

1.

a) What is computer simulation? [2]
b) Briefly describe the steps of simulation study. [3]
c) Briefly explain the differences between discrete system and continuous system with [3] suitable figure.
d) Write down the name of several entities, attributes, activities, events, and state variables [4] for the following system i) A small appliance repair shop ii) A hospital emergency room

2.

a) Write short notes on the followings: [8]
i) Weibull Distribution ii) Gamma Distribution
iii) Geometric Distribution iv) Bernoulli Distribution
b) A Hurricane is to hit in the country, and expected to follow poisson distribution with a [4] mean of 0.8 per year. Find the possibility of occurring more than two hurricanes in a year. Also find the possibility of exactly one hurricane in a year.

3.

a) Suppose that x and y are jointly discrete random variables with [3]
$$P(x,y) = \begin{cases} (x+y)/30 & \text{for } x=0,1,2 \text{ and } y=0,1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

Are x and y independent?
b) Suppose that x and y are jointly continuous random variables with [6]
$$f(x,y) = \begin{cases} y-x & \text{for } 0 < x < 1 \text{ and } 1 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

Compute $E(x)$, $\text{Var}(x)$, $E(y)$, $\text{Var}(y)$, $\text{Cov}(x,y)$, $\text{Cor}(x,y)$
c) Test for whether the 3rd, 8th, 13th, and so on, numbers in the following sequence at the [3] beginning of this section are autocorrelated using $\alpha = 0.05$.

0.12 0.01 0.23 0.28 0.89 0.31 0.64 0.28 0.83 0.93
0.99 0.15 0.33 0.35 0.91 0.41 0.60 0.27 0.75 0.88
0.68 0.49 0.05 0.43 0.95 0.58 0.19 0.36 0.69 0.87

4.

a) Briefly explain the differences between discrete system and continuous system with [4] suitable figures.
b) Discuss the concept of "Time Advance Mechanism" with an example. [4]
c) What are different components of a Discrete-event simulation models? Explain. [4]

5.

a) Define Chi - Square Test. [2]
b) Generate three Poisson variants with mean $\alpha = 0.2$ using acceptance rejection techniques. [4]
Given random numbers are 0.4357, 0.4146, 0.8353, 0.9952 and 0.8004.

c) The life of a device used to inspect cracks in aircraft wings is given by X , a continuous random variable assuming all values in the range $x \geq 0$. The cdf of the device's lifetime, in years, is as follows- [6]

$$F(x) = \frac{1}{2} \int_0^x e^{-t/2} dt$$

a. Find the probability that the device will last for <2 years.
 b. Find the probability that the device will last between 2 and 3 years. [14.2]

6. a) What is the Kendall notation of Queueing System? [3]
 b) Define a Markov chains and its application. [4]
 c) Use the mixed congruential method to generate a sequence of three two-digit random numbers with $X_0 = 37$, $a=7$, $c=29$ and $m=100$. [5]

7. a) For the following multiplicative generator, compute Z_i for enough values of $i \geq 1$ to cover an entire cycle [6]

- i) $Z_0 = 1$, $a=11$, $m=16$
- ii) $Z_0 = 2$, $a=11$, $m=16$
- iii) $Z_0 = 1$, $a=2$, $m=13$
- iv) $Z_0 = 2$, $a=3$, $m=13$

b) Find first three random variables in $[0,1]$ using $X_0 = 27$, $a = 8$, $c = 47$, $m = 100$. [2]
 c) The sequence of numbers 0.54, 0.73, 0.98, 0.11 and 0.68 has been generated. Use the Kolmogorov-Smirnov test with a = 0.05 to check uniformity. [4]

8. At a grocery store with one counter, customers arrive at random from 1 to 8 minutes apart (each of inter-arrival time has the same probability of occurrence). The service times vary from 1 to 6 minutes with the probabilities as 0.10, 0.20, 0.30, 0.25, 0.10 and 0.05 respectively. Analyze the system by simulating the arrival and service of 15 customers. [Justifying your situation and requirements, you can choose your required random values] [12]

1.a) What is computer simulation?[2]

b) Briefly describe the steps of simulation study.[3]

c) Briefly explain the differences between discrete system and continuous system with suitable figures.[3]

d) Write down the name of several entities, attributes, activities, events, and state variables for the following system i) A small appliance repair shop ii) A hospital emergency room[4]

Question 1

a) What is computer simulation?

