-1)(-1-4)$

$= 0$

$f(4) = (4+1)(4-3)(4-4)$

$= 0$

$\therefore f(-1) = f(4)$



So, the function $f(x) = (x+1)(x-1)(x-4)$ in $[-1,4]$ satisfies Rolle's theorem.

And, ~~f'(x)~~ If there is atleast one point on this function, where its differential value is 0. i.e. $f'(c) = 0$

(ii) $\Rightarrow f'(x) = 3x^2 - 8x - 1$

$f'(c) = 3c^2 - 8c - 1$

$\therefore f'(c) = 0$

$\Rightarrow 3c^2 - 8c - 1 = 0 \Rightarrow c = \frac{-(-8) \pm \sqrt{(-8)^2 - 4.3.(-1)}}{2.3}$

$= \frac{8 \pm \sqrt{64+12}}{6} = \frac{8 \pm \sqrt{76}}{6}$

$\therefore c = \frac{8 \pm \sqrt{76}}{6}$

or $c = \left(\frac{8+\sqrt{76}}{6}, \frac{8-\sqrt{76}}{6}\right)$ on those point, the different value is 0.

final (17-18)

Test Rolle's theorem for the function $f(x) = x^2+5x-6$ in the interval $(-6,1)$.

Soln: The function $f(x) = x^2+5x-6$ in the closed interval $[-6,1]$. Here, $a=-6$, $b=1$, According to Rolle's theorem,

$$f(a) = f(b).$$

$$\therefore f(x) = x^2+5x-6$$

$$f(-6) = (-6)^2+5.(-6)-6 = 36-30-6=0$$

$$f(1) = 1^2+5.1-6 = 1+5-6=0$$

$$\therefore f(-6) = f(1).$$

Now,

$$f(x) = x^2+5x-6$$

$$\Rightarrow f'(x) = 2x+5$$

$$\therefore f'(c) = 2c+5$$

Again, according to rolle's theorem,

$$f'(c) = 0$$

$$\Rightarrow 2c+5 = 0$$

$$c = -\frac{5}{2}$$

The function $f(x)$ is continuous on the closed interval $[-6,1]$ and also differentiable on the open interval $(-6,1)$. ~~The function f(x)~~ And the point where $f'(c)=0$ is $c=-\frac{5}{2}$. Thus, we can say that this function satisfies Rolle's theorem.

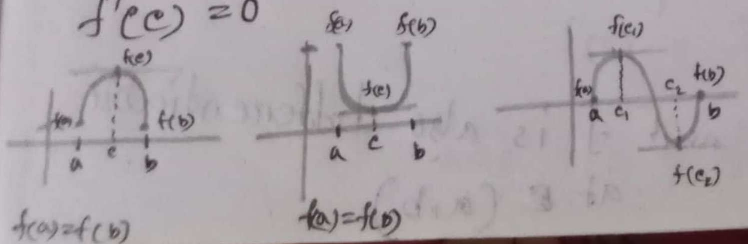
Mean value theorem

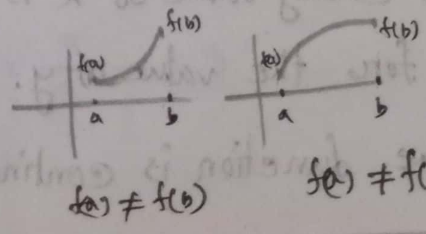
Theorem: Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). Then there is at least one point c in (a,b) such that,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

* Mean value theorem is the extension of rolle's theorem.

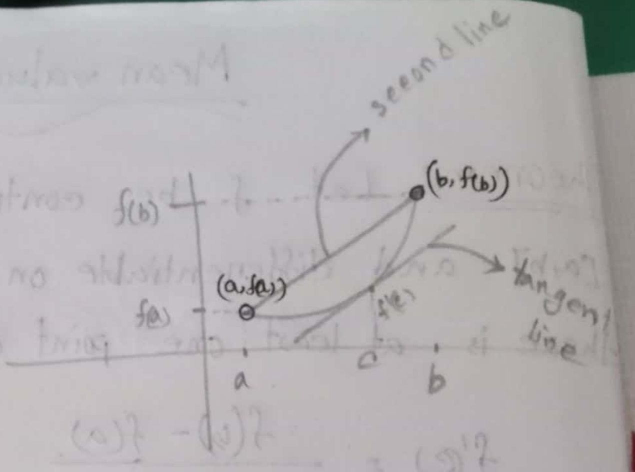
Condition for Rolle's Theorem	Condition for Mean Value Theorem
(i) Let, f a function in closed interval belongs to real number. $f: [a,b] \longrightarrow \mathbb{R}$	(i) Let, a function in closed interval $[a,b]$ belongs to real number. $f: [a,b] \rightarrow \mathbb{R}$
(ii) f is continuous at $[a,b]$	(ii) f is continuous at $[a,b]$
(iii) f is differentiable at (a,b)	(iii) f is differentiable at (a,b)
(iv) $f(a) = f(b)$ must be. most imp	(iv) $f(a) \neq f(b)$ most imp
(v) At least one point c at (a,b) where $f'(c) = 0$	(v) At least one point c at (a,b) where $f'(c) = \frac{f(b)-f(a)}{b-a}$.





