1. Big book maths
② Queuing theory
3. Recursive formula
4. Series
⑤ marcov chain/Hidden marcov chain/
monte carlo
⑥ Game theory
⑦ TOH & variants
⑧ Lines in the plain problem
9. Josephus intro
10. Josephus binary property
11. Josephus Repertoire method
12. Josephus reverse find N
13. Josephus Radix based properties
14. Josephus Customized m
15. Sum of recurrence
16. TOH recurrence to sum
17. QS recurrence to sum
18. Integer function
19. Number theory
(gcd, proofs, prime, era, fibonacci, FTA, PF)
20. factorial factors
21. Stern brocot tree.
22. Binomial coefficient
㉓ Bayes theorem.
24. Discrete random variabl
25. Continues random "
26. Expectation of random "
㉗ Confusing matrix
㉘ MMI queue

- * MLE
- * NBC
- * NBC and some issues temple univ

- Sum of recurrence
- TOH recurrence to sum
- QS recurrence to sum.

Mid-I

1. markov chain
2. Queuing theory
3. Confusion matrix
4. Precision, recall, f1 meas
5. Recursive formula

~~Markov chain:~~

Tower of Hanoi:- we are given a tower of eight disks, initially stacked in decreasing size on one of three pegs. The objective is to transfer the entire tower to one of the other pegs, moving only one disk at a time and never moving a large one onto a smaller.

T_n = minimum no of moves to solve problem

$T_0 = 0$

$T_1 = 1$

$T_2 = 3$

For n disks to transfer from one peg to another we require:

$$T_n = T_{n-1} + 1 + T_{n-1} = 2T_{n-1} + 1$$

Thus, the recurrence for tower of Hanoi stand

$$T_0 = 0$$

$$T_n = 2T_n + 1 \quad \text{for } n>0$$

we can find out the closed form of any recurrence to get quick result from the problem

$$T_n = 2T_{n-1} + 1$$

$$= 2(2T_{n-2} + 1) + 1$$

$$= 2^2 \{T_{n-2} + 2 + 1\}$$

$$= 2^2(2T_{n-3}+1)+2+1$$

$$= 2^3 T_{n-3}+2^2+2+1$$

$$= 2^n T_{n-n}+2^{n-1}+2^{n-2}+\cdots+2^2+2+1$$

$$T_n = 2^n T_0 + 2^{n-1} + 2^{n-2} + \cdots + 2^2 + 2^1 + 2^0$$

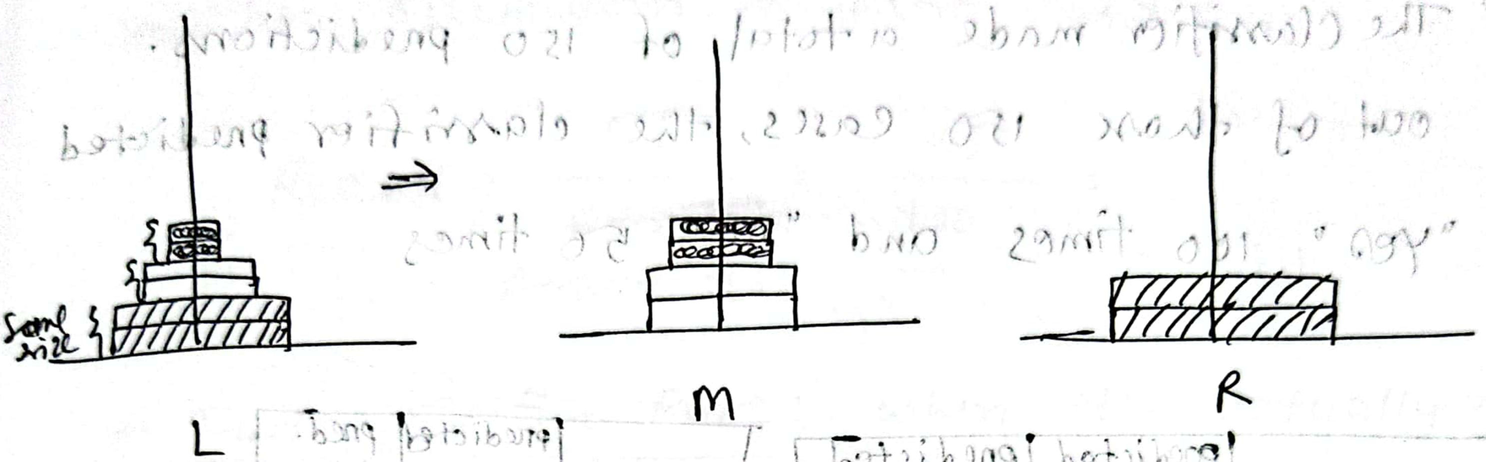
$$= 2^0+2^1+2^2+\cdots+2^{n-2}+2^{n-1}$$

$$= \frac{2^n-1}{2-1} \quad \left[a^0+a^1+a^2+\cdots+a^n = \frac{a^{n+1}-1}{a-1}\right]$$

$$= 2^n-1$$

$\therefore$ $T_n = 2^n - 1$ is called the closed form of solution of the tower of Hanoi problem.

Double tower of hanoi



$$T_n = T_{n-1} + 2 + T_{n-1} = 2T_{n-1} + 2$$

$$T_n = 2T_{n-1} + 2$$

$$= 2(2T_{n-2} + 2) + 2$$

$$= 2^2 T_{n-2} + 2^2 + 2$$

$$= 2^2(2T_{n-3} + 2) + 2^2 + 2$$

$$= 2^3 T_{n-3} + 2^3 + 2^2 + 2$$

$$\cdots\cdots$$

$$= 2^n T_{n-n} + 2^n + 2^{n-1} + \cdots + 2^1$$

$$= 0 + [2 + 2^2 + 2^3 + \cdots + 2^n]$$

$$= 2[2^0 + 2^1 + 2^2 + \cdots + 2^{n-1}]$$

$$= \frac{2(2^{n-1+1} - 1)}{2-1} = 2^{n+1} - 2$$

$$\boxed{T_n = 2^{n+1} - 2}$$

⊞ Confusion matrix:

* The Classifier made a total of 150 predictions. out of those 150 cases, the classifier predicted "yes" 100 times and "no" 50 times

	predicted No	predicted Yes
Actual No	TN = 45	FP = 5
Actual Yes	FN = 5	TP = 95

	predicted No	pred. Yes
Actual No	45	5
Actual Yes	5	95

$$\text{Accuracy} = \frac{TN+TP}{TN+FP+FN+TP} = \frac{45+95}{45+5+5+95} = \frac{140}{150}$$

$$= 93.33\%$$

Accuracy: How often is the classifier correct?

* **misclassification Rate:** Overall, how often is it wrong?

$$\text{misclassification rate} = \frac{FP+FN}{150} = \frac{10}{150}$$

$$= 6.67\%.$$

* True positive rate or Recall: when it's actually yes, how often does it predict yes?

$$\text{Recall} = \frac{TP}{\cancel{FN+TP}\ \text{Actual Yes}} = \frac{95}{100} =$$

* False positive Rate: when it's actually no, how often does it predict Yes?

$$\text{FP rate} = \frac{FP}{\text{Actual No}} = \frac{5}{50} = 10\%$$

* True Negative Rate:

$$\frac{TN}{\text{Actual NO}} = \frac{45}{50} = 90\%.$$

* precision: when it predicts yes, how often is it correct?

$$\text{precision} = \frac{TP}{\text{predicted Yes}} = \frac{95}{100} = 95\%.$$

F1 score prevalance: How often does the Yes condition actually occur in our sample?

$$F1\ score = \frac{Actual\ Yes}{Total} = \frac{100}{150}$$

$$= 66.67\%.$$

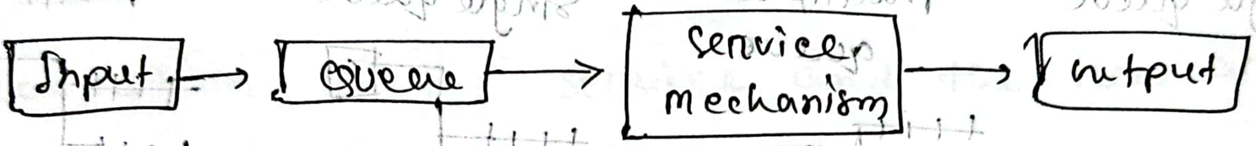
$$F_1\ Score = \frac{TP}{TP + \frac{1}{2}(FP+FN)}$$

* Queving theory:

1. **Customer**: One who requires service is called a customer.

2. **Server**: One who provides service is called server.

3. **Queue (waiting line)**: A group of customers at ~~a~~ some place to receive service called the queue.



