Solution to Selected Problems from the Numerical Methods Final Examination

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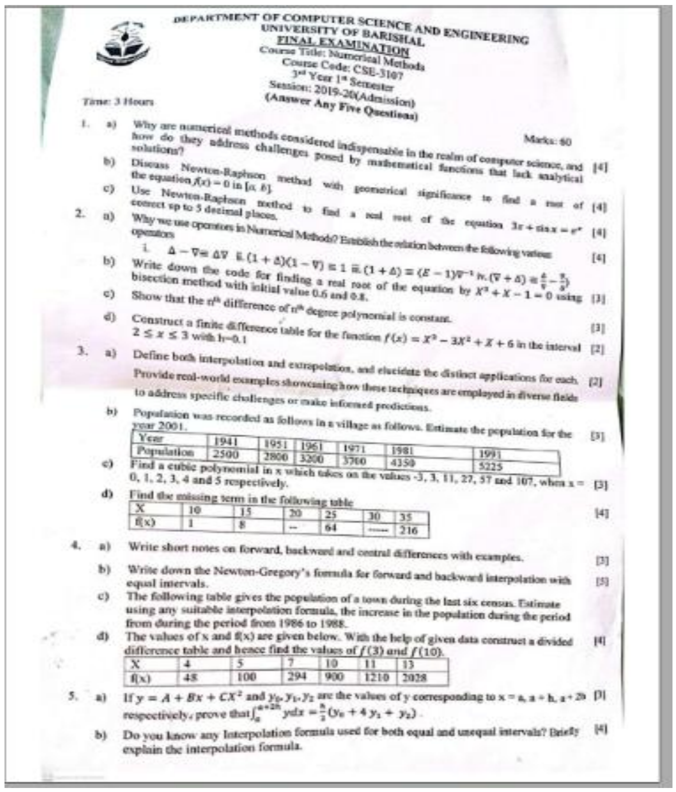

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
UNIVERSITY OF BARISHAL
FINAL EXAMINATION
Course Title: Numerical Methods
Course Code: CSE-3107
3rd Year 1st Semester
Session: 2019-20(Admission)
(Answer Any Five Questions)

Time: 3 Hours   Marks: 60

1. a) Why are numerical methods considered indispensable in the realm of computer science, and how do they address challenges posed by mathematical functions that lack analytical solutions? [4]
   b) Discuss Newton-Raphson method with geometrical significance to find a root of the equation $f(x) = 0$ in $[a, b]$. [4]
   c) Use Newton-Raphson method to find a real root of the equation $3x + \sin x = e^x$ correct up to 5 decimal places. [4]
2. a) Why we use operators in Numerical Method? Establish the relation between the following various operators [4]
      i. $\Delta - \nabla = \Delta\nabla$ ii. $(1+\Delta)(1-\nabla) \equiv 1$ iii. $(1+\Delta) = (E-1)\nabla^{-1}$ iv. $(\nabla + \Delta) = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$
   b) Write down the code for finding a real root of the equation by $X^3 + X - 1 = 0$ using bisection method with initial value 0.6 and 0.8. [3]
   c) Show that the n^{th} difference of n^{th} degree polynomial is constant. [3]
   d) Construct a finite difference table for the function $f(x) = X^3 - 3X^2 + X + 6$ in the interval $2 \le x \le 3$ with h=0.1 [2]
3. a) Define both interpolation and extrapolation, and elucidate the distinct applications for each. Provide real-world examples showcasing how these techniques are employed in diverse fields to address specific challenges or make informed predictions. [2]
   b) Population was recorded as follows in a village as follows. Estimate the population for the year 2001. [3]

Year	1941	1951	1961	1971	1981	1991
Population	2500	2800	3200	3700	4350	5225

   c) Find a cubic polynomial in x which takes on the values -3, 3, 11, 27, 57 and 107, when x = 0, 1, 2, 3, 4 and 5 respectively. [3]
   d) Find the missing term in the following table [4]

X	10	15	20	25	30	35
f(x)	1	8	--	64	-----	216

4. a) Write short notes on forward, backward and central differences with examples. [3]
   b) Write down the Newton-Gregory's formula for forward and backward interpolation with equal intervals. [5]
   c) The following table gives the population of a town during the last six census. Estimate using any suitable interpolation formula, the increase in the population during the period from during the period from 1986 to 1988.
   d) The values of x and f(x) are given below. With the help of given data construct a divided difference table and hence find the values of $f(3)$ and $f(10)$. [4]

X	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

5. a) If $y = A + Bx + CX^2$ and y_0, y_1, y_2 are the values of y corresponding to x = a, a + h, a + 2h respectively, prove that $\int_a^{a+2h} y\,dx = \frac{h}{3}(y_0 + 4y_1 + y_2)$. [3]
   b) Do you know any Interpolation formula used for both equal and unequal intervals? Briefly explain the interpolation formula. [4]

Question 1: Newton-Raphson Method

**a)

Why Numerical Methods are Indispensable in Computer Science:

1. **Handling Complex Problems:**
   Many real-world problems in science, engineering, and computer science result in mathematical models that **cannot be solved exactly** (analytically).
   Example: Nonlinear equations, differential equations, large data sets.
2. **Lack of Analytical Solutions:**
   Some mathematical functions or equations are so complicated that **no exact formula** exists to solve them.
   Example:

- Solving ∫e−x2dx\int e^{-x^2} dx — this integral has no exact solution in terms of elementary functions.
- Solving large systems of linear equations (like in graphics rendering or simulations).

