# Physics Improve

## Chapter 1: Heat and Thermodynamics

### 1. Write the difference between temperature and heat.

**Answer:** The temperature and heat have distinct meanings:

| Characteristic | Temperature | Heat |
|---|---|---|
| **Definition** | Measure of average kinetic energy of particles. | Transfer of thermal energy between systems |
| **Units** | Celsius (°C), Fahrenheit (°F), Kelvin (K) | Joules (J), calories |
| **Role** | Reflects how hot or cold an object is | Represents the energy transfer from hot to cold regions. |
| **Measurement** | Represents intensity of thermal energy | Represents the quantity of energy transferred |

### 2. Write short notes on i) Seebeck effect, ii) Peltier effect and iii) Pyrometer.

**Seebeck Effect:** The Seebeck effect is a phenomenon where a temperature difference between two different metals or semiconductors creates an electric voltage. This voltage is proportional to the temperature difference, allowing for the generation of electric power or the measurement of temperature. It forms the basis of thermo-electric devices like thermocouples and explains the principles behind thermoelectric thermometers.

**Peltier Effect:** The Peltier effect is the reverse of the Seebeck effect. When an electric current passes through the junction of two different metals or semiconductors, it either absorbs or releases heat, depending on the direction of the current. This effect is used in Peltier coolers, where the absorption of heat is exploited for cooling purposes.

**Pyrometer:** A pyrometer is a tool that measures really hot temperatures without touching the object by detecting the heat radiation it gives off, commonly used in industries. A pyrometer is a device used to measure high temperatures without direct contact. It works by detecting the infrared radiation emitted by a hot object, converting it into an electrical signal, and providing a non-contact temperature reading. Pyrometers are commonly used in industries like metal processing, glass manufacturing, and aerospace for accurate temperature monitoring in extreme conditions.

### Define entropy // What do you mean by entropy?

**Answer:**

Entropy is a measure of how things can get mixed up or disorganized in a system. It tells us how spread out or random the particles are within a substance. The mathematical expression for entropy change $\Delta S$ is given by the equation:

$$\Delta S = \int \frac{dQ}{T}$$

Where, $\Delta S$ is the change in entropy, $dQ$ is the heat added to or removed from the system, and $T$ is the temperature in Kelvin.

Write down the postulates of kinetic theory of gases/ State the fundamental postulates of the kinetic theory of gases/ State the fundamental assumptions of the kinetic theory of gases.

Answer:

The postulates of the kinetic theory of gases are as follows:

**1. Particles in Constant Motion:** Gas is composed of small particles (atoms or molecules) that are in constant, random motion.

**2. Negligible Volume:** The volume occupied by the gas particles themselves is negligible compared to the overall volume of the gas.

**3. Negligible Forces Between Particles:** The particles exert no forces on each other except during collisions. These collisions are assumed to be perfectly elastic.

**4. Constant Random Collisions:** Collisions between gas particles and with the walls of the container are perfectly elastic, meaning there is no loss of kinetic energy. The collisions happen randomly and result in changes in speed and direction.

**5. Average Kinetic Energy Proportional to Temperature:** The temperature of a gas is proportional to the average kinetic energy of its particles. The higher the temperature, the greater the average kinetic energy.

## Show from the kinetic theory that the mean kinetic energy of translation of one molecule of perfect gas is 3/2 $kT$. Where k is Boltzmann's constant and T is the absolute temperature of the gas?

**Answer:**

According to the kinetic theory, the kinetic energy (KE) of a molecule is given by:

$$KE = \frac{1}{2}mv^2$$

Where:

- $m$ is the mass of the gas molecule,
- $v$ is the velocity of the gas molecule.

Now, the average kinetic energy per molecule ($KE$) can be found by taking the average over all possible directions of motion. Since the molecule can move in three perpendicular directions in three-dimensional space, we have three components of velocity: $v_x, v_y, v_z$.

The mean kinetic energy per molecule is the average of the kinetic energy in each direction:

$$KE = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$$

Now, by the equipartition theorem, each degree of freedom contributes $1/2\ kT$ to the average energy, where $k$ is the Boltzmann constant and $T$ is the temperature.

In three dimensions, a molecule has three translational degrees of freedom. Therefore, we can write:

$KE = 1/2\ kT + 1/2\ kT + 1/2\ kT = 3/2\ kT$

So, the mean kinetic energy of translation of one molecule in a perfect gas is $3/2\ kT$.

## Define mean free path?

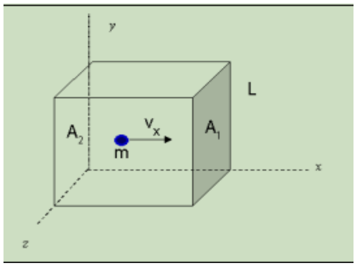
**Answer:**

The mean free path describes the average distance traveled by a gas molecule between successive collisions. It provides an indication of the distance over which a particle moves freely before interacting with another particle.

## Derive an expression for the pressure of a gas. / Prove that $P = \frac{1}{3}\rho c^2$

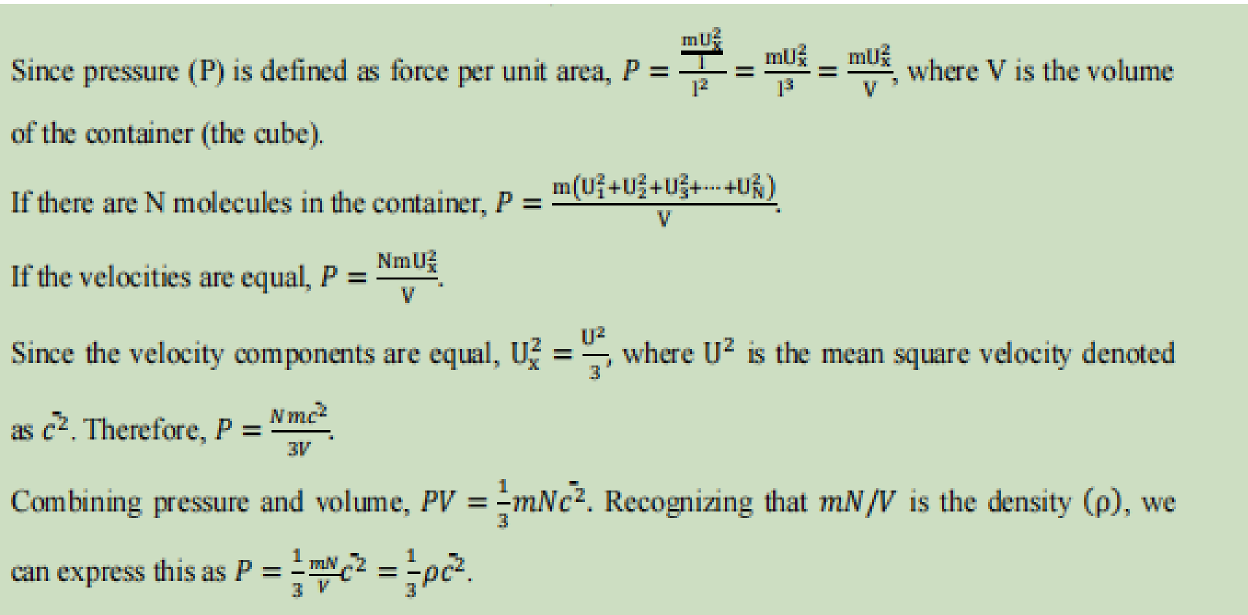
## Answer:

When an individual molecule collides with a wall, as shown in Fig, its momentum doubles. Considering the motion in the x-direction with a velocity $U_x$, where the length of the cube, mass of the molecule, and velocity are denoted by $l$, $m$, and $v$ respectively, the analysis proceeds as follows:



- Momentum in the x-direction: $mU_x$
- Momentum in the -x-direction: $-mU_x$
- Change in momentum: $2mU_x$
- Total time taken (from one end to the other and back): $\frac{2l}{U_x}$
- Rate of change in momentum: $\frac{2mUx}{\frac{2l}{Ux}} = 2mUx \text{ X } \frac{Ux}{2l} = \frac{mU_x^2}{l}$

According to Newton's Second Law, the rate of change of momentum is the force exerted by the molecule on the wall. Therefore, Force $F = mUx/l$.



Since pressure (P) is defined as force per unit area, $P = \frac{\frac{mU_x^2}{l}}{l^2} = \frac{mU_x^2}{l^3} = \frac{mU_x^2}{V}$, where V is the volume of the container (the cube).

If there are N molecules in the container, $P = \frac{m(U_1^2+U_2^2+U_3^2+\cdots+U_N^2)}{V}$.