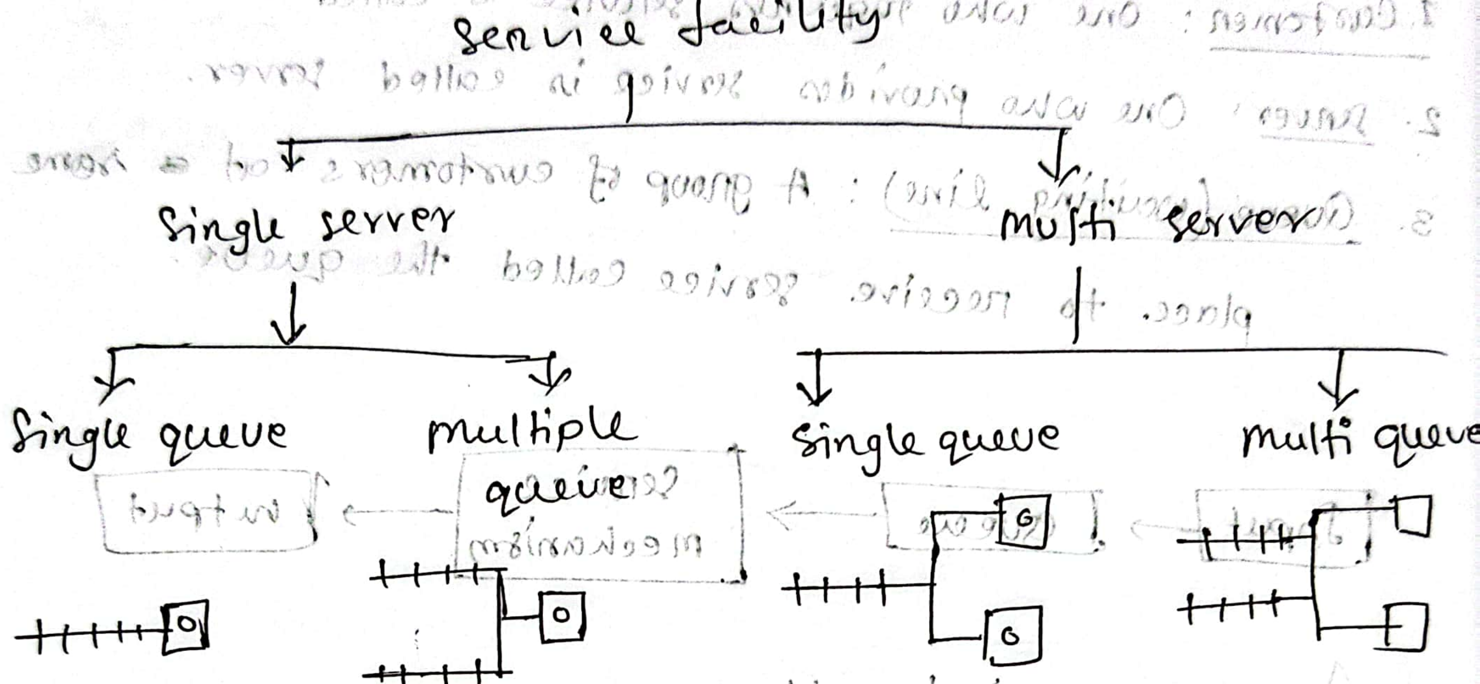
* Input or arrival pattern:

① **Balking**: If the customer return back without getting service because of long queue, is called balking.

② **Reneging**: This occur when the waiting customer leaves the queue due to impatient.

③ **Jocklying**: If there be more them one queue and customer leave one queue and join another queue is called Jocklying.

ⓑ Service pattern:



ⓒ Service desciplines

① FCFS

② LCFS

③ SIRO (service in random order)

* **Queue length:** The number of person in queue at any time.

* **Servicing time:** Time taken for servicing one unit of the queue. (u)

* **Mean arrival ~~time~~ rate:** The number of expected customers in one unit of time is known as mean arrival rate. denoted by (λ)

* **Mean servicing rate:** The expected complete service in one unit time is called mean servicing rate.

Denoted by μ.

$$\text{Traffic intensity } (\rho) = \frac{\lambda}{\mu}$$

* **Idle period:** Time interval in between the completion of the service and the new arrival is called the idle time.

Ⓠ On an average 96 patients per 24 hours day require the service of an emergency clinic. Also on average, a patient require 10 minutes of active attention. Find traffic intensity.

$\lambda = 96$ per day

$\mu = 24 \times 6$ per day

$$\rho = \frac{\lambda}{\mu} = \frac{96}{24 \times 6} = 2/3$$

Q customer arrive at a box office with one ticket window according to a poisson's input process with mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with mean 90 seconds. Traffic inten?

$$\lambda = 30 / \text{hour}$$

$$\mu = \frac{3600}{90} = 40 / \text{hour}$$

$$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = \frac{3}{4}$$

$$* \quad \lambda = 30 / \text{day}$$

$$\mu = \frac{24 \times 60}{36} = 40 / \text{day}$$

$$\therefore \rho = \frac{30}{40} = \frac{3}{4}$$

* Some important notation:

① λ – Mean arrival time rate

② μ → mean service rate

③ L_s → Expected service length

④ L_q → Expected queue length

⑤ W_s → " waiting time per customer in service

⑥ W_q → " " " " " " queue

⑧ L_n → Expected length of non-empty queue.

* Single server queuing model:

① mm1 : $(\infty|FIFO)$ → infinite queue length

② mm1 : $(N|FIFO)$ → finite queue length

① MM1:

Here, first M : poisson input

second M : poisson output

1 : single server

∞ : Infinite capacity of system.

formulas

① Average number of customer in service, $L_s = \frac{\lambda}{\mu-}$

② " " of customer in queue, $L_q = \frac{\lambda^2}{\mu(\mu-\lambda}$

③ " waiting time of customer in queue, $W_q = \frac{\lambda}{\mu(\mu-\lambda}$

④ " " " " " " " service, $W_s = \frac{1}{\mu-\lambda}$

⑤ Expected length of non-empty queue, $L_n = \frac{\mu}{\mu-\lambda}$

$$= \frac{1}{1-\rho}$$

⑥ probability that no customer at the counter,

$$P_0 = 1-\rho = 1-\frac{\lambda}{\mu}$$

Next formule

one

Ex Customer arrive at a box office with a ticke window according to a poisson's input process with mean rate of 30 per hour. The time required to serve a customer has an exponential distribut with mean 90 seconds.

Find the average —

① length of service (L_s)

② queue length (L_q)

③ waiting time in queue (W_q)

④ Time spent by a customer in the system.

$\lambda = 30 / hr$

$\mu = \frac{3600}{90} = 40 / hr$

① $L_s = \frac{\lambda}{\mu - \lambda} = \frac{30}{40-30} = 3$

② $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{30\times30}{40(40-30)} = \frac{\overset{3}{3\cancel{0}}\times 3\cancel{0}}{4\cancel{0}\times\cancel{10}} = \frac{9}{4}$

③ $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{30}{40(40-30)} = \frac{3}{40}$

④ $W_s = \frac{1}{\mu-\lambda} = \frac{1}{40-30} = \frac{1}{10} = \frac{60}{10} = 6$ min

---

probability of queue exceeds $\geq n$? $= \rho^n$:

→ $p(\text{queue size} \geq n) = \rho^n$

The time spent by a repairman on his jobs has an exponential distribution with mean 30 minutes. If the repairs sets in the order in which they come in and if the arrival of sets is approximately poisson with an average rate of 10 per 8 hour day, what is the repairmen expected idle time each day? How many jobs are ahead of the average set just brought in?

$\mu = \frac{60}{30} = 2$ per 8 hour day $= 2 \times 8 = 16$ per 8 hr a day

~~pe~~ $\lambda = 10$ per 8 hour a day

$\therefore P_0 = 1 - \rho = 1 - \frac{\lambda}{\mu} = 1 - \frac{10}{16} = \frac{6}{16} = \frac{3}{8}$

$\frac{3}{8}$ in a day $= \frac{3}{8} \times 8 = 3$ hour in a day

$L_s = \frac{\lambda}{\mu - \lambda} = \frac{10}{16-10} = \frac{10}{6} = \frac{5}{3}$ set

⑧ Arrivals at a telephone booth are considered to be poisson. with an average time of 10 minutes between one arrival and the next. The length of a phone call assumed to be distributed exponentially, with mean 3 minutes. Find the following—

① What is the probability that a person arriving at the booth will have to wait.

② What is the average length of the queues tha form from time to time?

③ the telephone department will install a secon booth when convinced that an arrival would expect to have to wait at least thre minutes for the phone. by how much must th flow of arrivals be increased in order to justify a second booth?

=

$$\lambda = \frac{60}{10} = 6 \,/\, hr$$

$$\mu = \frac{60}{3} = 20 \,/\, hr$$

অপেক্ষা করতে হলে queue তে অন্তত একজন লোক থাকতে হবে।

① $p(\text{queue size} \geq 1) = \rho^1 = \frac{\lambda}{\mu} = \frac{6}{20}$

$$= \frac{3}{10}$$

(ii) $L_n = \frac{\mu}{\mu - \lambda} = \frac{20}{20-6} = \frac{20}{14} = \frac{10}{7}$

নতুন booth install করতে হলে arrival কত পরিমাণ বাড়াতে হবে?

(iii) $\cancel{W_q = \frac{\mu}{\mu(\mu-\lambda)}}$

$$W_q = 3$$

$$\Rightarrow \frac{\lambda'}{\mu(\mu-\lambda')} = \frac{3}{60}$$

$$\Rightarrow \frac{\lambda'}{20(20-\lambda')} = \frac{3}{60}$$

$$\Rightarrow \frac{\lambda'}{20-\lambda'} = 1$$

$$\Rightarrow 2\lambda' = 20$$

$$\therefore \lambda' = 10.$$

$\therefore$ arrival বাড়াতে হবে $\lambda' - \lambda = 10 - 6 = 4\ /hr$

Quiz Q

Customer arrive at a Shop according to the poisson distribution with a mean of 10 customer/hr. The manager notes that no customer arrives for the first three minutes after the shop opens. the probability that a customer arrives within the next three minutes is?

$$\lambda = 10/hr$$

time intervals, $t = \frac{3}{60} = \frac{1}{20}$ hours

$$p(\text{no arrival in time } t) = e^{-\lambda t}$$

$$p(\text{at least one arrival in time } t) = 1 - e^{-\lambda t}$$

$$= 1 - e^{-10 \cdot \frac{1}{20}}$$

$$= 1 - e^{-0.5}$$

$$= 0.393469$$

Q - (18-19) final

The arrival rate of customers is 120 per hr and the service rate of each server is 180 per hour. Find the following. All time need to converted in minutes.