3. **Approximation Ability:**
   Numerical methods allow computers to **approximate solutions** to any desired level of accuracy.
   For example:

- Finding square roots using the Newton-Raphson method.
- Calculating values of sine, cosine using Taylor Series expansion.

4. **Efficiency and Speed:**
   With the help of algorithms (like bisection, iteration methods), computers can quickly and efficiently solve **huge, complex problems** that humans cannot solve manually.

How Numerical Methods Address These Challenges:

Challenge	**Numerical Solution**
No analytical solution exists	Numerical approximation methods (e.g., iteration) provide usable solutions.
Complex or infinite calculations	Computers apply numerical algorithms to simplify and compute results.
Large system of equations	Matrix methods (e.g., Gauss Elimination) solve efficiently.
Real-world data often noisy or incomplete	Numerical interpolation, regression handle such cases to find best fit or estimate missing values.

Real-World Examples:

1. **Weather Prediction:**
   Partial Differential Equations (PDEs) model weather — solved by numerical methods on supercomputers.

2. **Engineering Simulations:**
   Structural analysis using **Finite Element Methods (FEM)** relies entirely on numerical methods.

3. **Machine Learning:**
   Optimization algorithms like **Gradient Descent** use numerical methods to find the best model parameters.

4. **Graphics and Animation:**
   Rendering realistic images in games or movies uses numerical methods to solve lighting and shading models.

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b) Discuss the Newton-Raphson method with geometrical significance.

* Newton-Raphson's method :

1.8 ***Derive Newton-Raphson's iterative method to find an approximate root of an equation $f(x) = 0$.***

[$f(x) = 0$ সমীকরণের একটি আসন্ন মূল নির্ণয় করার জন্য নিউটন র‍্যাফসনের পুনঃকারী সূত্রটি প্রতিষ্ঠা কর।] **[Pre-'94, '97, '04; M.Sc-'95, '96, '98 '01, '02, '03; 3H-'96, '97, '99, 2000, '01, '02, '03]**

Solution : This method is generally used to improve the initial approximation, obtained by Bisection method or method of False Position. It is the most useful and well-known numerial method.

Suppose x_0 is an initial approximate value of the root $x_1 = x_0 + h$ of an equation $f(x) = 0$, where h being a small quantity.

$\therefore f(x_1) = f(x_0 + h) = 0$

Expanding by Taylor's series.

$$f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \ldots\ldots\ldots = 0$$

Since h is very small, neglecing the second and higher order terms of h.

$$\therefore f(x_0) + hf'(x_0) = 0$$

$$\Rightarrow h = -\frac{f(x0)}{f'(x_0)},\ f'(x_0) \neq 0$$

So first approximation, $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

succcessive approximations are given by $x_2, x_3, \ldots\ldots, x_{n+1}$, where $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$, $n = 0, 1, 2, \ldots\ldots$ which is the Newton-Raphson formula. [দ্বিধাবিভক্তি পদ্ধতি বা ফলস্ পজিশন পদ্ধতিতে নির্ণীত আদি আসন্নমানকে মানোন্নয়ন করার জন্য সাধারনত এই পদ্ধতি ব্যবহৃত হয়। ইহা খুব ব্যবহৃত এবং সুপরিচিত সাংখিক পদ্ধতি।

মনে করি $f(x) = 0$ সমীকরণের $x_1 = x_0 + h$ মূলের আদি আসন্নমান x_0, যেখানে h ক্ষুদ্র সংখ্যা।

$$\therefore f(x_1) = f(x_0 + h) = 0$$

টেলর ধারায় বিস্তৃতি করে পাই, $f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \ldots\ldots\ldots\ldots = 0$

যেহেতু h খুব ক্ষুদ্র, সুতরাং h এর দ্বিতীয় ও অধিক মাত্রার পদ উপেক্ষা করি।

$$\therefore f(x_0) + hf'(x_0) = 0$$

$$\Rightarrow h = -\frac{f(x_0)}{f'(x_0)},\ f'(x_0) \neq 0$$

সুতরাং প্রথম আসন্নমান, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

পর্যায়ক্রমিক আসন্নমান $x_2, x_3, \ldots\ldots\ldots\ldots, x_{n+1}$ হতে পাওয়া যায়, যেখানে