If the velocities are equal, $P = \frac{NmU_x^2}{V}$.

Since the velocity components are equal, $U_x^2 = \frac{U^2}{3}$, where $U^2$ is the mean square velocity denoted as $\bar{c}^2$. Therefore, $P = \frac{Nm\bar{c}^2}{3V}$.

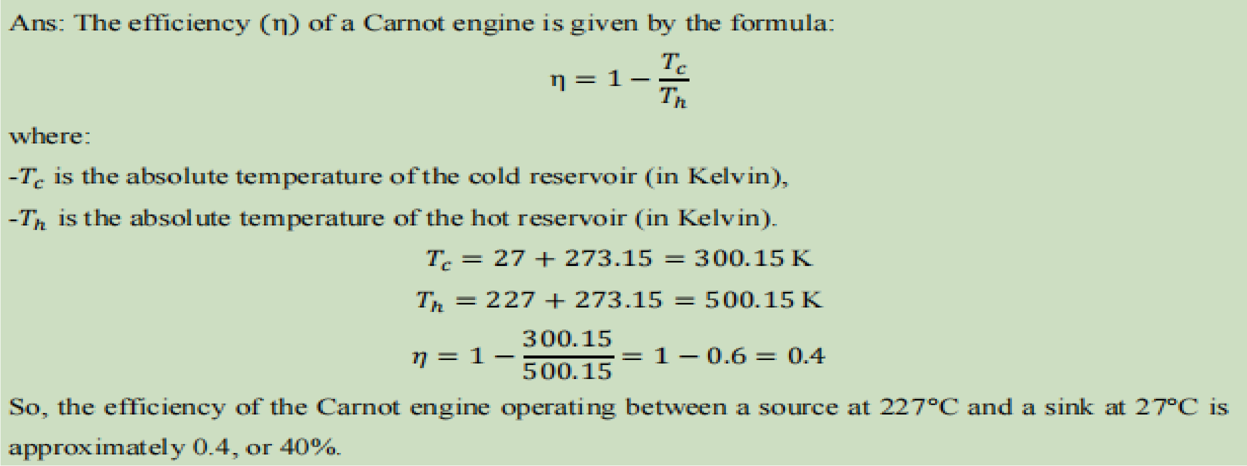
Combining pressure and volume, $PV = \frac{1}{3}mN\bar{c}^2$. Recognizing that $mN/V$ is the density (ρ), we can express this as $P = \frac{1}{3}\frac{mN}{V}\bar{c}^2 = \frac{1}{3}\rho\bar{c}^2$.

## Brownian Motion:

Brownian motion is the random and jumpy movement of tiny particles, like dust or pollen, when they are suspended in a fluid (like water or air). This erratic motion happens because these particles are constantly bumped and nudged by the surrounding molecules in the fluid. Brownian motion helped scientists confirm the existence of atoms and molecules and the dynamic behavior of small particles in various fields of science.

## Find the efficiency of a Carnot's engine working between 227 °C and 27°C. // If the temperature of a sink is 27 °C and that if the source 227°C, calculate the efficiency of the Carnot engine?

**Answer:**



Ans: The efficiency (η) of a Carnot engine is given by the formula:

$$\eta = 1 - \frac{T_c}{T_h}$$

where:

-$T_c$ is the absolute temperature of the cold reservoir (in Kelvin),

-$T_h$ is the absolute temperature of the hot reservoir (in Kelvin).

$$T_c = 27 + 273.15 = 300.15\,\mathrm{K}$$

$$T_h = 227 + 273.15 = 500.15\,\mathrm{K}$$

$$\eta = 1 - \frac{300.15}{500.15} = 1 - 0.6 = 0.4$$

So, the efficiency of the Carnot engine operating between a source at 227°C and a sink at 27°C is approximately 0.4, or 40%.

## Show that entropy remains constant in reversible process but increases in irreversible process?

**Answer:**

Reversible Process:

For a reversible process, the change in entropy ( $\Delta S$ ) is given by:

$$\Delta S = \int \frac{dQ}{T}$$

In a reversible process, we assume that the heat ($dQ$) is transferred at each step in such a way that the system remains in thermal equilibrium, and the temperature ($T$) is constant. This simplifies the integral to:

$$\Delta S = \frac{1}{T} \int dQ$$

The integral of $dQ$ represents the total heat added or removed during the reversible process. As T is constant, the change in entropy is simply proportional to the total heat transfer.

Irreversible Process:

For an irreversible process, the expression for entropy change is the same:

$$\Delta S = \int \frac{dQ}{T}$$

However, in an irreversible process, the temperature ($T$) may vary, and the heat transfer might not be happening in a way that the system stays in thermal equilibrium at each step. Therefore, the integral becomes more complex, and the change in entropy ($\Delta S$) is positive, indicating an increase in disorder:

$$\Delta S = \int \frac{dQ}{T} > 0$$

The inequality ($> 0$) emphasizes that the entropy change is positive for an irreversible process, reflecting the overall increase in disorder.

## Derive Maxwell's thermodynamic relations / Derive Maxwell's four thermodynamics relations.

**Answer:**

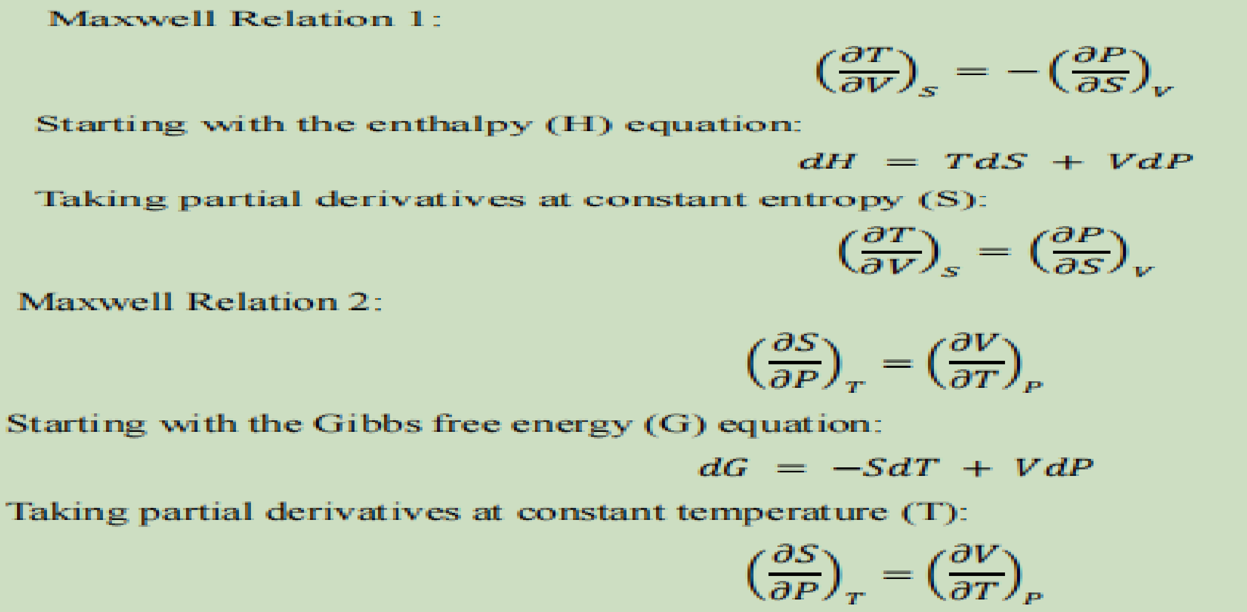
Maxwell's thermodynamic relations are four equations that relate various thermodynamic properties. These relations are derived from the fundamental equations of thermodynamics.

Derivation:

The four thermodynamic potentials:

1. **Internal Energy (U):**
$$dU = TdS - PdV$$
2. **Enthalpy (H):**
$$dH = TdS + VdP$$
3. **Helmholtz Free Energy (A):**
$$dA = -SdT - PdV$$
4. **Gibbs Free Energy (G):**
$$dG = -SdT + VdP$$

Now, let's derive the Maxwell relations.