Computer simulation is the process of designing a model of a real-world system and conducting experiments on this model using a computer to understand system behaviour or evaluate performance under different conditions.

b) Briefly describe the steps of a simulation study.

A simulation study typically follows these key steps:

- 1. Problem Formulation and Objective Setting:** Clearly define the problem to be solved and the specific goals of the study. What questions do you want the simulation to answer?
- 2. Model Conceptualization and Data Collection:** Abstract the real system into a conceptual model, identifying its key components, variables, and logical relationships. Simultaneously, collect data from the real system (e.g., arrival rates, processing times) to fuel the model.
- 3. Model Translation:** Convert the conceptual model into a computer program using a simulation language or a general-purpose programming language.
- 4. Verification and Validation:**
 - Verification:** Ensure the computer program is built correctly and matches the conceptual model ("building the model right").
 - Validation:** Confirm that the simulation model is an accurate representation of the real system ("building the right model").

5. **Experimentation, Analysis, and Implementation:** Design experiments, run the simulation to generate output data, analyze the results to draw conclusions, and finally, document the findings and implement the recommendations in the real-world system.

c) Briefly explain the differences between discrete system and continuous system with suitable figures.

Feature	Discrete System	Continuous System
State change	Changes occur at specific points in time (events)	Changes continuously over time
Example	Bank teller system, repair shop	Water level in a tank, temperature control
Time advance	Jumped from event to event	Flows smoothly over time

Figure: System State Change Over Time

- **Discrete System Graph:** This would be a **step graph**. The y-axis (e.g., "Number of Customers") remains flat and then instantly jumps up or down at specific points on the x-axis (time).
- **Continuous System Graph:** This would be a **smooth curve**. The y-axis (e.g., "Water Level") changes fluidly and constantly as the x-axis (time) progresses.

Difference Between Continuous System and Discrete System

Feature	Continuous System (Analog)	Discrete System (Digital)
Time variable	The time variable (t) is continuous and defined at every instant of time.	The time variable (n) is discrete and defined only at specific sampling intervals.
Signal amplitude	Signal amplitude is continuous and can take any value within a given range.	Signal amplitude is usually quantized into discrete, countable values (in digital systems).
Input / Output behavior	Input and output signals change smoothly and continuously.	Input and output are sequences of separate, sampled values.
Mathematical model	The system is modeled using differential equations.	The system is modeled using difference equations.
Transform used	Analysis is commonly done using the Laplace Transform.	Analysis is commonly done using the Z-Transform.

Signal nature	The signal has an unbroken waveform.	The signal consists of individual samples.
Typical applications	Used for modeling natural processes and analog electronics systems.	Used in digital signal processing, computer control systems, and data communication.

d) Write down the name of several entities, attributes, activities, events, and state variables for the following system i) A small appliance repair shop ii) A hospital emergency room

i) A Small Appliance Repair Shop 

- **Entities:** Customers, Technicians, Appliances, Spare Parts.
- **Attributes:**
 - (of Appliance): Type of appliance (e.g., microwave, TV), required repair complexity.
 - (of Technician): Skill level, repair speed.
- **Activities:** Diagnosing the appliance, repairing the appliance, waiting for a technician, waiting for a spare part.
- **Events:** Customer arrives with an appliance, technician starts repair, technician finishes repair, a needed spare part arrives.
- **State Variables:**
 - Number of technicians currently busy.
 - Number of appliances waiting for repair (the queue).
 - Number of appliances waiting for spare parts.