(a) average no of customer in the system.

$$\lambda = 120/hr = \frac{120}{60} = 2 /min$$

$$\mu = 180/hr = \frac{180}{60} = 3 /min$$

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{2}{3-2} = 2$$

(b) Average waiting (time in the queue)

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{2}{3(3-2)} = \frac{2}{3}$$

(c) The system utilization

$$\rho = \frac{\lambda}{\mu} = \frac{2}{3}$$

ⓓ $p(\text{queue size} \geq 1) = \rho' = \frac{\lambda}{\mu} = \frac{2}{3}$

ⓔ $p(\text{queue size} \geq 1) = \rho' = \frac{\lambda}{\mu} = \frac{2}{3} = 0.667$

ⓕ In mm1 | ∞ , the probability of customer being blocked is 0.

Game theory:

Game: Game is defined as an activity among two or more persons as per set of rules at the end of which each person gets some benefit or bears loss. The set of rules and ~~produce~~ procedure defines the game.

A competitive situation is called game.

The term game ~~theory~~ represents a conflic between two or mores. parties,

Game theory:

Solve the game whose pay-off matrix is given-

player-B

Player-A

$$\begin{array}{c|ccccc|c} & B_1 & B_2 & B_3 & B_4 & B_5 & \text{row minima} \\ \hline A_1 & -2 & 0 & 0 & 5 & 3 & -2 \\ A_2 & 3 & 2 & \textcircled{1} & 2 & 2 & 1 \\ A_3 & -4 & -3 & 0 & -2 & 6 & -4 \\ A_4 & 5 & 3 & -4 & 2 & -6 & -6 \\ \hline \text{Column maxima} & 5 & 3 & 1 & 5 & 6 & \end{array}$$

maxi (minimum) = $\underline{\gamma}$ = Max$(-2,1,-4,-6)$ = 1

mini (maximum) = $\bar{\gamma}$ = Min$(5,3,1,5,6)$ = 1

Since, $\underline{\gamma} = \bar{\gamma} = 1$, there exists a saddle point.

value of the game = 1

The position of saddle point is the optimal strategy and is given by $[A_2, B_3]$

Ⓠ Determine which of the following two person zero-sum games are strictly determinable and fain. Given the optimum strategy for each player in the case of strictly determinable games.

(i)

$$\begin{array}{c|c|cc|}
 & & \text{player B} & \\
 & & B_1 & B_2 \\
\text{player A} & A_1 & -5 & 2 \\
 & A_2 & -7 & -4
\end{array}$$

(ii)

$$\begin{array}{c|c|cc|}
 & & \text{player B} & \\
 & & B_1 & B_2 \\
\text{player A} & A_1 & 1 & 7 \\
 & A_2 & 4 & -3
\end{array}$$

①

$$\begin{array}{cc|cc|c}
 & & \text{P-B} & & \\
 & & B_1 & B_2 & \text{row minima} \\
\text{P-A} & A_1 & \textcircled{-5} & 2 & -5 \\
 & A_2 & -7 & -4 & -7 \\
 & \text{col maxima} & -5 & 2 &
\end{array}$$

maxi(minimum) = $\underline{\gamma}$ = Max (-5, -7) = -5

mini(Maximum) = $\overline{\gamma}$ = Min (~~ø~~ -5, 2) = -5

Since, $\underline{\gamma} = \overline{\gamma} = -5$, hence there exists a saddle point.

value of the game = −5.

Since, $\bar{\gamma} = \underline{\gamma} = -5 \neq 0$, hence, the game is strictly determinable and not fair.

∴ optimal strategy is the position (A, B_1)

⑪

$$\begin{array}{c|cc|c} & \multicolumn{2}{c}{\text{P-B}} & \\ & B_1 & B_2 & \text{row minima} \\ \hline \text{P-A} \; A_1 & 1 & \textcircled{1} & 1 \\ A_2 & 4 & -3 & -3 \\ \hline \text{col maxima} & 4 & 1 & \end{array}$$

maxi(minimum) = $\underline{\gamma}$ = max(1, −3) = 1

mini (maxima) = $\bar{\gamma}$ = min(4, 1) = 1

Since, $\underline{\gamma} = \bar{\gamma} = 1 \neq 0$, hence the game is strictly determinable and not fair.

The value of the game = 1

Optimal strategy is the position $(A_1 B_2)$

(8+6)

Ⓠ ~~⑦~~ 6-(C)

$$\begin{array}{ccccc|c} & & & \text{player-B} & & \\ & & B_1 & B_2 & B_3 & \text{Row minima} \\ \text{player-A} & A_1 & 6 & 8 & 6 & 6 \\ & A_2 & 4 & 12 & 2 & 2 \\ \text{Col maxima} & & 6 & 12 & 6 & \end{array}$$

$\text{Maxi(minimum)} = \underline{\gamma} = \max(6,2) = 6$

$\text{mini(maximum)} = \bar{\gamma} = \min(6,12,6) = 6$

Since, $\underline{\gamma} = \bar{\gamma} = 6 \neq 0$, hence there is a saddle point.

The saddle point is $A_1 B_3$ or $A_1 B_1$

⑪ Best strategy for player A_1 = strategy I

Best strategy for player B = strategy III or I

Q – 6(d) 6-(d)

Find a range of values of a and b for which the following pay-off matrix will a saddle point at (2,2) position.

$$\begin{array}{cc|ccc} & & & B & \\ & & B_1 & B_2 & B_3 \\ \hline & A_1 & 2 & 4 & 5 \\ A & A_2 & 10 & 7 & b \\ & A_3 & 4 & a & 6 \end{array}$$

For a saddle point at (2,2), the value of position (2,2) should be

① The maximum in its row (for player A's strategy)

(ii) The minimum in its column (for " B's strategy)

for (2,2) be a sadde point, value 7 must be greater the largest in its ~~[illegible]~~. ~~[illegible]~~ row

$7 \geq 10$ and $7 \geq b$

Since, 10 is not less than or equal to.

So, $7 \geq b$.

for (2,2) be a saddle point, the value 7 must be smaller in its ~~Row~~ column.

Hence, $7 \leq 4$ and $7 \leq a$

from this, we get,

$$7 \leq a$$

So, the range is

$$\boxed{a \geq 7 \text{ and } b \leq 7}$$

6-(a)

4 Importance of game theory in computer Science?

Game theory plays a significant role in computer science and engineering in the following ways —

① **Networks design and optimization:** Game theory helps in designing efficient networks, especially in distributed system where multiple users or devices compete for the resources.

② **Security:** In cybersecurity, game theory is used to model and analyze situations where attackers and defenders interact.

③ **Artificial Intelligence:** Game theory is fundamental in AI for designing systems that can compete or cooperate with other agents.

6-(b)

④ **Competitive situation:** This refers to a scenario where multiple players or participants compete for a limited resource or outcome.

6(b)

Strategy: A strategy is a predefined plan of action or a set of choices that a player follows throughout the game. It outlines how the player will act in response to any possible situation that may arise during the game.

Two-person zero-sum game:

A two-person zero-sum game is a situation where two players are involved, and the gain of one player results in an equivalent loss for the other player.

Maximin: This principle is used by a player to maximize their minimum gain.

minimax: This principle is used by a player to ~~maximize their min~~ minimize their maximum loss.

Saddle point: A saddle point in a pay-off matrix is an equilibrium point where the strategy chosen by the both players is optimal.

Markov Chain:

A random process in which the occurrence of future state depends on the currently preceding state and only on it, is known as markov chain.

Assumption:

① finite number of state

② State are mutually exclusive. [একই সময়ে একটি state সম্ভব]

③ State are collectively exhaustive.
[সব সম্ভাব্য state আমাদের জানা থেকেই থাকবে]

④ probability of moving from one state to another state is constant over time.

Transition probability:- The probability of moving from one state to another state or remaining in the same state during a single time period is called the transition probability

$$P_{ij} = P(\text{Next state } S_j \text{ at } t=1 \mid \text{initial state } S_i \text{ at } t=0)$$

Transition probability matrix:

With the help of TPM, we predict the movement of system from one state to the next state.

Next state(j) t=1

$$P = \begin{array}{c} \text{Initial} \\ \text{State (i)} \\ t=0 \end{array} \begin{array}{c|ccc|} & S_1 & S_2 & S_3 \\ \hline S_1 & P_{11} & P_{12} & P_{13} \\ S_2 & P_{21} & P_{22} & P_{23} \\ S_3 & P_{31} & P_{32} & P_{33} \\ \hline \end{array}$$

↓ retention & gain

→ retention & loss

Assumption :-

① Row sum = 1

② each element of TPM is probability

$0 \leq P_{ij} \leq 1$ & Non-negative.

③ Squere matrix because –

row show → initial state

Column show → Alternative state in next move

[Next state]

Ⓠ In a certain market, only two brand of col drinks A and B are sold. Given that a man last purchased brand A, there is 80% chance that he would buy the same brand in the next purchase, while if a man purchased brand B, there is 90% chance that he would buy brand B in his next purchase. Using this information —

① Develop transition probability matrix.

② ~~Interpotate~~ Interprete the State transition matrix in terms of (i) Retention & gain

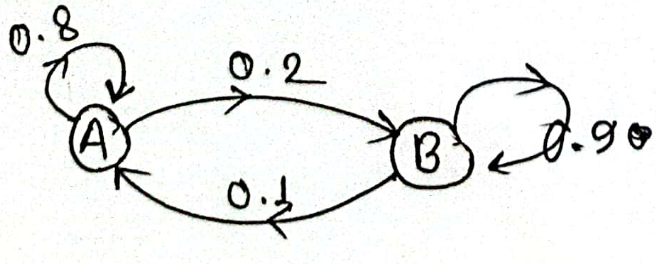
(ii) Retention & Loss.