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, $n = 0, 1, 2, \ldots\ldots\ldots$ ইহাই নিউটন রাফসন সূত্র।]

বিঃ দ্রঃ (1) Newton-Raphson's iterative method কে অনেক সময় Newton-Raphson's method অথবা Newton's method অথবা Tangential method হিসেবে প্রকাশ করা হয়।

(2) যদি $\phi'(x_0) = 0$ হয় তবে নিউটন র‍্যাফসন পদ্ধতি প্রয়োগযোগ্য হবে না।

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c) Use the Newton-Raphson method to find a real root of $3x + \sin x = e^x$.

1.8-12 ***Find a real root of the equation $3x + \sin x = e^x$ by using Newton-Raphson's method, correct up to three decimal places.*** *[নিউটন র‍্যাফসন পদ্ধতি ব্যবহার করে $3x + \sin x = e^x$ সমীকরণের একটি বাস্তব মূল নির্ণয় কর যা তিন দশমিক স্থান পর্যন্ত শুদ্ধ হবে।]*

Solution : Given equation, $3x + \sin x = e^x$ .... (1)

Let $f(x) = 3x + \sin x - e^x$

$\therefore f'(x) = 3 + \cos x - e^x$

Here $f(0).\ f(0.5) < 0$

So a real root of (1) lies between 0 and 0.5. Let the initial approximate value of the root, $x_0 = \frac{0 + 0.5}{2} = 0.25$

Now we get from Newton-Raphson's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.356753638$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.360416926$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.360421702$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 0.360421702$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 0.360421702$$

Hence the required root is 0.360, correct up to three decimal places.

Question 2

**a)

i)

(iii) We have,

$$\Delta\nabla y_n = \Delta(y_n - y_{n-1}) = \Delta y_n - (y_n - y_{n-1})$$

$$= \Delta y_n - \nabla y_n = (\Delta - \nabla)y_n$$

$$\therefore \Delta - \nabla \equiv \Delta\nabla$$

Ii)

$$(v)\ (1+\Delta)(1-\nabla)\,y_n = (1+\Delta)(y_n - \nabla y_n)$$
$$= (1+\Delta)(y_n - y_n + y_{n-1})$$

$$= (1+\Delta)\,y_{n-1}$$
$$= y_{n-1} + y_n - y_{n-1}$$
$$= 1.y_n$$
$$\therefore (1+\Delta)(1-\nabla) \equiv 1$$

iii)

$$(vii)\ (E-1)\,\nabla^{-1} y_n = (E-1)(1-E^{-1})^{-1}\,y_n$$
$$= (E-1)(1 + E^{-1} + E^{-2} + E^{-3} + E^{-4} + \ldots\ldots\ldots)y_n$$
$$= (E + 1 + E^{-1} + E^{-2} + E^{-3} + \ldots\ldots$$
$$- 1 - E^{-1} - E^{-2} - E^{-3} - E^{-4} - \ldots\ldots)\,y_n$$
$$= Ey_n$$
$$= (1+\Delta)\,y_n$$
$$\therefore 1 + \Delta \equiv (E-1)\,\nabla^{-1}$$

iv) pari na

**b)

b) Write code for finding a real root of $X^3 + X - 1 = 0$ using the bisection method (initial values: 0.6, 0.8).

Python Code:

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```python
def bisection(f, a, b, tol=1e-6, max_iter=100):
    for _ in range(max_iter):
        c = (a + b) / 2
        if f(c) == 0 or (b - a)/2 < tol:
            return c
        if f(a) * f(c) < 0:
            b = c
        else:
            a = c
    return (a + b) / 2