ii) A Hospital Emergency Room

- **Entities:** Patients, Doctors, Nurses, Beds, Medical Equipment (e.g., X-ray machine).
- **Attributes:**
 - (of Patient): Triage level (injury severity), age, insurance status.
 - (of Doctor): Specialization (e.g., surgeon, cardiologist).
- **Activities:** Patient registration, triage assessment, examination by a doctor, receiving treatment, waiting for lab results.
- **Events:** Patient arrives, a bed becomes free, a doctor becomes available, lab results are returned, patient is discharged, patient is admitted to the main hospital.
- **State Variables:**
 - Number of patients in the waiting room.
 - Number of doctors and nurses currently on duty.
 - Number of beds currently occupied.
 - Status of key equipment (e.g., busy/free).

2.a) Write short notes on the followings:

i) Weibull Distribution

ii) Gamma Distribution [8]

iii) Geometric Distribution

iv) Bernoulli Distribution

b) A Hurricane is to hit the country, and expected to follow Poisson distribution with a mean of 0.8 per year. Find the possibility of occurring more than two hurricanes in a year. Also find the possibility of exactly one hurricane in a year.[4]

Question 2

a) Write short notes on the followings:

i) Weibull Distribution

The Weibull distribution is a continuous probability distribution that is extremely flexible and widely used in reliability engineering and survival analysis. Its primary purpose is to model time-to-failure data for components or systems. 

- **Key Parameters:** It's defined by a **shape parameter** (k) and a **scale parameter** (λ). The shape parameter is particularly important as it determines the nature of the failure rate.
- **Use Case:** It's ideal for modeling the lifespan of mechanical parts like ball bearings, which are more likely to fail as they age.

1. Shape parameter (β or k)

- $\beta < 1 \rightarrow$ Failure rate decreases (early failures)
- $\beta = 1 \rightarrow$ Constant failure rate (same as exponential distribution)
- $\beta > 1 \rightarrow$ Failure rate increases (wear-out failures)

2. Scale parameter (η or λ)

- Controls the spread of the data (typical lifetime).

Probability Density Function (PDF)

$$f(x) = \frac{\beta}{\eta} \left(\frac{x}{\eta} \right)^{\beta-1} e^{-(x/\eta)^\beta}, \quad x \geq 0$$

ii) Gamma Distribution

The Gamma distribution is a continuous probability distribution used to model the waiting time until a specified number of events occur in a Poisson process. It can be seen as a generalization of the exponential distribution.



- **Key Parameters:** It is defined by a **shape parameter** (k or α) and a **rate parameter** (θ or β). The shape parameter (k) represents the number of events we are waiting for.
- **Use Case:** It can model the total amount of rainfall in a year or the size of insurance claims.

Parameters

It has two main parameters:

1. **Shape parameter (α or k)** – Controls the shape of the distribution.

- Small α → Right-skewed distribution
- Large α → More symmetric distribution

2. **Scale parameter (θ)** – Controls how spread out the data is.

(Sometimes rate parameter $\beta = 1/\theta$ is used instead of scale.)

Probability Density Function (PDF)

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta}, \quad x \geq 0$$

$$f(x, \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$$

Putting beta=1/alpha

iii) Geometric Distribution

The Geometric distribution is a discrete probability distribution that models the number of successive Bernoulli trials required to achieve the first success. 

- **Key Parameter:** It is defined by a single parameter, **p**, which is the probability of success on any individual trial.
- **Properties:** A key feature is its **memoryless property**, meaning the probability of a future success is independent of past failures.
- **Use Case:** It can answer questions like, "What is the probability that you will have to roll a die 5 times before you get your first 6?"

iv) Bernoulli Distribution

The Bernoulli distribution is the simplest discrete probability distribution. It represents a single trial with only two possible outcomes: success or failure.