③ Draw transition diagram.

Next purchase [n=1]

①

$$P = \text{present purchase } [n=0] \quad \begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix} \end{array}$$



Find out the transition matrix for —

ⓐ The president of the USA tells a person A his intention to run or not to run in the next election. Then A relays the news to B, who in torn relays the message to C and so fourth, always to some new person. we assume that there is a probability that a person will change the answer from yes to no when transmitting it to the next person.....

Solve: We are given two state —

State 1 — Yes
State 2 — No

probability a : changing the msg from "Yes" to "No"
" b : " " "No" to "Yes"

Next state

$$\begin{array}{ccc} & & \begin{array}{cc}\text{Yes} & \text{No}\end{array} \\ \text{initial state} & \begin{array}{c}\text{Yes}\\ \text{No}\end{array} & \left[\begin{array}{cc}1-a & a \\ b & 1-b\end{array}\right]\end{array}$$

7-(b)

(b) In the dark ages, Harvard, Darmouth and Yale ___ ___ ___ .

Solve: were three states —

① Harvard

② Darmouth

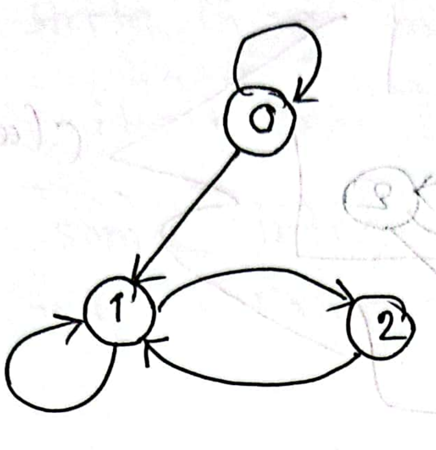
③ Yale

Next state

$$\begin{array}{cc|ccc} & & \text{Harvard} & \text{Dermouth} & \text{Yale} \\ \hline \text{Initial State} & \text{Harvard} & 0.8 & 0 & 0.2 \\ & \text{Darmouth} & 0.2 & 0.7 & 0.1 \\ & \text{Yale} & 0.3 & 0.3 & 0.4 \end{array}$$

Recurrence :

7-(a)



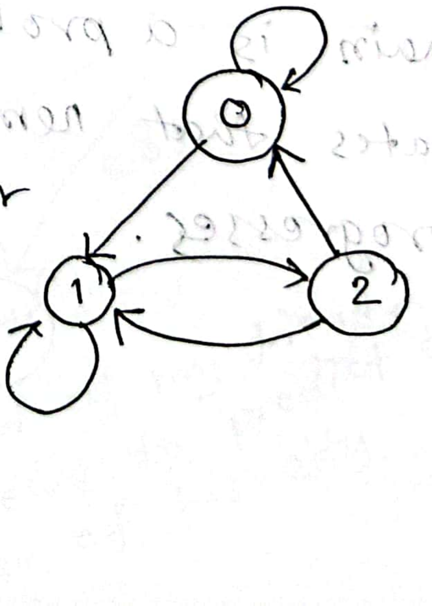
if we start a random walk from state 1 and go to state 2, we are bound to come back to the state 1. This type of state called recurrent state. Hence, this phenomenon is called recurrence.

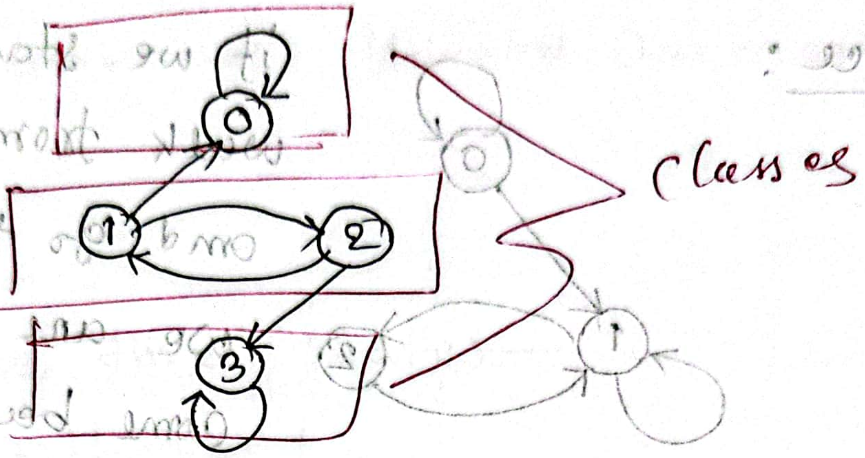
এটা একটা reducible chain

- **Ruducible:** If we can not go from every state to every other state / some states are not reachable from other state is called ruducible.

যদি কোনো স্টেট থেকে অন্য স্টেটগুলোতে না যাওয়া যায়, তখন সেটা Ruducible chain

- **Irreducibility:** If we can go from every state to every other state, we can call that chain irreducible.





Classes: States in a markov chain can be grouped into classes based on their ability to reach each other. It is possible to move from any state in the same class to any other state. State in different classes do not communicate with one other.

Stationary distribution:

A stationary distribution of a markov chain is a probability distribution over the states that remains unchanged as time progresses.

$\pi P = \pi$ [π = Stationary distribution, P = Transition matrix]

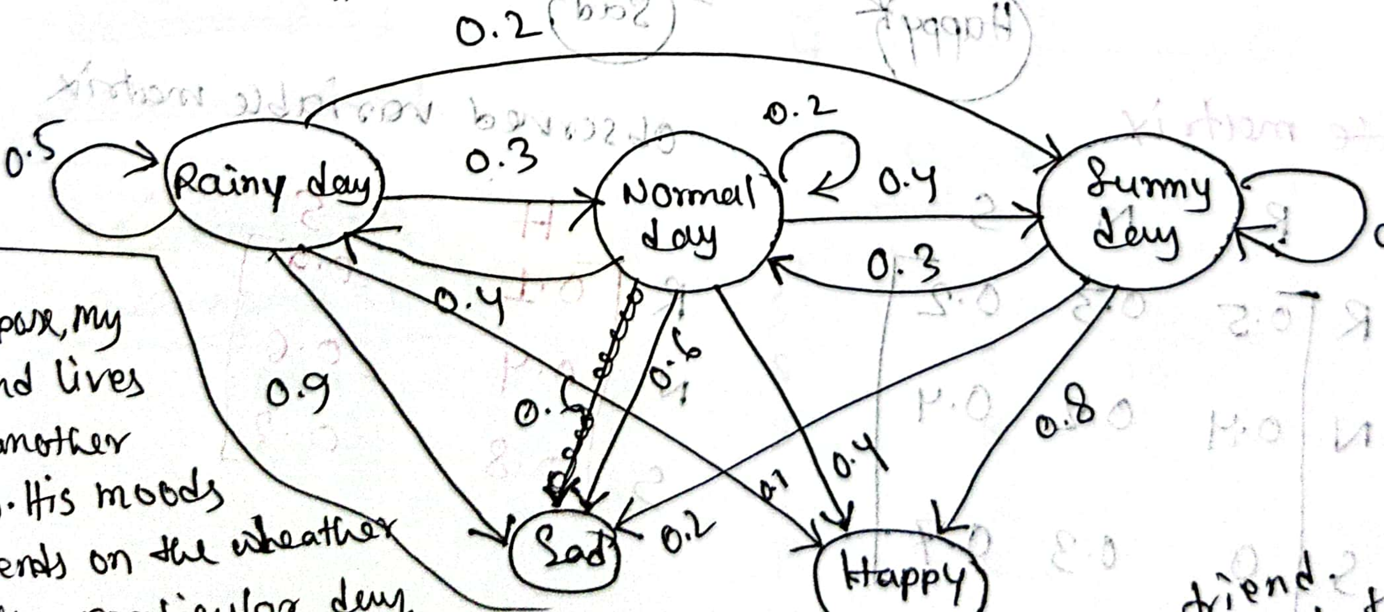
* **periodic**:- A state in a markov chain is said to be periodic if it can only be returned to at multiples of some integer greater than 1

n-step transition matrix:

কোনো একটি state থেকে অন্য state এ যদি n সংখ্যক step এর মাধ্যমে যেতে হয়, তাহলে probability calculation করতে হলে, matrix টাকে n গুণ করতে হবে।

অর্থাৎ, $\boxed{A^n}$

Hidden Markov chain:



Suppose, my friend lives in another city. His moods depends on the weather of the particular day.