f = lambda x: x**3 + x - 1
root = bisection(f, 0.6, 0.8)
print("Root:", root)
```

Output: The root is approximately 0.6823.

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c)

... are zero.

2.5 ***Show that the nth difference of nth degree polynomial is constant.*** [দেখাও যে, n-ঘাতের বহুপদীর n-তম পার্থক্য ধ্রুবক।]

OR, ***State and prove the fundamental theorem of difference calculus.***

Solution : Statement : The nth difference of nth degree polynomial is constant.

Proof : Let $y = f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n$ be a polynomial of nth degree.

$\therefore$ The first difference of the polynomial.

$$\Delta y = f(x + h) - f(x)$$

$$= a_0\{(x + h)^n - x^n\} + a_1\{(x + h)^{n-1} - x^{n-1}\} + a_2\{(x + h)^{n-2} - x^{n-2}\} + \ldots\ldots + a_{n-1}\{(x + h) - x\} + a_n - a_n$$

$$= a_0[\{x^n + nhx^{n-1} + \frac{n(n-1)}{2!}h^2x^{n-2} + \frac{n(n-1)(n-2)}{3!}h^3x^{n-3} + \ldots\ldots + h^n\} - x^n]$$

$$+ a_1[\{x^{n-1} + (n-1)hx^{n-2} + \frac{(n-1)(n-2)}{2!}h^2x^{n-3} + \ldots\ldots + h^{n-1}\} - x^{n-1}]$$

$$+ a_2[\{x^{n-2} + (n-2)hx^{n-3} + \frac{(n-2)(n-3)}{2!}h^2x^{n-4} + \ldots\ldots + h^{n-2}\} - x^{n-2}] + \ldots\ldots + a_{n-1}h$$

$$= a_0nhx^{n-1} + a_1'x^{n-2} + a_2'x^{n-3} + \ldots\ldots + a'_{n-2}x + a'_{n-1}$$

where $a_1' = \frac{a_0 n(n-1)}{2!}h^2 + a_1(n-1)h$

$$a_2' = a_0\frac{n(n-1)(n-2)}{3!}h^3 + a_1\frac{(n-1)(n-2)}{2!}h^2 + a_2(n-2)h$$ etc. are new coefficients.

$\therefore$ The second difference of the polynomial.

$$\Delta^2 y = a_0nh\{(x + h)^{n-1} - x^{n-1}\} + a_1'\{(x + h)^{n-2} - x^{n-2}\} + a_2'\{(x + h)^{n-3} - x^{n-3}\} + \ldots\ldots + a'_{n-2}\{(x + h) - x\} + a'_{n-1} - a'_{n-1}$$

$$= a_0n(n-1)h^2x^{n-2} + a_1''x^{n-3} + a_2''x^{n-4} + \ldots\ldots + a''_{n-3}x + a''_{n-2}$$

where $a_1'', a_2'', \ldots\ldots a''_{n-3}, a''_{n-2}$ are new coefficients.

Similarly continuing the above process, we have

$$\Delta^n y = a_0n(n-1)(n-2)\ldots\ldots 3.2.1\, h^n x^{n-n}$$

$$= a_0n!\, h^n \text{ (constant)}$$

Thus the nth difference of nth degree polynomial is constant.

**d)

d) Construct a finite difference table for $f(x) = X^3 - 3X^2 + X + 6$ in $[2, 3]$ with $h = 0.1$.
Example Table (partial):

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
2.0	4.0	-0.9	0.6	0.6
2.1	3.1	-0.3	1.2	0.6
...	...	...	...	...

Note: The third difference $\Delta^3 f$ is constant (0.6), confirming the polynomial is cubic.

**Question 3: **

**a)

1. Interpolation

Definition:
Interpolation is the process of **estimating unknown values** that fall **within the range** of known data points.

Application:
Used when you have data points in a specific range and want to find values in between them.

Real-World Example:
In **weather forecasting**, if you know the temperature at 10 AM and 12 PM, interpolation can help estimate the temperature at 11 AM.

Other Fields:

- **Engineering:** Predicting material stress levels between measured points.
- **Finance:** Estimating bond prices between known interest rates.

2. Extrapolation

Definition:
Extrapolation is the process of **estimating unknown values** that fall **outside the range** of known data points.

Application:
Used to **predict future trends** based on existing data.

Real-World Example:
In **population studies**, if a city's population was 2 million in 2020 and 2.2 million in 2024, extrapolation can predict the population in 2030.

Other Fields:

- **Sales Forecasting:** Predicting future sales based on past performance.
- **Climate Science:** Estimating future temperature changes based on historical climate data.

**b)

3.2-1 ***The population of a country in the decennial census were as under. Estimate the population for the year 1925.*** *দশবার্ষিক গণনায় কোন দেশের জনসংখ্যা নিচে দেওয়া হল। 1925 সালের জনসংখ্যা নির্ণয় কর।*

Year	1891	1901	1911	1921	1931
Population (in thousands)	46	66	81	93	101

Solution : First we form a backward difference table ;

Year x	Population y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
1901	66	20			
1911	81	15	−5		
1921	93	12	−3	2	
1931	101	8	−4	−1	−3