$$\boxed{f'(c)} = \underbrace{\frac{f(b)-f(a)}{b-a}}$$

It means the ~~slope~~ slope at tangent to the curve at point c is equal to zero.



The slope of ~~sec~~ second line is $= \frac{f(b)-f(a)}{b-a}$

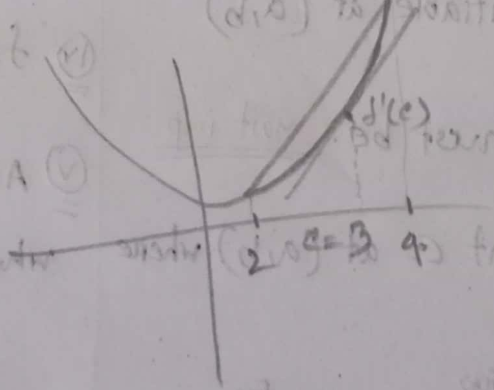
* The ~~equation~~ condition means,

in open interval (a,b), ~~where~~ there must be a ~~single~~ or some points c, ~~where~~ If we draw a tangent line at the point c, then the tangent line should be parrallal to the second line.

Example: proved that the function $f(x) = x^2$ in interval [2,4] is support the mean value theorem.

$f(x) = x^2$

or $y = x^2$



Here, every value of x is true for the value of y.

so, the function is continuous. and f is also differentiable at (a,b)

Now, $f(2) = 2^2 = 4$ $\qquad f(x) = x^2$

$f(4) = 4^2 = 16$ $\qquad f'(x) = 2x$

$f'(c) = 2c$ —— (1)

So, $f(a) \neq f(b)$ or $f(2) \neq f(4)$

$$\therefore f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{16-4}{4-2} = \frac{12}{2} = 6$$

$\therefore f'(c) = 6$

$\Rightarrow$ ~~f(c)=6~~ $2c = 6$

$\Rightarrow$ ~~2a~~ $\therefore c = 3$

So, we can say that the function $f(x) = x^2$ in the interval $[2, 4]$ is supported mean value theorem.

Semester final: (19-20)

Show that the function $f(x) = \frac{x^3}{4} + 1$ satisfies the hypotheses of Mean Value Theorem over the interval $[0, 2]$.

Solⁿ:

$a = 0$

$b = 2$

$$f(x) = \frac{x^3}{4} + 1$$

$$f(0) = \frac{0}{4} + 1 = 1$$

$$f(2) = \frac{2^3}{4} + 1 = \frac{8}{4} + 1 = 3$$

and $f(0) \neq f(2)$

So, $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Here,

$f(x) = \dfrac{x^3}{4} + 1$

$f'(x) = \dfrac{3x^2}{4}$

$\therefore f'(c) = \dfrac{3c^2}{4}$

$\Rightarrow \dfrac{3c^2}{4} = \dfrac{3-1}{2-0}$

$\Rightarrow 3c^2 = \dfrac{8}{2}$

$\Rightarrow c^2 = \dfrac{4}{3}$

$\therefore c = \pm \dfrac{2}{\sqrt{3}}$

i.e. the function is in the closed interval $[0,2]$ and $-\dfrac{2}{\sqrt{3}}$ don't exist in this interval, so we considered the value of c is $\dfrac{2}{\sqrt{3}}$.

This function is continuous at $[0,2]$ interval and also differentiable at $(0,2)$ interval and it has at least one ponit $c = \dfrac{2}{\sqrt{3}}$, where $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ is true.

So, the function satisfies the hypotheses of Mean value theorem.

final (17-18)

Test the Mean Value theorem for the function $f(x)=3+2x-x^2$ in the interval $[0,1]$.

Solⁿ: The function $f(x) = 3+2x-x^2$ in the interval $[0,1]$.