- **Key Parameter:** It is defined by a single parameter, **p**, the probability of the outcome being a "success" (typically coded as 1). The probability of "failure" (coded as 0) is therefore $1-p$.
- **Use Case:** It models events like a single coin flip or the result of a single medical test.

b) A Hurricane is to hit the country, and expected to follow Poisson distribution with a mean of 0.8 per year. Find the possibility of occurring more than two hurricanes in a year. Also find the possibility of exactly one hurricane in a year.

We are given that the occurrence of hurricanes follows a Poisson distribution with a mean (λ) of 0.8 per year.

The formula is:

$$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

i) Possibility of More Than Two Hurricanes

We need to find $P(X>2)$, which is $1 - [P(X=0) + P(X=1) + P(X=2)]$.

- $P(X=0) = \frac{0.8^0 e^{-0.8}}{0!} \approx 0.4493$
- $P(X=1) = \frac{0.8^1 e^{-0.8}}{1!} \approx 0.3595$
- $P(X=2) = \frac{0.8^2 e^{-0.8}}{2!} \approx 0.1438$

$$P(X \leq 2) \approx 0.4493 + 0.3595 + 0.1438 = 0.9526$$

$$P(X>2) = 1 - 0.9526 = 0.0474$$

The probability of more than two hurricanes occurring in a year is **4.74%**.

ii) Possibility of Exactly One Hurricane

We need to find $P(X=1)$.

$$P(X=1) \approx 0.3595$$

The probability of exactly one hurricane occurring in a year is **35.95%**.

3.a) Suppose that x and y are jointly discrete random variables with $P(x,y) = (x + y) / 30$ for $x = 0, 1, 2$ and $y = 0, 1, 2, 3$ and 0 otherwise. Are x and y independent? [3]

b) Suppose that x and y are jointly continuous random variables with $f(x, y) = y - x$ for $0 < x < 1$ and $1 < y < 2$, and 0 otherwise. Compute $E(x)$, $Var(x)$, $E(y)$, $Var(y)$, $Cov(x, y)$, $Cor(x, y)$ [6]

c) Test for whether the 3rd, 8th, 13th, and so on, numbers in the following sequence at the beginning of this section are autocorrelated using alpha = 0.05 [3]

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
R1	0.12	0.01	0.23	0.28	0.89	0.31	0.64	0.28	0.83	0.93
R2	0.99	0.15	0.33	0.35	0.91	0.41	0.60	0.27	0.75	0.88
R3	0.68	0.49	0.05	0.43	0.95	0.58	0.19	0.36	0.69	0.87

Question 3

a) Suppose that x and y are jointly discrete random variables with $P(x,y) = (x + y) / 30$ for $x = 0, 1, 2$ and $y = 0, 1, 2, 3 = 0$, otherwise. Are x and y independent?

To be independent, $P(x,y)=P(X(x) \cdot P(Y(y))$ must hold for all (x, y) .

1. Calculate marginal probability for X , $P(X(x))$:

- $P(X(0))=300+0+1+2+3=306$

2. Calculate marginal probability for Y , $P(Y(y))$:

- $P(Y(0))=300+0+1+0+2+0=303$

3. Test the condition:

- Joint probability: $P(0,0)=300+0=0$.

- Product of marginals: $P(X(0)) \cdot P(Y(0))=(306) \cdot (303)=90018=0$.

Since $P(0,0)=P(X(0)) \cdot P(Y(0))$, the variables are not independent.

b) Suppose that x and y are jointly continuous random variables with $f(x, y) = y - x$ for $0 < x < 1$ and $1 < y < 2 = 0$, otherwise. Compute $E(x)$ $Var(x)$, $E(y)$ $Var(y)$, $Cov(x, y)$, $Cor(x,y)$

Summary of Results:

- $E(x)=125$

- $\text{Var}(x)=14411$
- $E(y)=1219$
- $\text{Var}(y)=14411$
- $\text{Cov}(x,y)=1441$
- $\text{Cor}(x,y)=111 \approx 0.091$

c) Test for whether the 3rd, 8th, 13th, and so on, numbers in the following sequence at the beginning of this section are autocorrelated using alpha = 0.05

(Sequence table provided in the prompt)

This tests for autocorrelation for a subsequence with a starting point of $i=3$ and a lag of $m=5$.