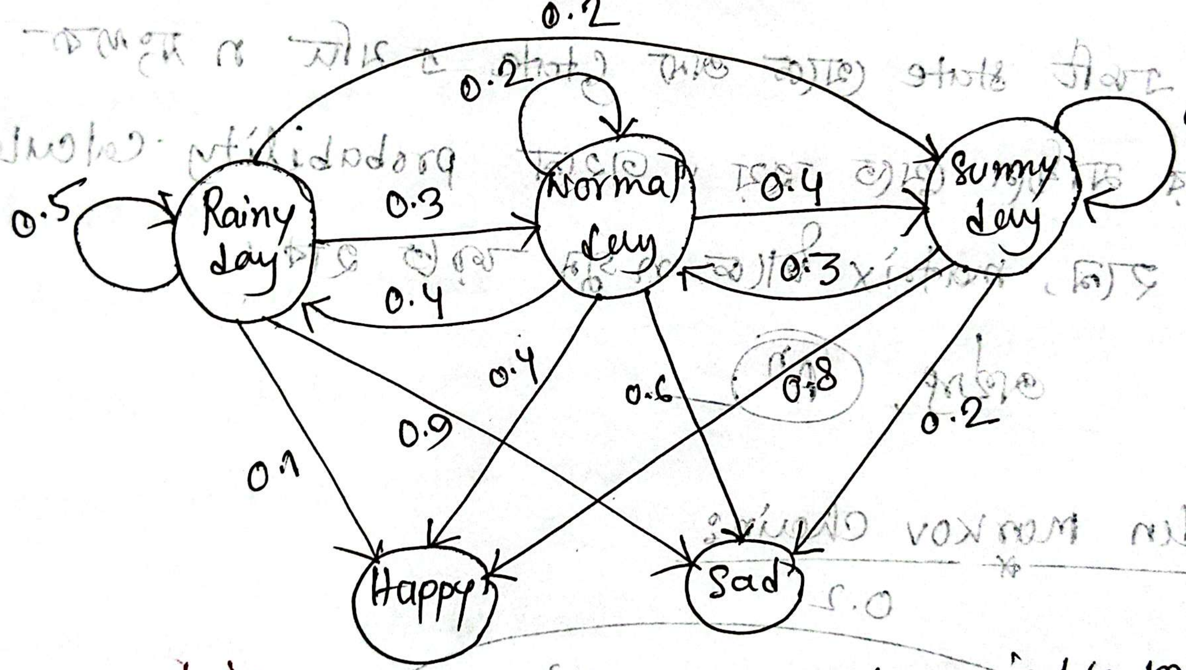
But we don't have the way to know the weather of the city.

We can know about the mood of my friend. knowing his mood we've to predict the

of his city.

So, states are hidden their, but we can observe variables like my friends mood.

[আমাদের অবশ্যই মানতে হবে যে বন্ধুর mood only depends on today weather but not the previous days.



State matrix

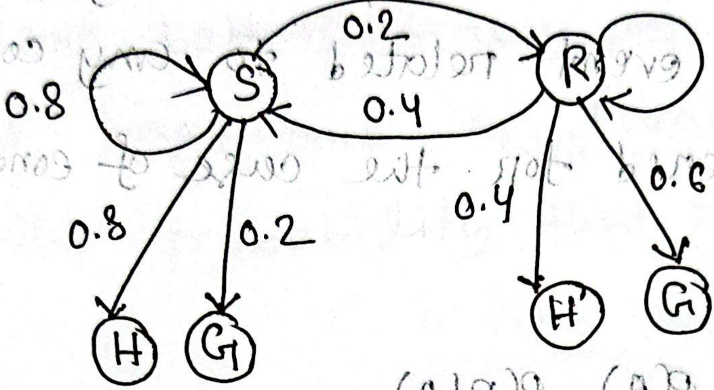
$$\begin{array}{c|ccc} & R & N & S \\ R & 0.5 & 0.3 & 0.2 \\ N & 0.4 & 0.2 & 0.4 \\ S & 0 & 0.3 & 0.7 \end{array}$$

transition matrix

Observed variable matrix

$$\begin{array}{c|cc} & H & S \\ R & 0.1 & 0.9 \\ N & 0.4 & 0.6 \\ S & 0.8 & 0.2 \end{array}$$

#



$\overset{2}{0.8S} + \overset{1}{0.4R} = S$

$\overset{1}{0.2S} + \overset{3}{0.6R} = R$

$S + R = S + R$

$= 1$

~~$S = \overset{2}{0.8S} + \overset{1}{0.4R}$~~

$S = \frac{2}{3}$

~~$R = \overset{1}{0.2S} + \overset{3}{0.6R}$~~

$R = \frac{1}{3}$

~~$S+R=1$~~

$\Rightarrow$ ~~$0.8S+0.4R+0.2S+0.6R=1$~~

~~$\Rightarrow 0.S+R=1$~~

~~$0.8S+0.4R=1$~~

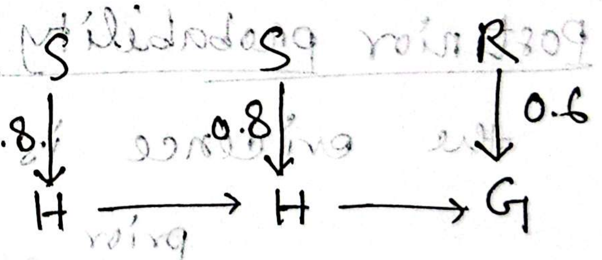
$4\times$ ~~$0.2S+0.6R=1$~~

~~$0.8S+2.4R=4$~~

~~$0.8S+0.4R=1$~~

~~$2R=3$~~

~~$R=\frac{3}{2}$~~



$= 0.384$

S	S	$\frac{6}{10} =$
S	S	
S	R	
S	R	
R	R	

⊞ Bayes theorem

7-(C)

Bayes theorem describes the probability of occurance of an event related to any condition. It is also considered for the case of conditional probability.

$$P(A|B) = \frac{P(A) \cdot P(B|A)}{P(B)}$$

⊞ **Prior probability:** The probability before the evidence is considered.

⊞ **Posterior probability:** The probability after the evidence is considered.

$$\underset{\text{posterior}}{\boxed{P(A|B)}} = \frac{\overset{\text{prior}}{\boxed{P(A)}}\ \boxed{P(B|A)}\ \text{Likelihood}}{\boxed{P(B)}\ \text{Marginal}}$$

Ex: A bag I contains 4 white ball and 6 black balls while enother bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and its found to be black. Find the probability that it was draw from bag I.

Let's define: B = probability of the drawn ball is black

$P(A_1)$ = probability of ball drawn from bag I

$P(A_2)$ = " " " " " bag II

$$\therefore P(A_1) = P(A_2) = \frac{1}{2}$$

$$P(B|A_1) = \frac{6}{10} = 0.6$$

$$P(B|A_2) = \frac{3}{7} = 0.4286$$

Total probability of drawing black ball, $P(B)$

$$= \frac{6}{10} \times \frac{1}{2} + \frac{3}{7} \times \frac{1}{2} = 0.3 + 0.2143 = 0.5143$$

$$P(A_1|B) = \frac{P(A_1) \times P(B|A_1)}{P(B)} = \frac{0.5 \times 0.6}{0.5143} = 0.5834$$

Ex-31 A man is known to speak the truth 2 out of 3 times. He throws a die and reports that the number obtained is a four. Find the probability that the number obtained is actually four.

Solve:

probability of ~~obtain~~ occurring four, $P(E_1) = \frac{1}{6}$

" " not occuring four, $P(E_2) = \frac{5}{6}$

probability of reporting actual four, $P(A|E_1) = \frac{2}{3}$

" " " not " ", $P(A|E_2) = \frac{1}{3}$

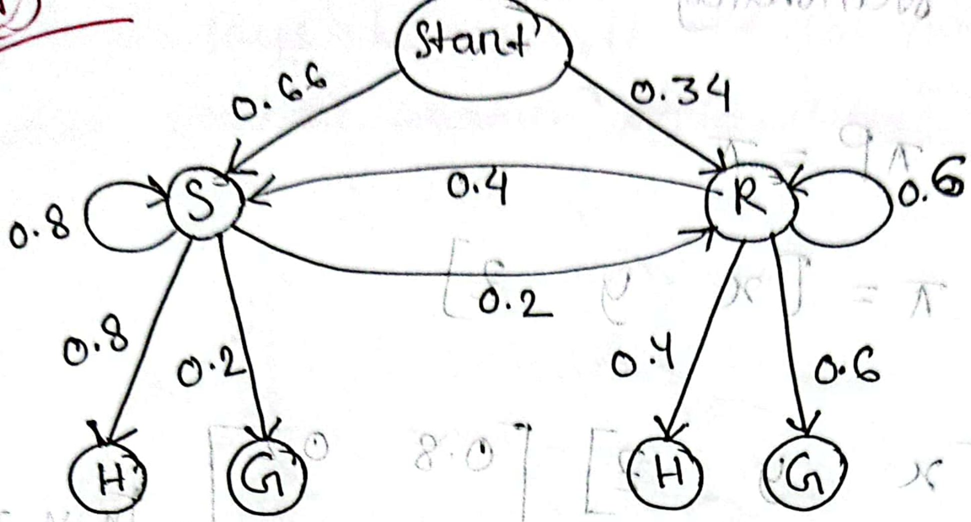
Using Bayes theorem :-

$$P(E_1|A) = \frac{P(E_1) \times P(A|E_1)}{P(B)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{3}}{\frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3}} = \frac{\frac{1}{9}}{\frac{1}{9} + \frac{5}{18}}$$

$$= \frac{1}{\cancel{9}} \times \frac{\overset{2}{\cancel{18}}}{\cancel{1}2+5} = \cancel{19 \times 18} = \frac{2}{7}$$

7 (d)



Hidden states: These states are not directly visible but can be inferred based on observed data.

~~Observed states~~: Here, sunny and rainny these two states are hidden states.

Observed ~~states~~ variable: These are the variable that we can actually observe.

Here, the moods or the man happy and grumpy are observed variables.