Maxwell Relation 1:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

Starting with the enthalpy (H) equation:

$$dH = TdS + VdP$$

Taking partial derivatives at constant entropy (S):

$$\left(\frac{\partial T}{\partial V}\right)_S = \left(\frac{\partial P}{\partial S}\right)_V$$

Maxwell Relation 2:

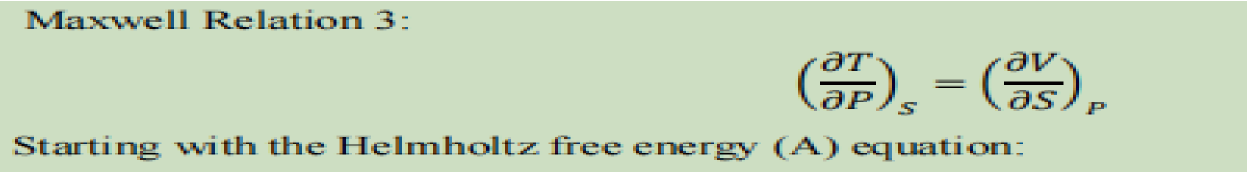
$$\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$$

Starting with the Gibbs free energy (G) equation:

$$dG = -SdT + VdP$$

Taking partial derivatives at constant temperature (T):

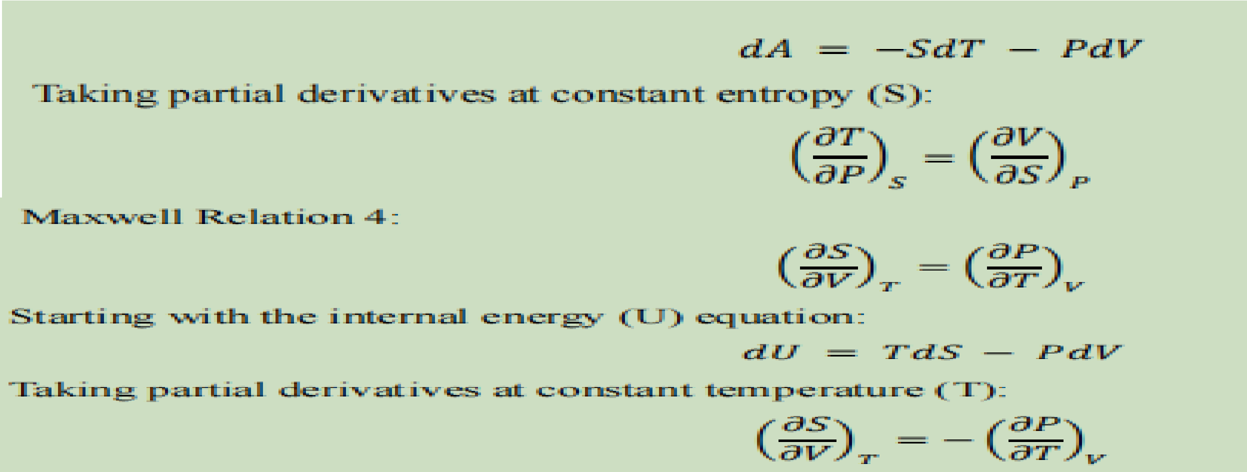
$$\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$$



Maxwell Relation 3:

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

Starting with the Helmholtz free energy (A) equation:



$$dA = -SdT - PdV$$

Taking partial derivatives at constant entropy (S):

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

Maxwell Relation 4:

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Starting with the internal energy (U) equation:

$$dU = TdS - PdV$$

Taking partial derivatives at constant temperature (T):

$$\left(\frac{\partial S}{\partial V}\right)_T = -\left(\frac{\partial P}{\partial T}\right)_V$$

## Use one of the Maxwell's four thermodynamics relations to obtain Clausius Clapeyron's latent heat equation.

**Answer:**

The Clausius-Clapeyron equation relates the latent heat (heat of transformation) of vaporization or condensation to the rate of change of vapor pressure with temperature. Let us imagine a vapor in equilibrium with its liquid held in a cylinder by a piston at the boiling point for that pressure. While a mass dm of the liquid vaporizes, so that the Gibbs free energy for the liquid decreases by $G1dm$ and the Gibbs free energy for the vapor increases by $G2dm$.

Where,

- $G1$ is the specific Gibbs free energy for the liquid and
- $G2$ is the specific Gibbs free energy for the vapor.

But the Gibbs free energy for the system is constant. That is:

$$H_1 - TS_1 = H_2 - TS_2,$$

Or,

$$T(S_2 - S_1) = H_2 - H_1$$

in which the enthalpy and entropy are specific. The left-hand side is the specific latent heat of vaporization The equality of the specific Gibbs free energies of liquid and vapor can also be written

$$U_1 - TS_1 + PV_1 = U_2 - TS_2 + PV_2$$

Or,

$$T(S_2 - S_1) = (U_2 - U_1) + P(V_2 - V_1)$$

This shows that the latent heat of vaporization goes into two things: To increase the internal energy upon vaporization and the $PdV$ work done against the external pressure as the volume increases. Thus, we could divide the latent heat into an internal latent heat and an external latent heat. Thus

$$-S_1dT + V_1dP = -S_2dT + V_2dP$$

Or,

$$\frac{dP}{dT} = \frac{S_2 - S_1}{V_2 - V_1}$$

While $S_2 - S_1 = L/T$, where L is the specific latent heat of vaporization. Thus, we arrive at the Clausius-Clapeyron equation:

$$\frac{dP}{dT} = \frac{L}{T(V_2 - V_1)}$$

# Chapter 2:Structure of Matter

## Define unit cell.

**Answer:**

A unit cell is the smallest repeating structural unit in a crystal lattice that, when stacked together in three-dimensional space and it builds up the entire crystal structure. The unit cell represents the arrangement of atoms, ions, or molecules in a crystal and provides the basis for describing the overall symmetry and properties of the crystal lattice.

## Define single crystal and polycrystalline solids.

**Answer:**

**Single Crystal:**

A single crystal is a solid material in which the arrangement of atoms or molecules is highly ordered and consistent throughout the entire structure. In a single crystal, the lattice structure extends without interruption, and there are no grain boundaries or defects in the atomic arrangement. The atoms or molecules in a single crystal align in a specific, repeating pattern, resulting in a homogeneous and uniform structure. Examples of single crystals include diamonds, quartz, and some metals under certain conditions.

**Polycrystalline Solids:**

Polycrystalline solids consist of multiple crystal domains or grains. Each grain is a small, well ordered crystalline structure, but the orientation of the crystals in adjacent grains may differ. The boundaries between these grains are called grain boundaries, where the atomic arrangement may be less ordered. Polycrystalline materials are more common than single crystals and exhibit a combination of properties associated with their individual grains.

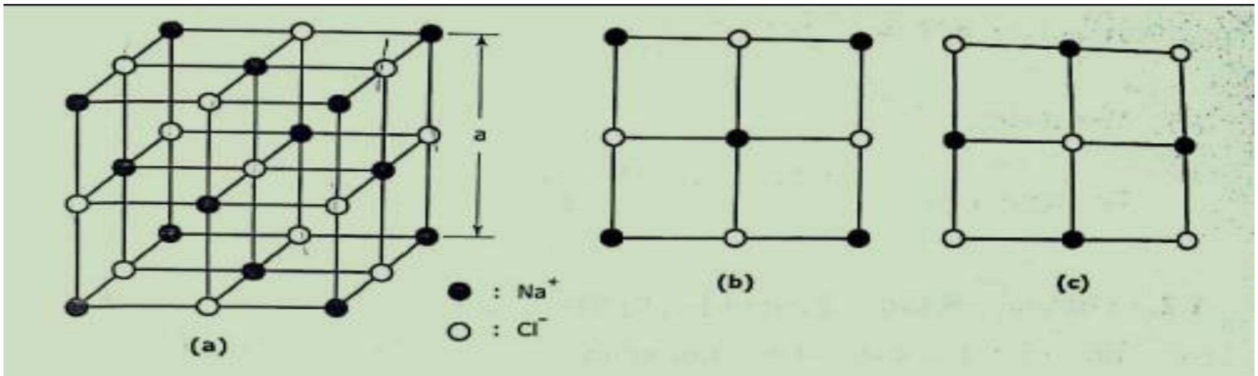
## Draw and explain the NaCl crystal structure and find the total number of Na ions and Cl ions per unit cell.

**Answer:**

The total number of sodium ions ($Na^+$) and chloride ions ($Cl^-$) per unit cell in the NaCl crystal structure:

In a face-centered cubic (FCC) unit cell:

- There is 1/8th of each corner ion within the unit cell.
- There is 1/2 of each face-centered ion within the unit cell.



**Sodium Ions ($Na^+$):**

1. From corners:
   - Each corner contributes 1/8th of an ion to the unit cell.
   - Total contribution from corners = 8 corners × (1/8) $Na^+$ ion/corner = 1 $Na^+$ ion.
2. From face centers:
   - Each face center contributes 1/2 of an ion to the unit cell.
   - Total contribution from face centers = 6 faces × (1/2) $Na^+$ ion/face = 3 $Na^+$ ions.