By Newton's formula for backward interpolation we have,

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_n + \ldots\ldots$$

where $u = \frac{x - x_n}{h}$

In this question, $x = 1925,\ x_n = 1931,\ h = 10$

$$\therefore u = \frac{1925 - 1931}{10} = -0.6$$

$$\therefore y(1925) = 101 + (-0.6)(8) + \frac{(-0.6)(-0.6+1)}{2!}(-4) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(-1) + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4!}(-1)$$

$$= 101 - 4.8 + 0.48 + 0.056 + 0.1008$$

$$= 96.8368$$

Hence the population for 1925 is estimated to be 96.84 thousand approx.

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b) Estimate the population for the year 2001 using given data.

Given Data:

| Year | 1941 | 1951 | 1961 | 1971 | 1981 | 1991 |

| Population | 2500 | 2800 | 3200 | 3700 | 4350 | 5225 |

Method: Use Newton's forward interpolation.

- Step size $h = 10$, base year 1941.
- For $x = 2001$, $u = (2001 - 1941)/10 = 6$.
- Compute differences and apply the formula:

(2001) \approx 2500 + 6(300) + \frac{6(6-1)}{2}(100) + \cdots \approx 2500 + 1800 + 1500 = 5800]

Final Answer: Estimated population in 2001 is approximately 5800.

**c) pari na

**d)

2.10-2 ***Complete the following table :***

x	10	15	20	25	30	35
f(x)	43	—	29	32	—	77

Solution : Since we are given four entries, f(x) can be represented by a third degree polynomical.

Consider, $y_0 = 43, y_2 = 29, y_3 = 32, y_5 = 77$

Now we have,

$\Delta^4 y_0 = 0 \Rightarrow (E-1)^4 y_0 = 0$

$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1) y_0 = 0$

$\Rightarrow y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$\Rightarrow y_4 - 4 \times 32 + 6 \times 29 - 4y_1 + 43 = 0$ ....(1)

$\Rightarrow y_4 - 4y_1 = -89$

Also we have

$\Delta^4 y_1 = 0$

$\Rightarrow y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$

$\Rightarrow 77 - 4y_4 + 6 \times 32 - 4 \times 29 + y_1 = 0$ ....(ii)

$\Rightarrow 4y_4 - y_1 = 153$

Solving (i) and (ii) we get,

$y_1 = \frac{509}{15}$ and $y_4 = \frac{701}{150}$

Thus the completed table becomes :

x	10	15	20	25	30	35
f(x)	43	$\frac{509}{15}$	29	32	$\frac{701}{15}$	77

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d) Find the missing term in the table:

| X | 10 | 15 | 20 | 25 | 30 | 35 |

| f(x) | 1 | 8 | -- | 64 | --- | 216 |

Observation: The values are perfect cubes ($f(x) = (X/5)^3$).

- Missing at $X = 20$: $f(20) = (20/5)^3 = 64$.
- Missing at $X = 30$: $f(30) = (30/5)^3 = 216$.

Final Answer: $f(20) = 64$, $f(30) = 216$.

**Question 4

**a)

Finite Differences in Numerical Methods

Finite difference methods are used to **approximate derivatives** of functions. These are important when dealing with data points rather than continuous functions.

1. Forward Difference:

- **Formula:**

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

- **Use:**
  When you want to estimate the derivative at a point using the value at that point and the next point.

- **Example:**
  Given: $f(x) = x^2$, find $f'(2)$ with $h = 1$.

$$f'(2) \approx \frac{f(3) - f(2)}{1} = \frac{9 - 4}{1} = 5$$

2. Backward Difference:

- **Formula:**

$$f'(x) \approx \frac{f(x) - f(x - h)}{h}$$

- **Use:**
  When you want to estimate the derivative using the point and its previous point.

- **Example:**
  Using the same function $f(x) = x^2$:

$$f'(2) \approx \frac{f(2) - f(1)}{1} = \frac{4 - 1}{1} = 3$$

3. Central Difference:

- **Formula:**

$$f'(x) \approx \frac{f(x + h) - f(x - h)}{2h}$$

- **Use:**
  More accurate than forward or backward difference because it uses points on both sides.

$$f'(2) \approx \frac{f(3) - f(1)}{2(1)} = \frac{9 - 1}{2} = 4$$

**b) Pari na

**c)

3.3-4 ***Find the number of men getting wages per hour between Tk. 10 and Tk. 15 from the following table.*** *নিচের তালিকা থেকে ঘন্টায় 10 টাকা থেকে 15 টাকা মজুরি প্রাপ্ত লোকের সংখ্যা নির্ণয় কর।*

Wages in Tk.	0–10	10–20	20–30	30–40
Number of men	9	30	35	42

Solution : Try yourself same as above. **Ans :** 15