Transition Matrix:

$$P = \begin{array}{c|cc} & S & R \\ S & 0.8 & 0.2 \\ R & 0.4 & 0.6 \end{array}$$

if π be a stationary distribution,

hence,

$$\pi P = \pi$$

if. $\pi = [x \quad y \quad z]$

then, if $[x \quad y \quad z] \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix} \Rightarrow [x, y, z]$

then, $\pi = [x \quad y \quad z]$ is the stationary distribution

(Ans)

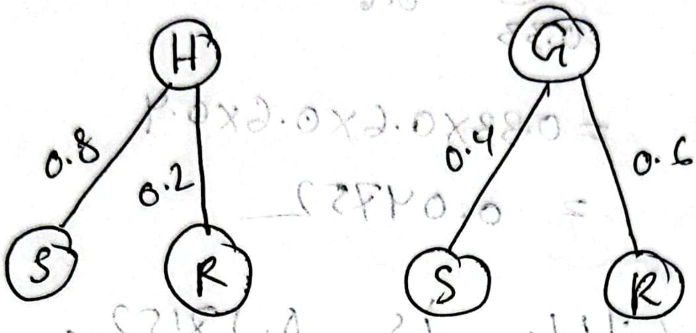
$$\begin{array}{ccc} S & S & R \\ \uparrow 0.8 & \uparrow 0.8 & \uparrow 0.6 \\ H & H & G \end{array}$$

Emission probability matrix:

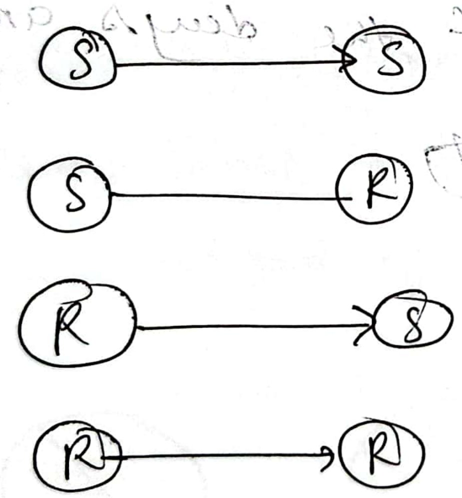
$$\begin{array}{c|cc} & H & G \\ \hline S & 0.8 & 0.2 \\ R & 0.4 & 0.6 \end{array}$$

∴ probability of HHG according to SSR is $= 0.66 \times 0.8 \times 0.8 \times 0.6 = 0.25344$

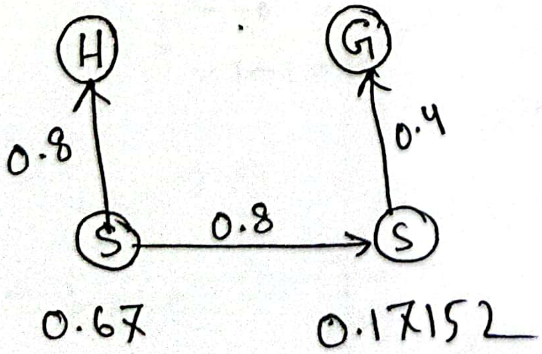
* If bob says he was H and G two days.
what was the weather probability?



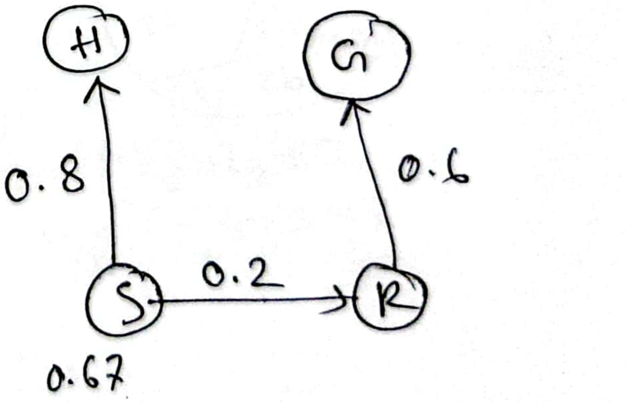
So, there are $2^n = 2^2 = 4$ possible permutation.



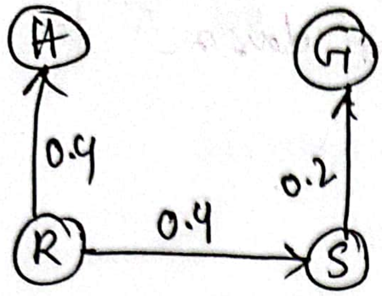
if we calculate:

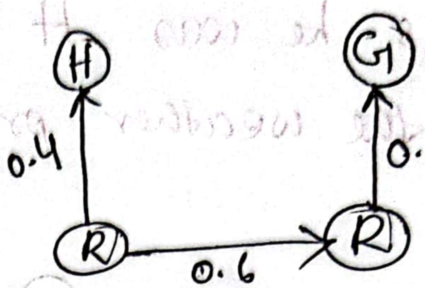


prior probability



$= 0.8 \times 0.2 \times 0.6 \times 0.67 = 0.064$





0.33

$= 0.33 \times 0.4 \times 0.4 \times 0.2$

$= 0.01056$

0.33

$= 0.33 \times 0.6 \times 0.6 \times 0.4$

$= 0.04752$

So, maximum probability is 0.17152.

And that is for $S \longrightarrow S$

So the probability of the days are

$\boxed{\text{sunny then sunny}}$

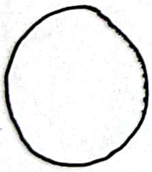
Lines in the plane problem:

L_n = Maximum no of regions in a circle formed by n straight lines.

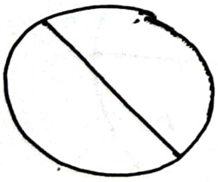
Ex: L_n = Maximum no of slices formed by n straight lines.

Line in the plane:

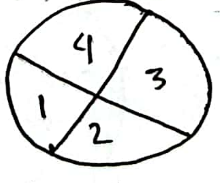
we have to find how many slices of pizza can a person obtain by making n straight cuts with a knife. Means what is the maximum number of regions, L_n defined by n lines in the plane?



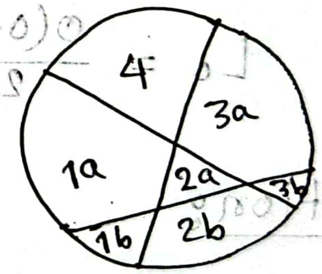
$L_0 = 1$



$L_1 = L_0 + 1$
$= 1 + 1 = 2$



$L_2 = L_1 + 2$
$= 2 + 2$
$= 4$



$L_3 = L_2 + 3$
$= 4 + 3 = 7$

So,

$$L_n = L_{n-1} + n$$
$$= L_{n-2} + (n-1) + n$$
$$= L_{n-3} + (n-2) + (n-1) + n$$
$$\vdots$$

$= L_{n-n} + (n-n+1) + \cdots + (n-2) + (n-1) + n$

$= L_0 + 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$

$= L_0 + S_n$

$= S_n + 1$

$$L_n = \frac{n(n+1)}{2} + 1 \quad \text{for. } n \geq 0$$

⊞ Induction Method prove:

Basis: For the lowest value of n which is 0, we show that our solution:

$$L_n = \frac{n(n+1)}{2} + 1$$

$$L_0 = \frac{0(0+1)}{2} + 1 = 0 + 1 = 1$$

Induction:

For all values between n to (n-1), we assume that our solⁿ is true.

$$L_n = L_{n-1} + n$$

$$= \frac{(n-1)(n-1+1)}{2} + 1 + n$$

$$= \frac{n^2 - n}{2} + 1 + n$$

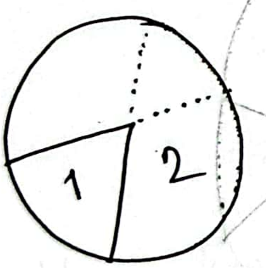
$$= \frac{n^2 - n + 2 + 2n}{2}$$

$$= \frac{n^2 + n + 2}{2}$$

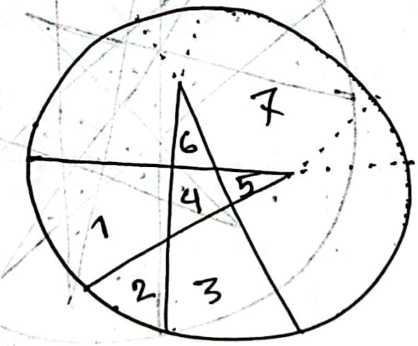
$$= \frac{n(n+1)}{2} + \frac{2}{2}$$

$$= \frac{n(n+1)}{2} + 1$$ (proved)

⊞ **Zig:** we're to find the maximum numbers of region while ~~zig~~ n numbers of zig crosses each other.



1 zig



2 zigs

Zig	Lines	Line region	Zig region	lost region
1	2	$\frac{n(n+1)}{2}+1 = 4$	2	2
2	4	11	7	4
3	6	22	16	6

Observing that, we are losting 2n numbers of zig regions.