Total $Na^+$ ions per unit cell = $Na^+$ ions from corners + $Na^+$ ions from face centers = 1 + 3 = 4 $Na^+$ ions.

**Chloride Ions ($Cl^-$):**

1. From corners:
   - Each corner contributes 1/8th of an ion to the unit cell.
   - Total contribution from corners = 8 corners × (1/8) $Cl^-$ ion/corner = 1 $Cl^-$ ion.
2. From face centers:
   - Each face center contributes 1/2 of an ion to the unit cell.
   - Total contribution from face centers = 6 faces × (1/2) $Cl^-$ ion/face = 3 $Cl^-$ ions.

Total $Cl^-$ ions per unit cell = $Cl^-$ ions from corners + $Cl^-$ ions from face centers = 1 + 3 = 4 $Cl^-$ ions.

Therefore, there are 4 $Na^+$ ions and 4 $Cl^-$ ions per unit cell in the NaCl crystal structure.

## Define packing factor.

**Answer:**

Packing factor is the fraction of the volume of a unit cell that is occupied by "hard sphere" atoms or ions. It is the sum of the sphere volumes of all atoms within a unit cell divided by the unit cell volume. It is dimensionless and always less than unity. The packing factor is expressed as a percentage.

The formula for calculating the packing factor is given by:

$$\text{Packing Factor} = \frac{\text{Volume of atoms,ions,or molecules}}{\text{Volume of the unit cell}} \times 100\%$$

## Prove that the packing fraction for body centered cubic (bcc) structure is 0.68 and for face centered cubic (fcc) structure is 0.74.

**Answer:**

The packing fraction is a measure of how efficiently spheres (atoms in a crystal) pack together in a crystal structure. It is defined as the ratio of the total volume occupied by the atoms to the volume of the unit cell.

**1. Body Centered Cubic (bcc) Structure:**

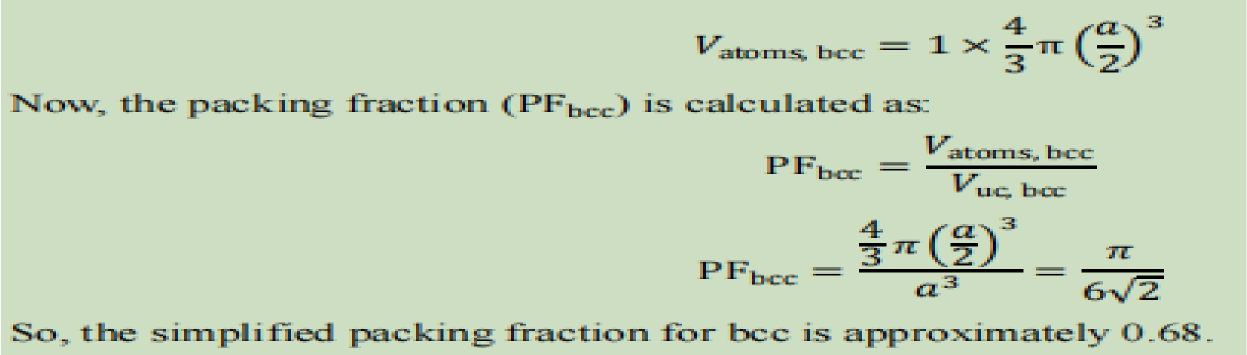
In a bcc structure, each corner atom is shared by eight-unit cells, and there is one additional atom at the center of the unit cell.

The volume of the unit cell ($V_{uc}$) in a bcc structure is given by:

$$V_{\text{uc, bcc}} = a^3$$

Where, a is the length of the edge of the cube.

The volume occupied by the atoms in one unit cell is given by:



$$V_{\text{atoms, bcc}} = 1 \times \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$$

Now, the packing fraction ($PF_{bcc}$) is calculated as:

$$PF_{bcc} = \frac{V_{\text{atoms, bcc}}}{V_{\text{uc, bcc}}}$$

$$PF_{bcc} = \frac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{a^3} = \frac{\pi}{6\sqrt{2}}$$

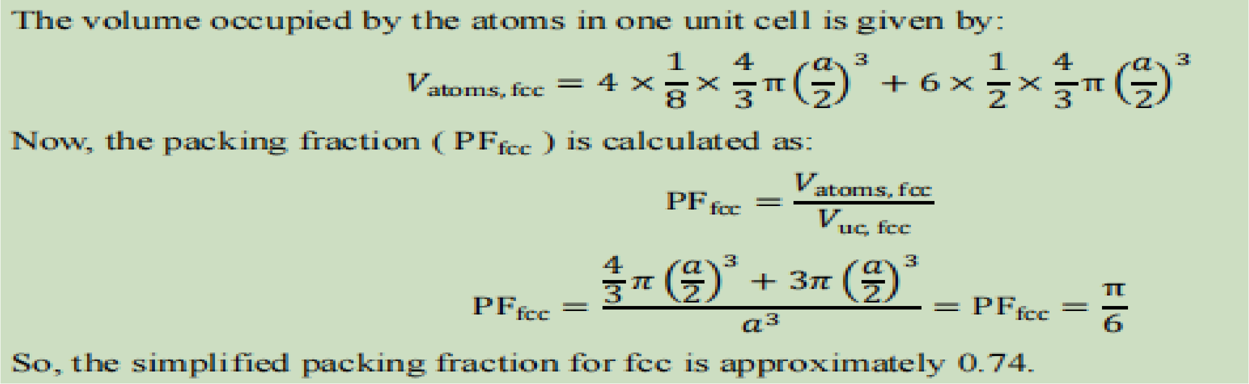
So, the simplified packing fraction for bcc is approximately 0.68.

**2. Face Centered Cubic (fcc) Structure:**

In an fcc structure, each corner atom is shared by eight-unit cells, and there are six additional atoms (at the centers of the faces).

The volume of the unit cell ($V_{uc}$) in an fcc structure is still given by:

$$V_{\text{uc, fcc}} =$$



The volume occupied by the atoms in one unit cell is given by:

$$V_{\text{atoms, fcc}} = 4 \times \frac{1}{8} \times \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 + 6 \times \frac{1}{2} \times \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$$

Now, the packing fraction ( $PF_{fcc}$ ) is calculated as:

$$PF_{fcc} = \frac{V_{\text{atoms, fcc}}}{V_{\text{uc, fcc}}}$$

$$PF_{fcc} = \frac{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3 + 3\pi\left(\frac{a}{2}\right)^3}{a^3} = PF_{fcc} = \frac{\pi}{6}$$

So, the simplified packing fraction for fcc is approximately 0.74.

## What are Miller indices?

**Answer:**

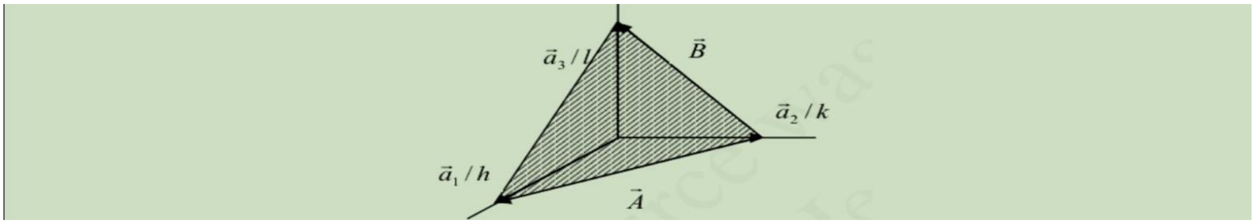
Miller indices are a symbolic notation used in crystallography to represent the orientation of crystal planes and directions within a crystal lattice. Miller indices are used to describe specific crystallographic planes and directions in a crystal structure. They involve three integers (h, k, l) that represent the intercepts of the plane or direction with the three axes of the crystal lattice.

## Show that the spacing d of plane (h, k, l) in a simple cubic lattice of side a is- $d = \frac{a}{\sqrt{h^2+k^2+l^2}}$

In a simple cubic lattice, the lattice vectors can be expressed as:

$a_1$ = ai^
$a_2$ = aj^
$a_3 = ak$^

where a is the lattice constant, and $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ are unit vectors along the x, y, and z directions, respectively.



The position vector of a general point (h, k, l) in the reciprocal lattice is given by:

$G = hb_1 + kb_2 + lb_3$

where $b_1$, $b_2$, and $b_3$ are the reciprocal lattice vectors. For a simple cubic lattice:

$$b_1 = \frac{2\pi}{a}\hat{\imath},$$

$$b_2 = \frac{2\pi}{a}\hat{\jmath},$$

$$b_3 = \frac{2\pi}{a}\hat{k}.$$

The magnitude of the reciprocal lattice vector is given by:

$$|G| = \frac{2\pi}{a}\sqrt{h^2 + k^2 + l^2}$$

Now, the spacing between crystal planes is related to the reciprocal lattice vector magnitude by:

d = 1/ $|G|$

Substitute the expression for $|G|$:

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

Therefore, we've derived the formula for the spacing d of a plane (h, k, l) in a simple cubic lattice of side a.