Here,
$a=0$
$b=1$

According to meanvalue theorem, $f(a) \neq f(b)$, so,

$$f(x) = 3+2x-x^2$$
$$f(0) = 3+2.0-0^2 = 3$$
$$f(1) = 3+2.1-1 = 4$$

$\therefore f(0) \neq f(1)$.

again,
$$f'(c) = \frac{f(b)-f(a)}{b-a}$$
$$\Rightarrow 2-2c = \frac{4-1}{1-0}$$
$$\Rightarrow 2(1-c) = \frac{3}{1}$$
$$\Rightarrow 1-c = \frac{3}{2}$$
$$-c = \frac{3-2}{2}$$
$$\therefore c = \frac{-1}{2}$$

Here,
$$f(x) = 3+2x-x^2$$
$$f'(x) = 0+2-2x$$
$$= 2-2x$$
$$\therefore f'(c) = 2-2c$$

The function $f(x)=3+2x-x^2$ is continuous at $[0,1]$ closed interval and differentiable at open interval $(0,1)$. And the point c is $-\frac{1}{2}$. So, this function satisfies meanvalue theorem.

Expansion of function:

Taylor Polynomial: If f function can be differentiable n times at a, then we define the n^{th} Taylor Polynomial f_m of f about $x=a$ to be,

$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots\cdots + \frac{f^n(a)}{n!}(x-a)^n$$

Mid (18-19) Just এখানের আসছে

Find the Taylor polynomials of $\frac{1}{x+2}$ order $n=0,1,2,3$ about $x=3$.

Soln: Here,

$$f(x) = \frac{1}{x+2} = (x+2)^{-1}$$

$$f'(x) = -1(x+2)^{-2} = \frac{-1}{(x+2)^2}$$

$$f''(x) = 2(x+2)^{-3} = \frac{2}{(x+2)^3}$$

$$f'''(x) = -6(x+2)^{-4} = \frac{-6}{(x+2)^4}$$

Now,

$$f(3) = \frac{1}{3+2} = \frac{-1}{5}$$

$$f'(3) = \frac{-1}{(3+2)^2} \cdot \frac{1}{1!} = \frac{-1}{25}$$

$$f''(3) = \frac{2}{(3+2)^3} \cdot \frac{1}{2!} = \frac{1}{125}$$

$$f^{(3)}(3) = \frac{-6}{(3+2)^4} \cdot \frac{1}{3!} = \frac{-1}{625}$$

So, the Taylor polynomial is

$$P_3 = \frac{1}{5} + \frac{-1}{25}(x-3) + \frac{1}{125}(x-3)^2 + \frac{-1}{625}(x-3)^3$$

$$= \frac{1}{5} - \frac{x-3}{25} + \frac{(x-3)^2}{125} - \frac{(x-3)^3}{625} \quad \text{Ans}$$

Taylor series: If f function has derivative of all orders at 'a', then we call the series,

$$\sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots \cdots + \frac{f^n(a)}{n!}(x-a)^n + \cdots$$

Maclaurin Polynomial: If f function can be differentiable n times at zero (0), then we define the nth maclaurin polynomial for f about $x=0$ to be,

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots + \frac{f^n(0)}{n!}x^n$$

Maclaurin series: If f has derivatives of all orders at zero (0), then we call the series,

$$\sum_{n=0}^{\infty} \frac{f^n(0)}{n!}x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^n(0)}{n!}x^n + \cdots$$

Convergence of series:

The limit comparision test: Let Σa_k and Σb_k series with positive terms and suppose that,

$$\rho = \lim_{k\to\infty} \frac{a_k}{b_k}$$

if ρ is finite and $\rho > 0$, then the series both converge or both diverge.

Example: Use the limit comparision test to determine the series $\sum_{k=1}^{\infty} \frac{1}{2k^2+k}$ converge or diverge.

Soln: Let $a_k = \frac{1}{2k^2+k}$ — (i) be have like the series,

$b_k = \frac{1}{2k^2}$ — (ii)

Now, $\rho = \lim_{k\to\infty} \frac{a_k}{b_k}$

$$= \lim_{k\to\infty} \frac{\frac{1}{2k^2+k}}{\frac{1}{2k^2}}$$

$$= \lim_{k\to\infty} \frac{2k^2}{2k^2+k}$$

$$= \lim_{k\to\infty} \frac{\frac{2k^2}{k^2}}{\frac{2k^2}{k^2}+\frac{k}{k^2}}$$

$$= \lim_{k\to\infty} \frac{2}{2+\frac{1}{k}}$$

$$= \frac{2}{2+\frac{1}{\infty}}$$

$$= \frac{2}{2+0} = 1$$

Since ρ is finite and positive, then the given series converge.