∴ Zig regions, $Z_n = L_{2n} - 2n$

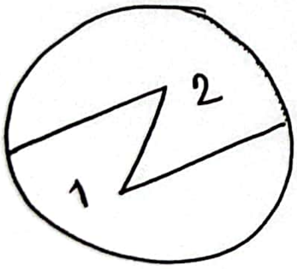
$$= \frac{2n(2n-1)}{2} + 1 - 2n$$

$$= (2n^2 - n + 1) - 2n$$

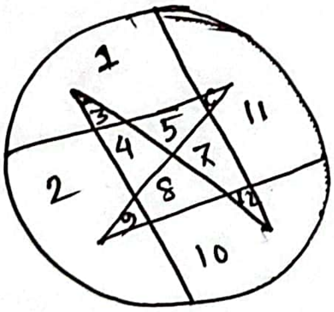
∴ $Z_n = 2n^2 - n + 1 - 2n$

Ans

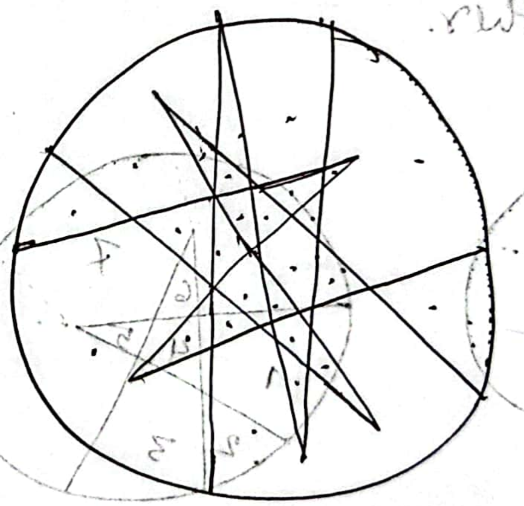
Zig-Zag :



$ZZ_1 = 2$



$ZZ_2 = 12$



$ZZ_3 = 31$

Zig Zag	lines	line regions	Zigzag regions	Lost regions
1	3	7	2	5
2	6	22	12	10
3	9	46	31	15

$$\therefore ZZ_n = L_{3n} - 5n$$

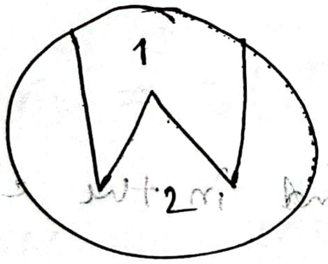
$$= \frac{3n(3n+1)}{2} + 1 - 5n$$

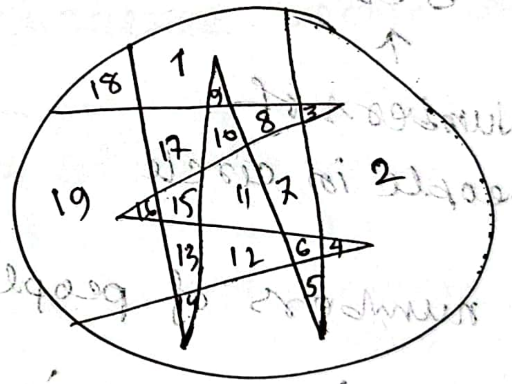
$$= \frac{9n^2+3n}{2} + 1 - 5n$$

$$= \frac{9n^2+3n+2-10n}{2}$$

$$= \frac{9n^2-7n+2}{2} \quad \boxed{\text{Ans}}$$

W object:





$W_1 = 2$

W	lines	line region	W region	lost region
1	4	11	2	9
2	8	37	19	18

$$\therefore W_n = L_{4n} \cancel{00} - 9n$$

$$= \frac{4n(4n+1)}{2} + 1 - 9n$$

$= 8n^2 + 2n + 1 - 9n$

$= 8n^2 - 7n + 1$

Ans

☐ Josephus problem:

একটি circle এ n সংখ্যক মানুষ দাঁড়াবে এবং প্রত্যেকে তার পরের জনকে মেরে ফেলবে। তাহলে finally কোন position এর ব্যক্তিটি বেঁচে থাকবে।

$J(1) = 1$ ← survivor's number

↑ Numbers of people in circle

if even numbers of people stand in the circle then the survivor's no is, $J(2n) = 2J(n) - 1$

if odd numbers of people stand in the circle then the survivor's no is, $J(2n+1) = 2J(n) + 1$

So,

$$J(1) = 1$$

$$J(2n) = 2J(n) - 1$$

$$J(2n+1) = 2J(n) + 1$$

Using these recurrence if we build a table :-

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$J(n)$	1	1	3	1	3	5	7	1	3	5	7	9	11	13	15	1

$J(n)$ is always 1 at the begeining of a group and it increases by 2 within the group. So, if we write n in form of $n = 2^m + l$, where 2^m is the largest power of 2 not exceeding n, where $l = n - 2^m$.

The solution to our recurrence would be,

$$J(n) = J(2^m + l) = 2l + 1$$

So, if there are 100 peoples in the circle,

$$J(100) = J(2^6 + 36) = 2 \times 36 + 1 = 73$$

then, $J(41) = J(2^5 + 9) = 2 \times 9 + 1 = 19$

$$J(10) = J(2^3 + 2) = 2 \times 2 + 1 = 5$$

B) Inductive proof of : $J(2^m+l) = 2l+1$

Basis: we prove, the closed form on m for the lowest value of m which is zero.

we show that,

$$J(2^m+l) = 2l+1 \text{ is true.}$$

if m is 0 l will also zero

$$\therefore J(2^0+0) = 2\times0+1 = 1$$

therefore, $J(1) = 1$

Hypothesis: for all values of m between 1 to (m−1)

$J(2^m+l) = 2l+1$ is true.

for even case:

$$J(2n) = 2J(n)-1$$

here, $2n = 2^m + l$

$$\Rightarrow n = 2^{m-1} + \frac{l}{2}$$

$$J(2n) = \cancel{2J\left(\frac{2^m+l}{2}+2l\right)} - 1$$

$$= 2J\left(2^{m-1} + l/2\right) - 1$$

$$= 2\left[2\times\frac{l}{2}+1\right] - 1$$

$$= 2l + 2 - 1$$

$$= 2l + 1$$

ODD case:

$$J(2n+1) = 2J(n) + 1$$

here, $2n+1 = 2^m + l$

$$\Rightarrow 2n = 2^m + l - 1$$

$$\Rightarrow n = 2^{m-1} + \frac{l-1}{2}$$

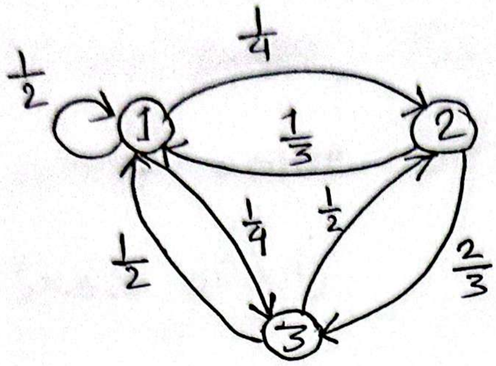
$$\therefore J(2n+1) = 2J\left(2^{m-1} + \frac{l-1}{2}\right) + 1$$

$$= 2\left[2 \times \frac{l-1}{2} + 1\right] + 1$$

$$= 2[l - 1 + 1] + 1$$

$$= 2l + 1$$

4) previous Q. solve :



ⓐ Is this chain irreducible?

Yes! this chain is irreducible because we can go from any state to any other state.

(b) Is this chain aperiodic?

The chain is aperiodic since there is a self loop in transition.

ⓒ find the stationary distribution:

Let's generate a transition matrix :-

$$A = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1/2 & 1/4 & 1/4 \\ 2 & 1/3 & 0 & 2/3 \\ 3 & 1/2 & 1/2 & 0 \end{array}$$

for stationary distribution, $\pi A = \pi$, here π is the stationary distribution.

$$\pi A = [\pi_1 \quad \pi_2 \quad \pi_3] \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$\frac{\pi_1}{2} + \frac{\pi_2}{3} + \frac{\pi_3}{2} = \pi_1$$

$$\frac{\pi_1}{4} + \pi_2 \cdot 0 + \frac{2\pi_3}{2} = \pi_2$$

$$\frac{\pi_1}{4} + \frac{2\pi_2}{3} + \pi_3 \cdot 0 = \pi_3$$

Simplifying these equations —

$$\frac{\pi_2}{3} + \frac{\pi_3}{2} = \frac{\pi_1}{2} \quad \text{——— (I)}$$

$$\frac{\pi_1}{4} + \frac{\pi_3}{2} = \pi_2 \quad \text{——— (II)}$$

$$\frac{\pi_1}{4} + \frac{2\pi_2}{3} = \pi_3 \quad \text{——— (III)}$$

putting value of π_2 in eqn (III)

$$\frac{\pi_1}{4} + \frac{2}{3}\left(\frac{\pi_1}{4} + \frac{\pi_3}{2}\right) = \pi_3$$

$$\Rightarrow \frac{\pi_1}{4} + \frac{2\pi_1}{12} + \frac{2\pi_3}{6} = \pi_3$$

$$\Rightarrow \frac{3\pi_1 + 2\pi_1}{12} = \frac{6\pi_3 - 2\pi_3}{6}$$

$$\Rightarrow \frac{2\pi_3}{3} = \frac{5\pi_1}{12}$$

$$\pi_3 = \frac{15}{24}\pi_1$$

putting this value into eqn (ii)

$$\frac{\pi_1}{4} + \frac{1}{2}\left(\frac{15\pi_1}{24}\right) = \pi_2$$

$$\Rightarrow \frac{\pi_1}{4} + \frac{\pi_1}{48} = \pi_2$$

$$\Rightarrow \frac{12\pi_1 + \pi_1}{48} = \pi_2$$

$$\Rightarrow \pi_2 = \frac{13\pi_1}{48}$$

Similarly, put ~~π_2 and~~ π_3 into the equation no (1) $\rightarrow$

$$\frac{\pi_2}{3} + \frac{1}{2}\left(\frac{15\pi_1}{24}\right) = \frac{\pi_1}{2}$$

$$\Rightarrow \frac{\pi_2}{3} + \frac{15\pi_1}{24} = \frac{\pi_1}{2}$$

$$\Rightarrow \frac{\pi_2}{3} = \frac{\pi_1}{2} - \frac{15\pi_1}{24}$$

$$= \frac{12\pi_1 - 15}{24}$$

[illegible]

Convergence: A series or sequence is said to converge if its terms approach a single finite value as the number of terms goes to infinity.