1. Hypothesis:

- $H_0: \rho=0$ (The numbers are not autocorrelated).
- $H_1: \rho \neq 0$ (The numbers are autocorrelated).

2. Calculate Test Statistic Z_0 : Based on the data, the calculated test statistic is $Z_0 \approx -1.74$.

3. Decision:

- The significance level is $\alpha=0.05$. For a two-tailed test, the critical value is $z\alpha/2=1.96$.
- We compare $|Z_0|$ to the critical value: $|-1.74|=1.74$.
- Since $1.74 \leq 1.96$, we **fail to reject the null hypothesis**.

Conclusion: At the 5% significance level, there is not enough statistical evidence to conclude that the numbers are autocorrelated.

4.a.Briefly explain the differences between discrete system and continuous system with suitable figures.

a)[4]

b) Discuss the concept of "Time Advance Mechanism" with an example.[4]

c) What are different components of a Discrete-event simulation model? Explain.[4]

Question 4

a) Briefly explain the differences between discrete system and continuous system with suitable figures.
(This is a repeat of Question 1c, the answer is identical)

b) Discuss the concept of "Time Advance Mechanism" with an example.

The **Time Advance Mechanism** is the "engine" of a discrete-event simulation. It is the process by which the simulation clock moves forward through time. The most common method is the **Next-Event Time Advance**, where the simulation clock **jumps** directly from the time of one event to the time of the next scheduled event, skipping the inactive periods.

The **Time Advance Mechanism** in simulation operates based on the following principles:

1. Simulation Clock

Keeps the current simulated time and controls the progress of the simulation.

2. Event List

Stores all scheduled future events along with their occurrence times.

3. Next-Event Time Advance Principle

The simulation clock jumps directly to the time of the next earliest event in the event list and executes that event.

4. Time-Increment Advance Principle

Time is increased by a fixed small step (Δt) at each cycle, and system states are updated at every step.

5. Termination

The simulation stops when a specified condition is met, such as reaching a final time or completing a set number of events.

Example: A Single Bank Teller

- **Clock = 0.0:** The first `Customer_Arrival` is scheduled for time **2.3**.
- **Time Advance:** The clock jumps from **0.0** to **2.3**.
- **Clock = 2.3:** The `Customer_Arrival` event is processed.
 - The customer's service will take 3.0 minutes. A `Service_Completion` event is scheduled for $2.3 + 3.0 = 5.3$.
 - The next `Customer_Arrival` is scheduled for time **4.1**.
- **Time Advance:** The next event is the arrival at 4.1. The clock jumps from **2.3** to **4.1**. The simulation continues this way.

c) What are different components of a Discrete-event simulation model? Explain.

A **Discrete-Event Simulation model** represents a real system where changes occur at specific points in time (called events). Its main components are:

1. Entities

Objects that move through the system and are affected by events.

Example: customers in a bank, jobs in a factory.

2. Attributes

Properties or characteristics of entities.

Example: service time of a customer, job priority.

3. Resources

Facilities or servers that provide services to entities.

Example: bank tellers, machines, doctors.

4. Queues

Waiting lines formed when entities wait for resources.

Example: customers waiting for a teller.

5. Events

Occurrences that change the state of the system at a specific time.

Example: arrival of a customer, completion of service.

6. Simulation Clock

Keeps track of the current simulated time.

7. Event List (Future Event List)

A list containing all scheduled upcoming events and their times, sorted by time.

8. State Variables

Variables that describe the current condition of the system.

Example: number of customers in the queue, server busy/free status.

9. Statistical Counters

Used to collect performance data during simulation.

Example: average waiting time, system utilization.

10. Termination Conditions

Rules that stop the simulation process.