The ratio test: Let Σa_k be a series with positive terms and suppose that,

$$\rho = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$$

ⓐ If $\rho < 1$, the series converge

ⓑ If $\rho > 1$ and $\rho = \infty$, the series diverge

ⓒ If $\rho = 1$, the series may converge or diverge. So that another test must be tried.

Example: Use the ratio test to determine whether the following series converge or diverge.

ⓐ $\sum_{k=1}^{\infty} \frac{1}{k!}$ ⓑ $\sum_{k=1}^{\infty} \frac{k^k}{k!}$

Solⁿ: ⓐ $\sum_{k=1}^{\infty} \frac{1}{k!}$

Let, $a_k = \frac{1}{k!}$ and $a_{k+1} = \frac{1}{(k+1)!}$

$$\rho = \lim_{k\to\infty} \frac{a_{k+1}}{a_k}$$

$$= \lim_{k\to\infty} \frac{\frac{1}{(k+1)!}}{\frac{1}{k!}}$$

$$= \lim_{k\to\infty} \frac{k!}{(k+1)!}$$

$$= \lim_{k\to\infty} \frac{1}{k+1}$$

$$= 0$$

or $0<1$. Therefore the series converge, since $\rho < 1$.

The root test: Let $\sum a_k$ be a series with positive terms and suppose that,

$$\rho = \lim_{k\to\infty} \sqrt[k]{a_k}$$
$$= \lim_{k\to\infty} (a_k)^{\frac{1}{k}}$$

(a) If $\rho < 1$, the series converge

(b) If $\rho > 1$ and $\rho = \infty$, the " diverge

(c) If $\rho = 1$, the series may converge or diverge, so that another test must be tried.

Example: Use the root test to determine whether the following series converge or diverge.

(a) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\{\ln(k+1)\}^k}$

Sol^n: (a) $\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$

Let,

$$a_k = \left(\frac{3k+2}{2k-1}\right)^k$$

$$\rho = \lim_{k\to\infty} (a_k)^{\frac{1}{k}}$$

$$= \lim_{k\to\infty} \frac{3k+2}{2k-1}$$

$$= \lim_{k\to\infty} \frac{3+\frac{2}{k}}{2-\frac{1}{k}}$$

$$= \frac{3}{2} > 1$$

$\therefore$ The given series diverge, since $\rho > 1$.

Example: 5 Find the nth Maclaurin polynomial for $\frac{1}{1-x}$ and express it in sigma notation.

Sol^n: Let. $f(x) = \frac{1}{1-x}$, The values of f and its first derivatives at $x=0$ are as follows.

$f(x) = \frac{1}{1-x} = (1-x)^{-1}$ $\quad f(0) = \frac{1}{1-0} = 1$

$\cancel{f'(x) = \frac{-1}{(1-x)^2}}$ $\quad \cancel{f'(0) = \frac{-1}{(1-0)^2} \cdot \frac{1}{1!} = -1}$

$f'(x) = -1(1-x)^{-2} \cdot -1$

$= (1-x)^{-2}$ $\quad f'(0) = \frac{1}{(1-0)^2} \cdot \frac{1}{1!} = 1$

$f''(x) = -2(1-x)^{-3} \cdot -1$

$= 2(1-x)^{-3}$ $\quad f''(0) = \frac{2}{(1-0)^3} \cdot \frac{1}{2!} = 1$

$f'''(x) = -6(1-x)^{-4} \cdot -1$

$= 6(1-x)^{4}$ $\quad f'''(0) = \frac{6}{(1-0)^4} \cdot \frac{1}{3!} = 1$

$f^{k}(x) = \frac{k!}{(1-x)^{k+1}}$ $\quad f^{k}(0) = \frac{k!}{(1-0)^{k+1}} \cdot \frac{1}{k!} = \frac{1}{1^{k+1}}$

Thus substituting $f^{(k)}(0) = 1^{k+1}$ into nth Maclaurin polynomial for $\frac{1}{1-x}$:

$= \sum_{k=0}^{n} x^k =$

$P_n(x) = 1 + 1 \cdot x + 1 \cdot x^2 + 1 \cdot x^3 + \text{------} + \frac{1}{1^{n+1}} x^n$

$= 1 + x + x^2 + x^3 + \text{-----} + x^n \quad (n = 1, 2, 3 \text{---})$