## What is Bragg’s law? Deduce Bragg’s equation// State and derive Bragg’s law.

### Answer:

**Bragg’s Law:**

When the X-ray is incident onto a crystal surface, its angle of incidence, $\theta$, will reflect with the same angle of scattering, $\theta$. And, when the path difference, d is equal to a whole number, n, of wavelength, $\lambda$, constructive interference will occur.

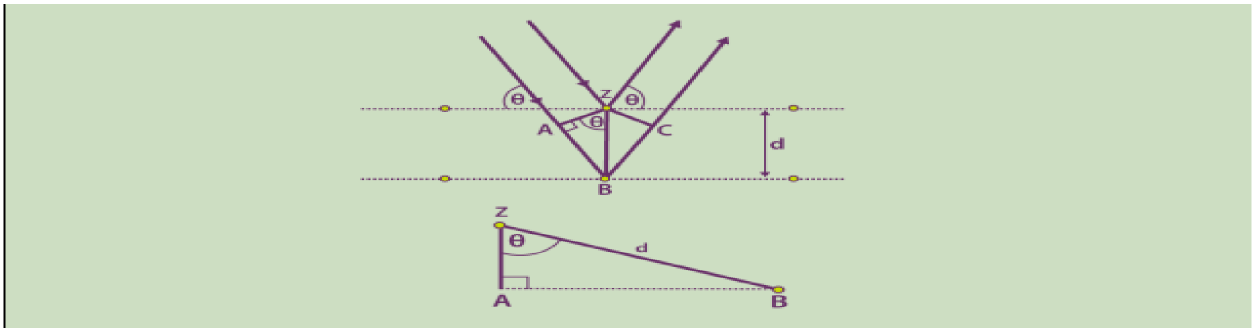
Bragg's Law is used to explain the phenomenon of X-ray diffraction by crystals.

The equation for Bragg's Law is given by:

$$n\lambda = 2d\sin(\theta)$$

Where:

- $n$ is an integer (known as the order of reflection).
- $\lambda$ is the wavelength of the incident X-rays.
- $d$ is the spacing between crystal planes.
- $\theta$ is the angle of incidence



Let’s consider the condition for constructive interference of X-rays scattered by different crystal planes.

1. Consider two parallel crystal planes separated by a distance d.
2. An incident X-ray beam with wavelength $\lambda$ strikes the crystal planes.
3. X-rays are diffracted by the crystal lattice, and if the path difference between the X-rays scattered by adjacent planes is a multiple of the wavelength, constructive interference occurs.

The condition for constructive interference is given by:

$$n\lambda = 2d\sin(\theta)$$

For constructive interference, the angle of incidence is such that the path difference between the rays scattered by adjacent planes is a multiple of the wavelength.

## What are the point defects?

### Answer:

Point defects are imperfections in the regular arrangement of atoms within a crystalline structure, occurring at specific points in the lattice. There are several types of point defects:

**Vacancy Defects**:

- Vacancies occur when an atom is missing from its regular lattice position. This creates an empty space or "hole" in the crystal structure.

**Interstitial Defects:**

- Interstitial defects happen when atoms occupy positions in the crystal structure that are not part of the regular lattice sites.

**Substitutional Defects:**

- Substitutional defects occur when an atom is replaced by a different type of atom in the crystal lattice.

**Interstitialcy:**

- Interstitialcy involves an atom moving from its regular lattice position to an interstitial site and back.

**Frenkel Defects:**

- Frenkel defects consist of a cation vacancy and a displaced cation that occupies an interstitial position.

## What are the line defects?

**Answer:**

Line defects, also known as dislocations, are imperfections that occur in the arrangement of atoms along a line or a plane within a crystalline structure. There are two main types of line defects:

1. Edge dislocations.
2. Screw dislocations.

**1. Edge Dislocations:**

An edge dislocation occurs when an extra plane of atoms, often called a "burger's vector," is introduced into the crystal lattice. This extra plane creates a mismatch between the upper and lower sections of the crystal, leading to a line defect.

**2. Screw Dislocations:**

A screw dislocation results from the shear or rotational movement of one part of a crystal lattice with respect to the other. This movement creates a spiral or helical pattern along the dislocation line.

## Distinguish among an insulator, a semiconductor and a conductor.

**Answer:**

Insulator, semiconductor, and conductor are classifications of materials based on their electrical conductivity. The distinction among these three types is primarily determined by the behavior of electrons in their atomic or molecular structures.

| 1. Conductor | 2. Semiconductor | 3. Insulator |
|---|---|---|
| **Characteristics** | **Characteristics** | **Characteristics** |
| 1. High electrical conductivity. | 1. Moderate electrical conductivity, between conductors and insulators. | 1. Very low electrical conductivity. |
| 2. Electrons in the outermost energy level (valence band) are loosely bound | 2. The energy gap between the valence band and the conduction band is relatively small | 2. Electrons in the valence band are tightly bound to atoms. |
| 3. Allows the easy flow of electric current. | 3. Conductivity can be altered by introducing impurities (doping) or changing temperature. | 3. Insulates against the flow of electric current. |
| 4. Example: Metals like copper, aluminum, silver. | 4. Example: Silicon (Si), germanium (Ge). | 4. Example: Rubber, glass, wood, ceramics. |

A table that distinguishes among an insulator, a semiconductor, and a conductor:

| Property | Insulator | Semiconductor | Conductor |
|---|---|---|---|
| Electrical Conductivity | Very low | Moderate | High |
| Band Gap | Wide (typically > 3 eV) | Moderate (typically 1-3 eV) | Very small or none (0 eV) |
| Temperature Dependence | Generally low | Moderate | High |
| Examples | Rubber, glass, wood | Silicon, germanium | Copper, aluminum, silver |
| Application | Electrical insulation | Electronic devices, diodes | Wires, electrical conductors |
| Mobility of Charge Carriers | Very low | Moderate | High |

# Chapter 3 : Waves and Oscillations

**A simple harmonic motion is represented by the equation y = 10 sin (10t +π/6) in SI units. Calculate the i)frequency and time period, ii) maximum acceleration and velocity, iii) velocity after 2 seconds, and iv) acceleration after 3 seconds.**

**Answer:**

Given that simple harmonic motion equation $y = 10\sin(10t + \frac{\pi}{6})$.

i) Frequency and Time Period:

The equation for simple harmonic motion is given by, $y = A\sin(\omega t + \phi)$, where:

- $A$ is the amplitude,
- ω is the angular frequency,
- ϕ is the phase angle.

In the given equation:

- Amplitude ($A$) = 10,
- Angular frequency (ω) = 10.

The frequency ($f$) is $f = \frac{\omega}{2\pi}$ and the time period ($T$) is ($T = \frac{1}{f}$)

$$\therefore f = 10/2\pi \approx 1.5915 \text{ Hz}$$

$$\therefore T = 1/\text{f} \approx 1/1.5915 \approx 0.6283 \text{ s}$$

ii) Maximum Acceleration and Velocity:

The maximum acceleration ($a_{\text{max}}$) and maximum velocity ($v_{\text{max}}$) are -

$$a_{\text{max}} = A\omega\hat{}2$$
$$v_{\text{max}} = \omega A$$
$$a_{\text{max}} = 10 \times (10)\hat{}2 = 1000 \text{ m/s}\hat{}2$$
$$v_{\text{max}} = 10 \times 10 = 100 \text{ m/s}$$

iii) Velocity after 2 seconds:

The velocity ($v$) at any time t is given by -

$$v(t) = A\omega\cos(\omega t + \phi)$$

Substitute t = 2 seconds into this equation

$$v(2) = 10 \times 10 \times \cos(10 \times 2 + \pi/6)$$
$$v(2) \approx -92.74 \text{ m/s}$$

iv) Acceleration after 3 seconds:

The acceleration (a) at any time t is given by-

$$\text{a}(t) = -A\omega\hat{}2 \sin(\omega t + \phi)$$

Substitute t = 3 seconds into this equation

$$a(3) = -10 \times (10)\hat{}2 \times \sin(10 \times 3 + \pi/6)$$

$$a(3) \approx -500 \text{ m/s}\hat{}2$$

In summary:

i) Frequency = 1.5915 Hz, Time Period = 0.6283 s

ii) Maximum Acceleration = 1000 m/s², Maximum Velocity = 100 m/s

iii) Velocity after 2 seconds ≈ -92.74 m/s

iv) Acceleration after 3 seconds ≈ 500 m/s²

## Calculate the total energy of a body executing simple harmonic motion // Show that the total energy of the particle in SHM is constant and it is independent of the instantaneous displacement?