3.3-5 ***The following table gives the population of a town during the last six censuses. Estimate using any suitable interpolation formula, the increase in the population during the period from 1986 to 1988 :*** *কোন শহরের শেষ ছয় শুমারির জনসংখ্যা নিচের তালিকায় দেওয়া হল। যে কোন সুবিধাজনক অন্তঃপাতন সূত্র ব্যবহার করে 1986 হতে 1988 সালের বর্ধিত জনসংখ্যা নির্ণয় কর।*

Year	1951	1961	1971	1981	1991	2001
Population (In thousands)	12	15	20	27	39	52

Solution : To find the population of 1986 and 1988 we form a backward difference table :

Year x	Population y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1951	12					
1961	15	3				
1971	20	5	2			
1981	27	7	2	0		
1991	39	12	5	3	3	
2001	52	13	1	−4	−7	−10

By Newton's formula for backward interpolation we get;

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_n + \frac{u(u+1)(u+2)(u+3)(u+4)}{5!}\nabla^5 y_n + \ldots \quad (1)$$

where $u = \frac{x - x_n}{h}$

For $x = 1986$: $u = \frac{1986 - 2001}{10} = -1.5$

$$\therefore y(1986) = 52 + (-1.5)(13) + \frac{(-1.5)(-1.5+1)}{2!}(1) + \frac{(-1.5)(-1.5+1)(-1.5+2)}{3!}(-4) + \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)}{4!}(-7) + \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)}{5!}(-10)$$

$= 52 - 19.5 + 0.375 - 0.25 - 0.1640625 - 0.1171875$

$= 32.34375$

For $x = 1988$; $u = \frac{1988 - 2001}{10} = -1.3$

$$\therefore y(1988) = 52 + (-1.3)(13) + \frac{(-1.3)(-1.3+1)}{2!} + \frac{(-1.3)(-1.3+1)(-1.3+2)}{3!}(-4) + \frac{(-1.3)(-1.3+1)(-1.3+2)(-1.3+3)}{4!}(-7)$$

$$+ \frac{(-1.3)(-1.3+1)(-1.3+2)(-1.3+3)(-1.3+4)}{5!}(-10)$$

$= 52 - 16.9 + 0.195 - 0.182 - 0.1353625 - 0.1044225$

$= 34.873215$

Thus increase in the population during the period from 1986 to 1988 = y (1988) − y(1986)

$= 34.873215 - 32.34375$

$= 2.529465$

$= 2.53$ thousand approx.

**d) man vinno

3.10-2 ***The values of x and f(x) are given below :***

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

With the help of given data construct a divided difference table and hence find the values of f(8) and f(15).

Solution : The divided difference table of the given data is as follows :

i	x_i	$f(x_i)$	$f(x_i, x_{i+1})$	$f(x_i, x_{i+1}, x_{i+2})$	$f(x_i, x_{i+1}, x_{i+2}, x_{i+3})$
0	4	48	$\frac{100-48}{5-4} = 52$	$\frac{97-52}{7-4} = 15$	$\frac{21-15}{10-4} = 1$
1	5	100	$\frac{294-100}{7-5} = 97$	$\frac{202-97}{10-5} = 21$	$\frac{27-21}{11-5} = 1$
2	7	294	$\frac{900-294}{10-7} = 202$	$\frac{310-202}{11-7} = 27$	$\frac{33-27}{13-7} = 1$
3	10	900	$\frac{1210-900}{11-10} = 310$	$\frac{409-310}{13-10} = 33$	
4	11	1210	$\frac{2028-1210}{13-11} = 409$		
5	13	2028			

By Newtons general interpolation formula, we have

$$f(x) = f(x_0) + (x - x_0)\, f(x_0, x_1) + (x - x_0)(x - x_1)\, f(x_0, x_1, x_2)$$
$$+ (x - x_0)(x - x_1)(x - x_2)\, f(x_0, x_1, x_2, x_3) + \ldots\ldots$$

$$\therefore f(8) = 48 + (8 - 4) \times 52 + (8 - 4)(8 - 5) \times 15 + (8 - 4)(8 - 5)(8 - 7) \times 1$$
$$= 448$$

and
$$f(15) = 48 + (15 - 4) \times 52 + (15 - 4)(15 - 5) \times 15$$
$$+ (15 - 4)(15 - 5)(15 - 7) \times 1$$
$$= 3150.$$

... the following

c) Find the cubic polynomial which takes the following values $y(0) = 1,\ y(1) = 3,\ y(3) = 31,\ y(6) = 223,\ and\ y(10) = 1011$ then find y(2.5) using Newton's interpolation formula. [3]

d) With the help of given data construct a divided difference table and hence find the values of f(8) and f(15). [2]

X	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

6. a) Explain the concept of numerical integration. Compare and contrast the trapezoidal rule and Simpson's rule. Provide an example scenario where numerical integration is necessary in computer science or engineering. [5]

b) Find the value of $I = \int_{0.2}^{1.4} (\sin x - x^{-1} + e^{x})dx$ by using trapezoidal rule along with error in trapezoidal rule. [5]

c) Write down the error of Simpson's 3/8 rule. [2]

a) Find the first and second derivatives for Newton's forward and backward interpolation formula. [4]

b) Solve the system of linear equations by cremer's method [4]

$$X - Y + Z = 2, \qquad X + Y + Z = 6, \qquad 2X - Y + 3Z = 9$$

c) Solve the following system of linear equations by Gauss-Jordan method [4]