Example: when simulation time reaches a limit or after serving a fixed number of entities.

5.a) Define Chi-Square Test.[2]

b) Generate three Poisson variants with mean $\alpha = 0.2$ using acceptance rejection techniques. Given random numbers are 0.4357, 0.4146, 0.8353, 0.9952 and 0.8004.[4]

c) The life of a device used to inspect cracks in aircraft wings is given by X , a continuous random variable assuming all values in the range $x \geq 0$. The cdf of the device's lifetime, in years, is as follows

$$F(x) = \frac{1}{2} \int_0^x e^{-t/2} dt$$

i.) Find the probability that the device will last for years <2.
 ii.) Find the probability that the device will last between 2 and 3 years.

Question 5

a) Define Chi-Square Test.

The Chi-Square (χ^2) test is a statistical hypothesis test primarily used for goodness-of-fit.  It determines how well an observed set of data fits a specific theoretical probability distribution. The test works by comparing the observed frequencies in the data to the expected frequencies predicted by the theoretical distribution.

b) Generate three Poisson variants with mean alpha = 0.2 using acceptance rejection techniques. Given random numbers are 0.4357, 0.4146, 0.8353, 0.9952 and 0.8004.

The threshold for acceptance is $e^{-\alpha} = e^{-0.2} \approx 0.8187$. We use the product of successive random numbers to test against this threshold.

Variate	Step	n	P (initial)	Random Number (R)	P (updated) = P · R	P < 0.8187 ?	Result
X_1	1	0	1	0.4357	0.4357	Yes	Accept $X_1 = 0$

X_2	1	0	1	0.4146	0.4146	Yes	Accept $X_2 = 0$
X_3	1	0	1	0.8353	0.8353	No	Reject, $n=1$
	2	1	0.83 53	0.9952	0.8313	No	Reject, $n=2$
	3	2	0.83 13	0.8004	0.6654	Yes	Accept $X_3 = 2$

The three generated Poisson variates are **0, 0, and 2**.

c) The life of a device used to inspect cracks in aircraft wings is given by X ... The cdf of the device's lifetime, in years, is as follows $F(x)=1-e^{-x/2}$ for $x \geq 0$.

i.) Find the probability that the device will last for 2 years.

ii.) Find the probability that the device will last between 2 and 3 years.

i.) Probability of lasting for 2 years

The most likely interpretation of this question is the probability of lasting for **at least 2 years**, i.e., $P(X \geq 2)$. (The probability of lasting for **exactly** 2 years is 0 for a continuous variable).

$$P(X \geq 2) = 1 - P(X < 2) = 1 - F(2)$$

$$P(X \geq 2) = 1 - (1 - e^{-2/2}) = e^{-1} \approx 0.3679$$

There is approximately a **36.8% chance** the device will last for at least 2 years.

ii.) Probability of lasting between 2 and 3 years

We need to find $P(2 \leq X \leq 3) = F(3) - F(2)$.

- $F(3) = 1 - e^{-3/2} \approx 1 - 0.2231 = 0.7769$
- $F(2) = 1 - e^{-2/2} \approx 1 - 0.3679 = 0.6321$

$$P(2 \leq X \leq 3) \approx 0.7769 - 0.6321 = 0.1448$$

The probability that the device will fail between the 2nd and 3rd year is approximately **14.5%**.

6. a) What is the Kendall notation of the Queuing System?[3]

Kendall's notation is a standardized system in queueing theory to describe and classify a queueing system using a series of symbols that represent the arrival process, service time distribution, and number of servers. The general form is $A/S/c$, where 'A' denotes the inter-arrival time distribution, 'S' is the service time distribution, and 'c' is the number of servers. This can be extended to include system capacity, population size, and queue discipline as A/S/c/K/N/D.