### Answer:

The total energy (E) in simple harmonic motion (SHM) can be expressed as the sum of kinetic energy (KE) and potential energy (PE). In the case of a mass-spring system undergoing SHM, where a mass (m) is attached to a spring with a force constant (k), the expression for total energy is as follows:

$$E = KE + PE$$

1. Kinetic Energy ($KE$):
    - In SHM, the velocity (v) of the mass is given by $v = \omega\sqrt{A^2 - x^2}$, where ω is the angular frequency and $x$ is the displacement from the equilibrium position.
    - The kinetic energy is expressed as $KE = 1/2\,mv^{\wedge}2$ .
2. Potential Energy ($PE$):
    - In SHM, the potential energy is stored in the spring and is given by $PE = 1/2\,kx^{\wedge}2$ , where $k$ is the spring constant.

Putting it all together:

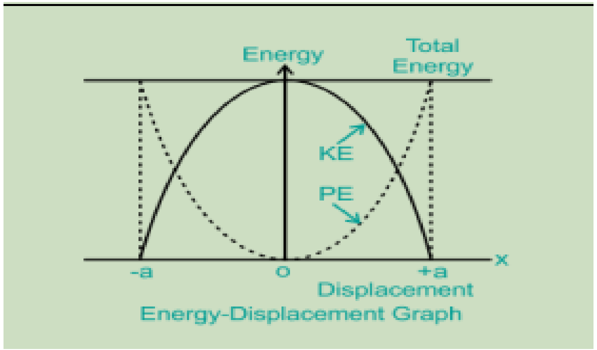
$$E = \frac{1}{2}m\left(\omega\sqrt{A^2 - x^2}\right)^2 + \frac{1}{2}kx^2$$

Simplifying further, you can express the angular frequency ω in terms of the spring constant $k$ and mass m using the relation ω = √ (k/m)

$$E = \frac{1}{2}m\left(\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}\right)^2 + \frac{1}{2}kx^2$$

$$E = \frac{1}{2}kA^2$$

So, in the case of simple harmonic motion, the total energy E is - $E = \frac{1}{2}kA^{\wedge}2$



Since both $A$ and $k$ are constants and do not depend on the instantaneous displacement $x$, the total energy E remains constant throughout the motion. This means that E is independent of the instantaneous displacement of the mass at any given time.

## Explain how Lissajous' figures are useful in the laboratory?

**Answer:**

In the laboratory, Lissajous figures are useful for:

**1. Frequency and Phase Measurement:** Comparing frequencies and phases of oscillating signals.

**2. Instrument Calibration**: Verifying the accuracy of instruments like oscilloscopes.

**3. Resonance and Harmonic Analysis:** Identifying resonance frequencies and harmonic components in systems.

**4. PLL Tuning:** Optimizing and tuning phase-locked loops for synchronization.

**5. FM and AM Analysis:** Analyzing frequency and amplitude modulation.

**6. Education and Demonstration:** Visualizing complex oscillatory concepts for educational purposes.

**7. Nonlinear Dynamics and Chaos**: Revealing intricate patterns in experiments related to chaotic behavior.

## What are the free, damped and forced vibrations?

**Answer:**

**Free Vibrations:**

- Free vibrations occur when a system is allowed to oscillate without any external influence or driving force.
- In the absence of damping and external forces, a vibrating system will continue oscillating indefinitely with constant amplitude
- The natural frequency of the system determines the rate at which these free vibrations occur.

**Damped Vibrations:**

- Damped vibrations occur when an external force opposes the motion and reduces the amplitude of oscillation over time.
- Damping can be caused by various factors, such as friction, air resistance, or other dissipative forces.
- The damping effect is characterized by a damping ratio. Overdamped systems decay more slowly, critically damped systems decay optimally fast, and underdamped systems exhibit oscillatory behavior before decaying.

**Forced Vibrations:**

- Forced vibrations occur when an external force is applied to a system, driving it to oscillate at a frequency different from its natural frequency.
- The external force is typically periodic and has a frequency known as the forcing frequency.
- Resonance can occur when the forcing frequency matches the natural frequency of the system, leading to large amplitude vibrations.

## Explain the principal of superposition of two waves.

**Answer:**

Principle of Superposition:

The principle of superposition says that when two waves meet, their effects add up. If you have two waves traveling through the same medium, at any point, the total displacement is the sum of the displacements caused by each wave individually.

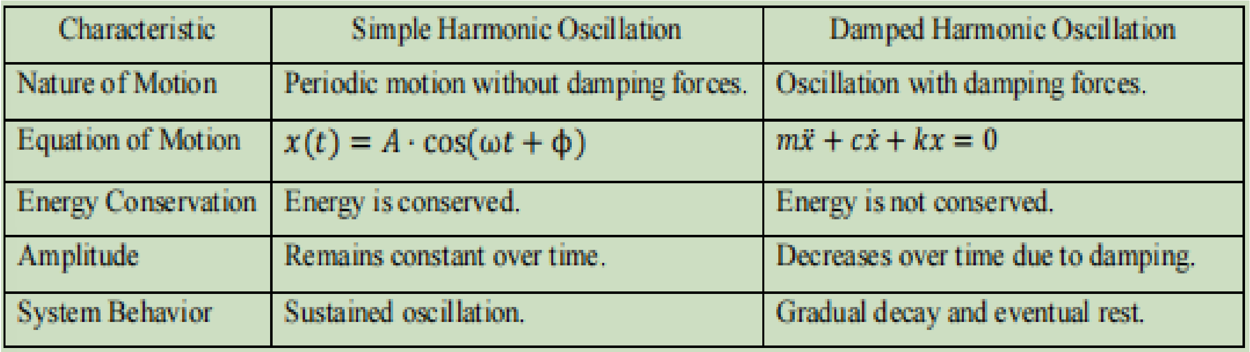
## Distinguish between simple harmonic oscillation and damped harmonic oscillation?

**Answer:**

Simple harmonic oscillation and damped harmonic oscillation are two types of oscillatory motion, and they differ primarily in the presence of damping forces. **Descriptive:**

| Properties | Simple harmonic oscillation | Damped harmonic oscillation |
|---|---|---|
| 1. Nature of Motion | Simple harmonic oscillation is a type of periodic motion where an object oscillates back and forth about a central point (equilibrium position) under the influence of a restoring force that is directly proportional to the displacement from the equilibrium. | Damped harmonic oscillation occurs when there is an additional damping force acting on the oscillating object. This force is proportional to the velocity of the object and acts opposite to the direction of motion. |
| 2. Equation of Motion | The displacement (x) of the object in simple harmonic motion is described by a sinusoidal function, typically expressed as,<br>x(t) = = $A\cos(\omega t + \phi)$, where $A$ is the amplitude, ω is the angular frequency, $t$ is time, and ϕ is the phase angle. | The equation of motion for damped harmonic oscillation is more complex and includes a damping term. A commonly used equation is,<br>$m\ddot{x}+ c\dot{x}+ kx = 0$, where m is the mass, $\dot{x}$ is the velocity, $c$ is the damping coefficient, and k is the spring constant. |
| 3. Energy Conservation | In simple harmonic motion, energy is conserved, and the amplitude remains constant over time. | In damped harmonic oscillation, energy is not conserved due to the presence of damping forces. The amplitude of oscillation decreases over time, and the system eventually comes to rest. |

The differences between simple harmonic oscillation and damped harmonic oscillation:



| Characteristic | Simple Harmonic Oscillation | Damped Harmonic Oscillation |
|---|---|---|
| Nature of Motion | Periodic motion without damping forces. | Oscillation with damping forces. |
| Equation of Motion | $x(t) = A \cdot \cos(\omega t + \phi)$ | $m\ddot{x} + c\dot{x} + kx = 0$ |
| Energy Conservation | Energy is conserved. | Energy is not conserved. |
| Amplitude | Remains constant over time. | Decreases over time due to damping. |
| System Behavior | Sustained oscillation. | Gradual decay and eventual rest. |

## Derive an expression of stationary waves and also discuss the conditions for nodes and Antinodes?

**Answer:**

Consider two waves traveling in opposite directions:

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx - \omega t)$$

Now, the total displacement $y$ is the sum of these two waves:

$$y = y1 + y2$$

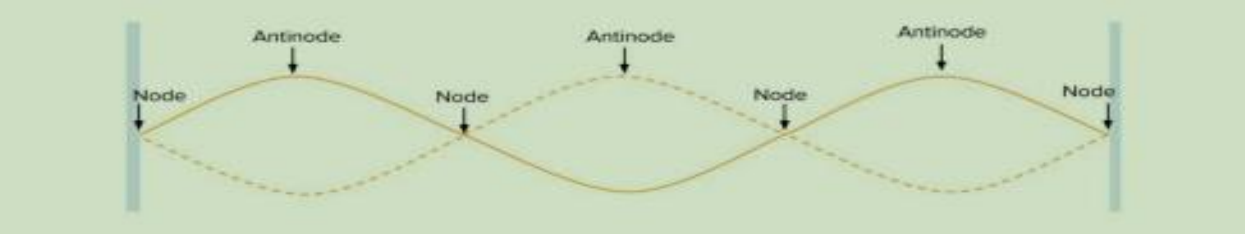
$$y = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$y = 2A\sin\left(\frac{\mathrm{kx} - \omega\mathrm{t} + \mathrm{kx} + \omega\mathrm{t}}{2}\right)\cos\left(\frac{(\mathrm{kx} - \omega\mathrm{t}) - (\mathrm{kx} + \omega\mathrm{t})}{2}\right)$$

$$y = 2A\sin\left(\frac{2\mathrm{kx}}{2}\right)\cos\left(\frac{-2\omega\mathrm{t}}{2}\right)$$

$$y = 2A\sin(kx)\cos(\omega t)$$

So, the expression for a stationary wave is: $y = 2A\sin(kx)\cos(\omega t)$.



Conditions for Nodes and Antinodes:

Nodes:

- For nodes, $\cos(kx) = 0$, Where $k = \frac{2\pi}{\lambda}$
- This happens when $kx = (2n+1)\frac{\pi}{2}$, where n is an integer.
- Solving for x:

$$x = \frac{(2n + 1)\pi}{2\ x\ \frac{2\pi}{\lambda}} = (2n+1)\frac{\lambda}{4}$$

This equation represents the positions along the wave where the displacement is always zero (nodes).

Antinodes:

- For antinodes, $\cos(kx) = \pm 1$.
- This happens when $kx = n\pi$, where n is an integer.
- Solving for x:

$$x = \frac{n\pi}{k} = \frac{n\pi}{\frac{2\pi}{\lambda}} = n\frac{\lambda}{2}$$

This equation represents the positions along the wave where the displacement is maximum (antinodes).

The superposition of two waves traveling in opposite directions creates a stationary wave with nodes and antinodes at specific positions.

# Chapter 4: Physical Optics

## What is coherent source? // When two sources are said to be coherent?

**Answer:**

Coherent sources are two or more wave sources that maintain a constant phase relationship between their waves. This coherence is essential for phenomena like interference, where waves combine either constructively or destructively.

Lasers are an example of coherent light sources.

Two sources are said to be coherent when they meet the following conditions:

1. **Constant Frequency:** The two sources must emit waves of the same frequency.
2. **Constant Phase Difference:** The phase relationship between the waves from the two sources should be constant.
3. **Stability Over Time:** Coherence requires the maintenance of a constant phase difference over time.

## Discuss the conditions for bright and dark bands // For Young's double slit experiment, establish the condition for bright fringes // Briefly describe the condition of bright and dark fringe from Young's double slit experiment?

**Answer:**

The condition for bright (constructive interference) and dark (destructive interference) fringes in Young's double-slit experiment can be expressed mathematically as follows:

1. Bright Fringes (Maxima):

The condition for a bright fringe is that the path difference ($\Delta x$) between the waves from the two slits is an integer multiple ($m$) of the wavelength ($\lambda$) of the light.

Mathematically, this is expressed as:

$$\Delta x = \mathrm{m}\,\lambda$$

2. Dark Fringes (Minima):

The condition for a dark fringe is that the path difference ($\Delta x$) between the waves from the two slits is a half-integer multiple ($m + 1/2$) of the wavelength ($\lambda$) of the light.

Mathematically, this is expressed as:

$$\Delta x = (\mathrm{m} + \tfrac{1}{2})\,\lambda$$

Here, $\Delta x$ is the path difference between the waves arriving at a point on the screen from the two slits, $m$ is an integer, and $\lambda$ is the wavelength of the light. These conditions ensure that the waves either reinforce (bright fringe) or cancel each other out (dark fringe) due to their phase relationship.

## In Young's double slit experiment, the distance between the two slits 0.8mm and the distance of the screen from the slits is 1m. Calculate the width of the bright band when the slits are illuminated by a monochromatic light of wavelength of 5890Å?

**Answer:**

The width of a single bright fringe ($W$):

$$W = D\lambda/2d$$

Given that:

- Wavelength ($\lambda$) $= 5890$ Å $= 5890 \times 10^{-10}\, m$
- Distance from slits to screen ($D$) = $1m$
- Separation between the slits($d$) $= 0.8\, mm = 0.8 \times 10^{-3}\, m$.

$$W = \frac{1 \times 5890 \times 10^{-10}}{2 \times 0.8 \times 10^{-3}} = 3.67625 \times 10^{-4}\, m$$

So, the width of a single bright fringe is approximately $3.68 \times 10^{-4}\, m$ or $0.368\, mm$.

## Why the fringes are circular in Newton’s ring experiment?

**Answer:**

In Newton's ring experiment, circular fringes are observed due to the interference of light waves reflecting between a spherical lens and a flat surface. When light strikes the thin air film formed between the lens and the surface, some waves reflect from both the upper and lower interfaces. Circular symmetry arises because of the spherical lens shape, causing variations in the path length difference and resulting in concentric circles of constructive and destructive interference. The circular fringes are a manifestation of the interference patterns, forming a distinct and recognizable pattern in the experiment.

## What do you mean by diffraction? // What is the diffraction of light? // Define diffraction of light with example.

**Answer:**

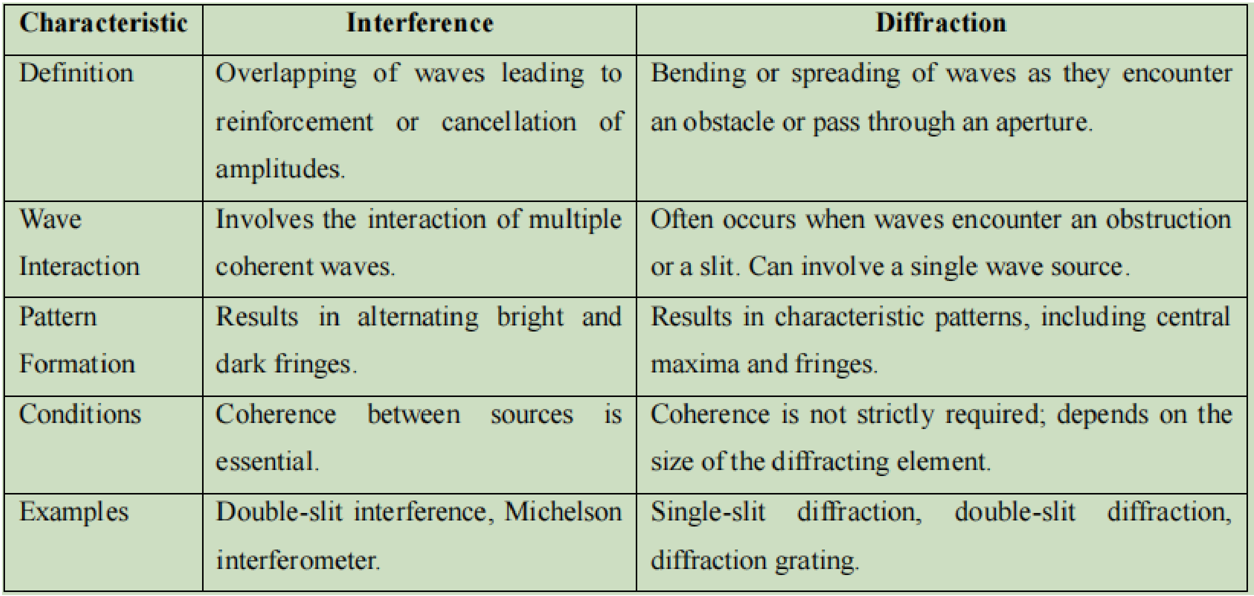
Diffraction of light refers to the bending or spreading of light waves as they encounter an obstacle or pass through an aperture. This phenomenon is a result of the wave nature of light, and it leads to the distribution of light in different directions beyond the geometric shadow of an obstacle or aperture. **Example,**

When sunlight passes through a small gap between two buildings, it diffracts around the edges, creating patterns of light and shadow on the ground. This effect is also observable when laser light passes through a narrow slit, producing a diffraction pattern on a screen placed behind the slit.