$$3X + 2Y + Z = 6, \qquad 4X + 3Y + 2Z = 9, \qquad 7X + 5Y - Z = 11$$

a) Using Euler method, find an approximate value of y corresponding to $x = 2$, given that $\frac{dy}{dx} = x + 2y$ and $y(1) = 1$. [4]

b) Use Taylor's series method to solve the equation [4

$$\frac{dy}{dx} = -xy, \qquad y(0) = 1$$

c) Use picard's method to find a solution of $\frac{dy}{dx} = 1 + xy$ up to third approximation, when $y(0) = 1$. [4

*** Good Luck ***

--

####**Question 5

**a) pari na

**b)

1. For Equal Intervals:

Newton's Forward Interpolation Formula:

Used when data points are equally spaced.

$$f(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \cdots$$

Where:

- $u = \frac{x - x_0}{h}$
- $h = x_1 - x_0$ (interval size)
- Δ is the forward difference operator.

Newton's Backward Interpolation Formula:

Used when interpolating near the **end of the table**.

$$f(x) = f(x_n) + u\nabla f(x_n) + \frac{u(u+1)}{2!}\nabla^2 f(x_n) + \cdots$$

Where:

- $u = \frac{x - x_n}{h}$
- ∇ is the backward difference operator.

---

2. For Unequal Intervals:

Lagrange's Interpolation Formula:

Suitable for **unequally spaced points**.

$$f(x) = \sum_{i=0}^{n} \left[f(x_i) \prod_{j=0, j \neq i}^{n} \frac{x - x_j}{x_i - x_j} \right]$$

- No requirement for equal spacing.
- Each term adjusts for the position of known points.

---

3. For Both Equal and Unequal Intervals:

Lagrange's Interpolation Formula works for **both equal and unequal intervals**, but it is generally preferred for unequal spacing.

**c)

= 3150.

3.10-3 ***Find the cubic polynomial which takes the following values $f(0) = 1, f(1) = 3, f(3) = 31, f(6) = 223$ and $f(10) = 1011$. Hence or otherwise obtain $f(2.5)$.***

[একটি ত্রিঘাত বহুপদী নির্ণয় কর যাহা $f(0) = 1, f(1) = 3, f(3) = 31, f(6) = 223$ এবং $f(10) = 1011$ মান গ্রহণ করে। অতঃপর অথবা অন্যভাবে $f(2.5)$ নির্ণয় কর।]

Solution: The divided difference table is as given below :

i	x_i	$f(x_i)$	$f(x_i, x_{i+1})$	$f(x_i, x_{i+1}, x_{i+2})$	$f(x_i, x_{i+1}, x_{i+2}, x_{i+3})$
0	0	1	$\frac{3-1}{1-0}=2$	$\frac{14-2}{3-0}=4$	$\frac{10-4}{6-0}=1$
1	1	3	$\frac{31-3}{3-1}=14$	$\frac{64-14}{6-1}=10$	$\frac{19-10}{10-1}=1$
2	3	31	$\frac{223-31}{6-3}=64$	$\frac{197-64}{10-3}=19$	
3	6	223	$\frac{1011-223}{10-6}=197$		
4	10	1011			

Now from Newton's interpolation formula for unequal intervals, we get

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2)$$
$$+ (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + \dots\dots$$
$$= 1 + 2x + x(x-1) \times 4 + x(x-1)(x-3) \times 1$$
$$= 1 + 2x + 4x^2 - 4x + x^3 - 4x^2 + 3x$$
$$= x^3 + x + 1$$

Again $f(2.5) = 1 + (2.5) \times 2 + (2.5)(2.5-1) \times 4 + (2.5)(2.5-1)(2.5-3) \times 1$
$= 19.125$

Putting the value $x = 2.5$ in the required cubic polynomial we get the same result.

**d)

3.10-2 ***The values of x and f(x) are given below :***

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

With the help of given data construct a divided difference table and hence find the values of f(8) and f(15).

Solution : The divided difference table of the given data is as follows :

i	x_i	$f(x_i)$	$f(x_i, x_{i+1})$	$f(x_i, x_{i+1}, x_{i+2})$	$f(x_i, x_{i+1}, x_{i+2}, x_{i+3})$
0	4	48	$\frac{100-48}{5-4} = 52$	$\frac{97-52}{7-4} = 15$	$\frac{21-15}{10-4} = 1$
1	5	100	$\frac{294-100}{7-5} = 97$	$\frac{202-97}{10-5} = 21$	$\frac{27-21}{11-5} = 1$
2	7	294	$\frac{900-294}{10-7} = 202$	$\frac{310-202}{11-7} = 27$	$\frac{33-27}{13-7} = 1$
3	10	900	$\frac{1210-900}{11-10} = 310$	$\frac{409-310}{13-10} = 33$	
4	11	1210	$\frac{2028-1210}{13-11} = 409$		
5	13	2028			

By Newtons general interpolation formula, we have

$$f(x) = f(x_0) + (x - x_0)\, f(x_0, x_1) + (x - x_0)(x - x_1)\, f(x_0, x_1, x_2)$$
$$+ (x - x_0)(x - x_1)(x - x_2)\, f(x_0, x_1, x_2, x_3) + \ldots\ldots$$

$$\therefore f(8) = 48 + (8 - 4) \times 52 + (8 - 4)(8 - 5) \times 15 + (8 - 4)(8 - 5)(8 - 7) \times 1$$
$$= 448$$

and
$$f(15) = 48 + (15 - 4) \times 52 + (15 - 4)(15 - 5) \times 15$$
$$+ (15 - 4)(15 - 5)(15 - 7) \times 1$$
$$= 3150.$$

... the following

Question 6: PARI NA

**Question 7: PARI NA **

**Question 8: PARI NA **

6th batch