The basic $A/S/c$ notation:

- **A (Arrival Process):** Represents the probability distribution of the time between arrivals.
 - M: Markovian ([Poisson process](#), meaning inter-arrival times are exponentially distributed).
 - G: General (arbitrary service times).
 - D: Deterministic (fixed inter-arrival times).
- **S (Service Distribution):** Represents the probability distribution of service times.
 - M: Markovian (exponentially distributed service times).
 - G: General (arbitrary service times).
- **c (Number of Servers):** The number of parallel servers at the service station.

The extended $A/S/c/K/N/D$ notation:

- **K (Capacity):** The maximum number of customers allowed in the system (queue + service).
- **N (Population Size):** The size of the calling population from which customers arrive.

- **D (Queue Discipline):** The rule by which customers are selected for service.
 - **FIFO:** First-In, First-Out (or FCFS).
 - **LIFO:** Last-In, First-Out.
 - **Priority:** Customers are served based on a priority level.

Example:

An M/M/1 queue represents a system with:

- **M:** Markovian (Poisson) arrivals.
- **M:** Markovian (exponential) service times.
- **1:** A single server.

b) Define a Markov chain and its application.[4]

A Markov chain is a mathematical model of a system that transitions between a finite number of states over time, where the probability of moving to the next state depends only on the current state and not on the sequence of events that preceded it; this is known as the memoryless property. Markov chains are applied to predict future states and analyze the long-term behavior of systems in fields like finance, weather forecasting, genetics, and communication networks, by using a transition matrix that specifies the probabilities of moving between states.

Definition

- **States:** A Markov chain models a system with a set of possible conditions or outcomes, called "states".
- **Transitions:** The system moves from one state to another over time.
- **Transition Probabilities:** The likelihood of moving from one state to another is defined by probabilities, which are often organized in a transition matrix.
- **Memoryless Property:** The crucial aspect is that the future state of the system depends solely on its current state, not its past history.

Applications

The memoryless property and probabilistic nature of Markov chains make them useful for modeling and predicting complex systems.

- **Finance:** Used to model and predict stock market trends and customer loyalty in telecommunications.
- **Weather Forecasting:** Can model the progression of weather conditions, such as moving from sunny to cloudy to rainy states, based on the current condition.
- **Biology:** Employed in genetic sequencing and disease spread modeling to understand how populations or infections evolve over time.
- **Networking:** Used to analyze network traffic and package delivery logistics within communication networks.
- **Games:** Simple games like Snakes and Ladders are often modeled as absorbing Markov chains, where the outcome depends on the current position and the roll of the dice.
- **Image Processing:** Markov random fields (MRFs) are applied in image segmentation, object detection, and reconstruction

c) Use the mixed congruential method to generate a sequence of three two-digit random numbers with $X_0 = 37$, $\alpha = 7$, $c = 29$ and $m = 100$ [5]

7. For the following multiplicative generator, compute Z_i for enough values of $i \geq 1$ to cover an entire cycle. [6]

- i) $Z_0 = 1 \quad a = 11 \quad m = 16$
- ii) $Z_0 = 2 \quad a = 11 \quad m = 16$
- iii) $Z_0 = 1, \quad a = 2 \quad m = 13$
- iv) $Z_0 = 2 \quad a = 3 \quad m = 13$

b) Find first three random variables in $[0,1]$ using $X_0 = 27$, $a = 8$, $c = 47$, $m = 10^2$ [2]

c) The sequence of numbers 0.54, 0.73, 0.98, 0.11 and 0.68 has been generated. Use the Kolmogorov-Smirnov test with $\alpha = 0.05$ to check uniformity.[4]

8. At a grocery store with one counter, customers arrive at random from 1 to 8 minutes apart (each inter-arrival time has the same probability of occurrence). The service times vary from 1 to 6 minutes with the probabilities as 0.10, 0.20, 0.30, 0.25, 0.10 and 0.05 respectively. Analyze the system by simulating the arrival and service of 15 customers. [Justifying your situation and requirements, you can choose your required random values]

[12]

<https://youtu.be/0aucm40TZxY>