## Distinguish between interference and diffraction?

**Answer:**

The differences between interference and diffraction in a table:



| Characteristic | Interference | Diffraction |
|---|---|---|
| Definition | Overlapping of waves leading to reinforcement or cancellation of amplitudes. | Bending or spreading of waves as they encounter an obstacle or pass through an aperture. |
| Wave Interaction | Involves the interaction of multiple coherent waves. | Often occurs when waves encounter an obstruction or a slit. Can involve a single wave source. |
| Pattern Formation | Results in alternating bright and dark fringes. | Results in characteristic patterns, including central maxima and fringes. |
| Conditions | Coherence between sources is essential. | Coherence is not strictly required; depends on the size of the diffracting element. |
| Examples | Double-slit interference, Michelson interferometer. | Single-slit diffraction, double-slit diffraction, diffraction grating. |

## State the followings: Brewster’s law.

**Answer:**

Brewster’s Law: If the tangent of the angle of incidence of incident light is numerically equal to the refractive index of the medium, the light wave experiences maximum plane polarization. In other terms, when the angle between the reflected angle and the refracted angle is 90°, maximum polarization is achieved. The polarization angle is known as Brewster’s angle.

The law is expressed mathematically as: $\tan \theta_B = n2 / n1$

where:

- $\theta B$ is the Brewster angle,
- $n1$ is the refractive index of the first medium (from which light is coming),
- $n2$ is the refractive index of the second medium (through which light is transmitted)

The Brewster angle is the angle of incidence at which the reflected light is completely polarized perpendicular to the plane of incidence.

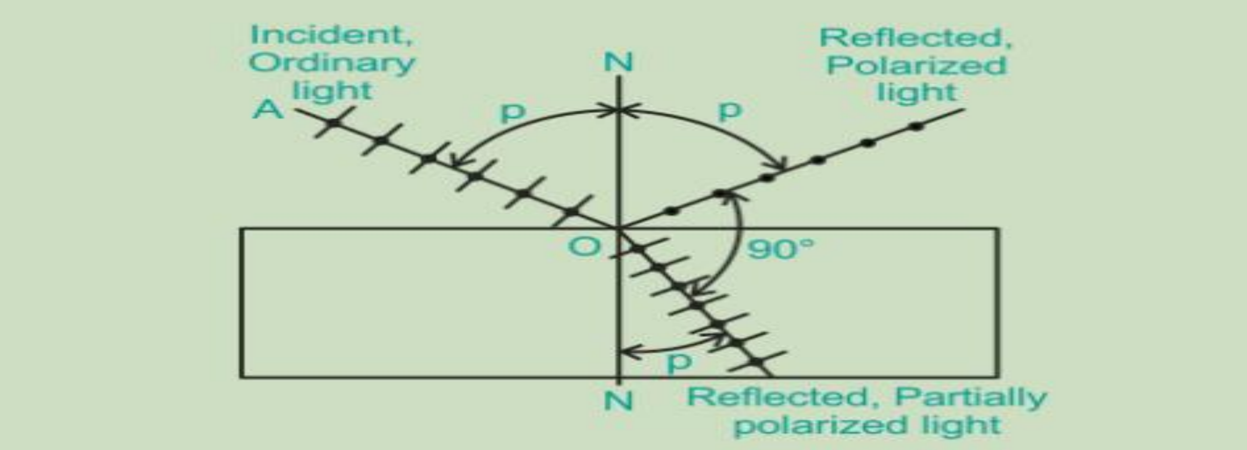
## According to Brewster law, prove that the reflected light and the refracted light are remained at right angle to each other?

**Answer:**

Mathematically, Brewster's law is stated as: $\mu = tan\, ip$

Where

- $\mu$ = Refractive index of the medium
- $i_p$ = Brewster's angle



The Brewster's law can be proved mathematically in the following method:

From Snell's law, $\mu = sini \,/\, sinr$

From Brewster's law, $\mu = tani = sini \,/\, cosi$

Combining both equations we get,

$$\frac{sini}{sinr} = \frac{sini}{cosi}$$

$$cosi = sinr = cos(\frac{\pi}{2} - r)$$

Thus, $i = \frac{\pi}{2} - r$ or $i + r = \frac{\pi}{2}$

Thus, the incident ray is perpendicular to the refracted ray.

## State and explain Malus law of polarization.

**Answer:**

**Malus law:** The intensity of the transmitted light varies as the square of the cosine of the angle between the two planes of transmission.

Malus's Law is expressed mathematically as: $I = I_0 \cdot \cos^{\wedge}2\,(\theta)$

where:

- $I$ is the intensity of the light transmitted through the polarizer,
- $I_0$ is the initial intensity of unpolarized light incident on the polarizer,
- $\theta$ is the angle between the transmission axis of the polarizer and the plane of polarization of the incident light.

## Discuss about the polarization by double refraction method?

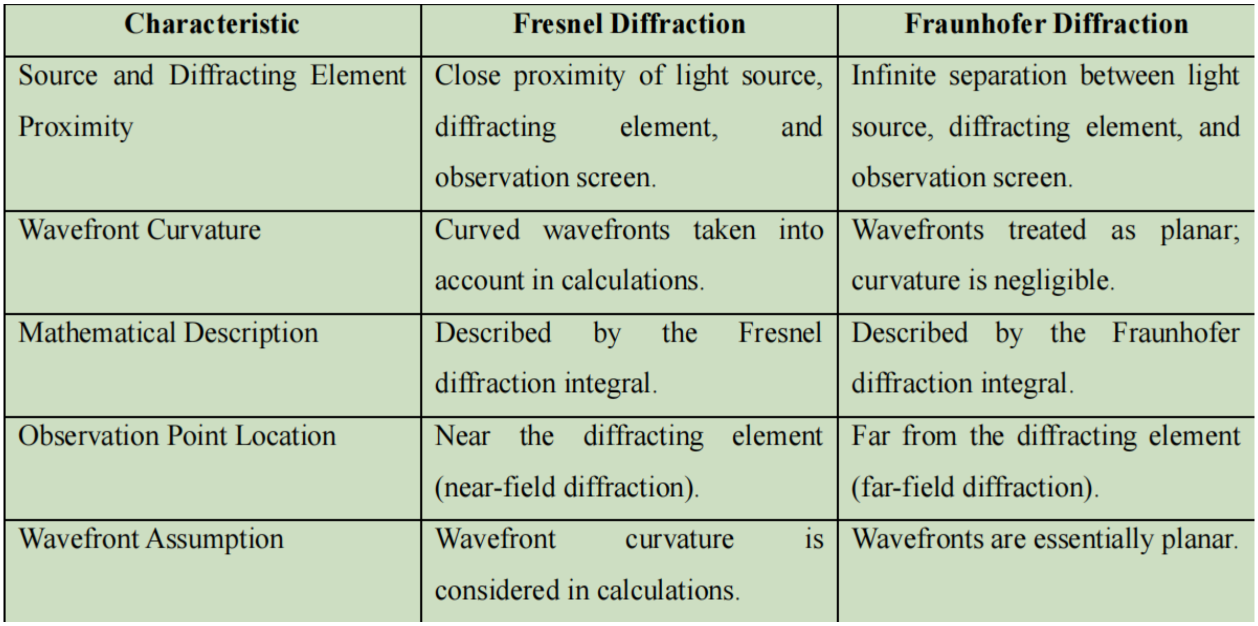
**Answer:**

**Double refraction** is an optical property in which a single ray of unpolarized light entering an anisotropic medium is split into two rays, each traveling in a different direction.

## Distinguish between Fresnel Diffraction and Fraunhofer Diffraction?

**Answer:**

The differences between Fraunhofer Diffraction and Fraunhofer Diffraction in a table:



| Characteristic | Fresnel Diffraction | Fraunhofer Diffraction |
|---|---|---|
| Source and Diffracting Element Proximity | Close proximity of light source, diffracting element, and observation screen. | Infinite separation between light source, diffracting element, and observation screen. |
| Wavefront Curvature | Curved wavefronts taken into account in calculations. | Wavefronts treated as planar; curvature is negligible. |
| Mathematical Description | Described by the Fresnel diffraction integral. | Described by the Fraunhofer diffraction integral. |
| Observation Point Location | Near the diffracting element (near-field diffraction). | Far from the diffracting element (far-field diffraction). |
| Wavefront Assumption | Wavefront curvature is considered in calculations. | Wavefronts are essentially planar. |