**Question 1: **

**a)

🔷 **Importance of Numerical Methods in Computer Science:**

1. Solving Complex Mathematical Problems:

Many real-world problems (like weather prediction, fluid flow, machine learning, simulations) involve **equations that can't be solved exactly**. Numerical methods help approximate these solutions.

2. Handling Functions Without Analytical Solutions:

Some mathematical functions or equations **do not have simple formulas** (like solving $\int e^{(-x^2)}dx$). In such cases, numerical methods (like numerical integration) are essential.

3. Useful in Engineering & Science:

Used in **engineering simulations, physics, biology, finance**, etc., where practical problems are solved using computers.

4. Efficient and Practical:

Even when exact solutions exist, they may be **too complex or time-consuming** to compute manually. Numerical methods give quick, usable approximations.

5. Foundation for Algorithms:

They are the **core of algorithms** used in:

- Machine learning
- Data analysis
- Computer graphics

- Cryptography
- Scientific computing

🔷 Floating Point Form of Numbers:

In computer systems, **real numbers** are represented in the following form:

Number=±(m×be)\text{Number} = \pm (m \times b^e)

Where:

- **m = Mantissa (or Significand)**
- **b = Base** (usually 2 for binary)
- **e = Exponent**

Example:

325.6=0.3256×103325.6 = 0.3256 \times 10^3

Here:

- Mantissa = 0.3256
- Base = 10
- Exponent = 3

In computers, numbers are usually stored in **IEEE Floating Point format** (like 32-bit or 64-bit).

🔷 Significant Digits (or Figures) of a Number:

Significant digits are the **meaningful digits in a number** that represent its precision.

Example:

- 0.00456 has **3 significant digits**: 4, 5, 6.
- 123.450 has **6 significant digits**.

✅ **Zeros before the first non-zero digit are NOT significant**.
✅ **Zeros after a decimal and non-zero digits ARE significant**.

🔷 **Round-off Error:**

Definition:

Round-off error happens when a number is **rounded to fit the computer's limited precision**.

Why does this happen?

Computers cannot store an infinite number of digits. So numbers are stored with limited digits, and the remaining part is discarded or adjusted, causing a **small error**.

Example:

π=3.1415926535...\pi = 3.1415926535...

But in a computer, it may be stored as **3.14159** only.

So the **difference between the true value and stored value** is the **round-off error**.

**b)

**c)

Question 2: 7th batch ar 2 no set same to same

**Question 3: 7th batch ar 3 no set ar b,c same to same **

**a)

🔹 **Interpolation**

Meaning:
Interpolation means **estimating a value** that lies **within** two known values in a data set.

In Simple Words:
Suppose you know the temperature at 10 AM and 12 PM. If you want to find the temperature at 11 AM, you use **interpolation**.

Formula Used:
If the data points are evenly spaced, you can use linear or polynomial interpolation formulas (like Newton's or Lagrange's).

📍 **Uses of Interpolation:**

- Filling in missing data in a chart or graph.

- Estimating values between known measurements in science and engineering.
- Creating smooth curves in computer graphics.
- Weather forecasting for short time gaps.

🔹 **Extrapolation**

Meaning:
Extrapolation means **predicting a value** that lies **outside** the range of known values.

In Simple Words:
Suppose you know the population in 2010 and 2020, and you want to predict the population in 2030. That's **extrapolation**.

Formula Used:
Same methods like linear or polynomial functions can be extended beyond known data points.

📍 **Uses of Extrapolation:**

- Forecasting future sales, population, or trends.
- Predicting stock market movements.
- Estimating future temperatures or climate changes.
- Extending scientific experiment results to future conditions.

**d) 7th batch ar 3 no set d ar same niom

**Question 4: 7th batch ar 4 no set same to same **

**Question 5: 7th batch ar 5 no set same to same **

Question 6: PARI NA

Question 7: PARI NA

Question 8: PARI NA