Divergence: Divergence occurs when a series or sequence does not approach a finite limit, meaning if either increases without bound, decreases without bound, or oscillates without settling on a value.

Divergent	Convergent
① Divergent don't have finite sum	① Has a finite sum
② Terms increases indefinitely	② Terms approache a specific value
③ Fails convergence test	③ passes, convergence test
④ Exhibits unbounded growth	④ Reaches a definite value
⑤ No specific sum	⑤ Has a calculable finite sum.

Divergent series:

$1+2+4+8+16+\cdots$ $\quad 1+0+1+0+1+\cdots$

$0+1+0+2+0+4+\cdots$ $\quad 1-2+3-4+5-6+\cdots$

Convergent series:

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

A harmonic series is divergent:

Because It doesnot have a finite sum because its terms grow without bound as more terms are added. specifically, the sum of the harmonic series is infinite, as it can be expressed as $\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)$, where n ranges from 1 to infinity.

harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{n} = \int_1^{\infty} \frac{1}{x}\,dx$$

$$= [\ln x]_1^{\alpha}$$

$$= \ln\infty - \ln 1$$

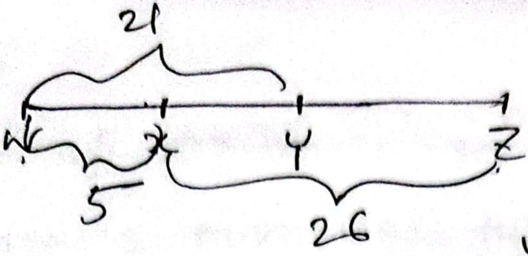
$$= \infty$$

$\therefore$ hence, harmonic series is infinity.

$\sqrt{x^2+x^2}$

$=\sqrt{2x^2}$

$=\sqrt{2}x$



GRE

$WY=21$

$WX+XY=21$

$YZ=2WX$

$XZ=26$

$XY+YZ=26$

$\cancel{XY}+2WX=26$

$\cancel{XY}+WX=21$

$WX=5$

$YZ=10$

$xY=xZ-YZ$

$=26-10=16$

$\frac{60-36}{60+36}=\frac{24}{96}=\frac{1}{4}$

BIG

BOOK

MATHS

$\omega=\frac{r}{2}$

$\boxed{\omega\times2\pi r=\pi r^2}$

$2\omega=r$

$\omega=\frac{r}{2}$

March

$d=2c$

~~d=40~~

April, 2d

3c

$(d+c)+(2d+3c)=500$

$\Rightarrow 3d+4c=500$

$\Rightarrow 3\times2c+4c=500$

$\Rightarrow 6c+4c=500$

$10c=500$

$c=50$

$\therefore d=100$

~~d=20~~

$(d+c)+(3c+2d)$ 2500

~~⇒ 2c+c+3c+2c~~

18-19 (Q-2)

(b) $w \times 2\pi r = \pi r^2$

$$w = \frac{r}{2}$$

(c) কুকুর/বিড়ালের বেচার

Dog sold in march $= d$

cats " " " $= c$

In march, $d = 2c$

In april, 2d dog and 3c cat.

total sell $= 500$

$\therefore (d+c) + (2d+3c) = 500$

$\Rightarrow 3d + 4c = 500$

$\Rightarrow 3 \times 2c + 4c = 500$

$\Rightarrow 10c = 500$

$c = 50$

$\therefore d = 2 \times 50 = 100$

So, In march total 100 dogs were sold.

ⓓ Dog show problem:

The number of ways to choose 2 distinct finalists from 5 for the two awards is given by.

$$\boxed{{}^5P_2 = 20}$$

Ans

ⓔ Expectation:

Expectation is a concept in probability that represents the long-term average or mean of a random variable.

Mathematically, the expectation $E(x)$ of a discrete random variable x, which takes values $x_1, x_2, \ldots x_n$ with probabilities $P_1, P_2 \ldots P_n$ is defined as,

$$E(x) = \sum_{i=1} x_i \cdot p_i$$

18-19

4 (b) Recursive formula for

① fibonacci number

$$F(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ F(n-1) + f(n-2) & , n > 1 \end{cases}$$

(II) Factorial

```
fact (n)
if (n= =0 || n==1)
    return n;
else return n* fact(n-1);
```

$$n! = \begin{cases} 1 & , \text{if } n = 0, 1 \\ n(n-1)! & , \text{if } n > 1 \end{cases}$$

(III) GCD:

$$gcd(a,b) = \begin{cases} a & ; \text{if } b = 0 \\ gcd(b, a \% b) & ; \text{if } a \geq b \text{ and } b> \end{cases}$$

⊞ A number in decimal notation is divisible by 3 if and only if the sum of the digits is divisible 3. prove this well known rule and generaliz it.

⇒ first, let's split the number in the form of a power of 10s. lets take an example of a 3 digit number 'abc', where 'a' hundred's digit, 'b' is ten's digit and 'c' is unit's digit.

Therefore, $abc = a \times 10^2 + b \times 10^1 + c \times 10^0$

$$= (99+1)a + (9+1)b + c$$

$$= 99a + 9b + a + b + c$$

when we devide the number by 3 we get,

$$abc/3 = 33a + 3b + (a+b+c)/3$$

Hence, abc is divisible by 3 only when (a+b+c) is divisible by 3.

⊞ Complex numbers:

Complex numbers are numbers of the form $a+bi$, where:

(a) a and b are real numbers

(b) i is the imagenary unit, defined by

$$i^2 = -1$$

⊞ Proof that $\sqrt{2}$ is irrational:-

suppose $\sqrt{2}$ is an irrational number.

Then there are two mutually prime numbers $p, q > 1$ so that $\sqrt{2} = \frac{p}{q}$

$$\text{or, } 2 = \frac{p^2}{q^2}$$

$$\text{or, } 2q = \frac{p^2}{q}$$

Obviously $2q$ is an integer but $\frac{p^2}{q}$ is not an inte

because p and q are natural numbers and the are mutually prime and $q > 1$.

$\therefore$ $2q$ and $\frac{p^2}{q}$ can not be equal.

That is, $2q \neq \frac{p^2}{q}$

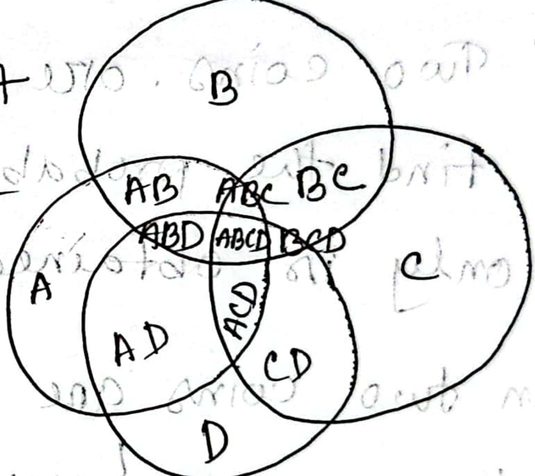
∴ $\sqrt{2}$ can not be expressed in the form $\frac{p}{q}$.

That is, $\sqrt{2} \neq \frac{p}{q}$

∴ $\sqrt{2}$ is an irrational number.

4 overlaping ven diagram:

we can draw a venn diagram with six regions. It's just that we can't acheive it in the expected way by putting two overlapping circle over two other overlapping. That method end up missing two regions. Here, there is no AC and BD



16-17

1 (d) ① A die is rolled, find the probability tha the number obtained is greater than 4.

there are 2 numbers those are greater than 4.

Total probability = 6

$$\therefore \text{probability} = \frac{2}{6} = \frac{1}{3}$$

(ii) Two coins are tossed. find the probability that one head only is obtained.

when two coins are tossed, the all possible outcomes are, {HH, HT, TH, TT} = 4

one head only = {HT, TH} = 2

$$\therefore \text{probability} = \frac{2}{4} = \frac{1}{2}$$

(iii) Two dice are rolled. probability that the sum is equal to 5.

* probable combinations are:—

$1+4=5$

$2+3=5$

$4+1=5$

$3+2=5$

Total combinations: $6 \times 6 = 36$

$\therefore$ probability $= \frac{4}{36} = \frac{1}{9}$

$\boxed{E}$ probability of getting king of heart.

All card $= 52$

King of heant $= 1$

$\therefore \; p = \frac{1}{52}$

18-19

প্র 5(b):

p-values: In statistics, a p-value is the probability of obtaining a result at least as extreme as the one observed in your sample data, assuming that the null hypothesis is true.

Harmonic Series: The harmonic series is an infinite series that is the sum of the reciprocals of the positive integers.

mathematically,

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

Harmonic series is divergence.

[proof লিখতে হবে নাই।

5(c) Collatz conjecture:

Collatz conjecture describes a process for a positive integer n.

$$f(n) = \begin{cases} n/2 \; ; \text{ if } n \text{ is even} \\ 3n+1 \; ; \text{ if } n \text{ is odd} \end{cases}$$

(i) $n = 22$; $22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$.

The cycle length is = 16

(ii) $n = 1000$; $1000 \rightarrow 500 \rightarrow 250 \rightarrow 125 \rightarrow 376 \rightarrow 188 \rightarrow \cdots$

the cycle length is = 112

(iii) $n = 1$; The cycle length = 1

(iv) $n = 7$; $7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$

The cycle length = 17

18-19

5(d)

(i) $\sum_{n=0}^{\infty} (-1)^n \frac{5}{n}$; This is a divergent series because the term $\frac{5}{n}$ approaches to infinity.

(ii) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$; $\frac{1}{n^2+n} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

$$= \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+ \cdots$$

This series is a convergent series, and it converge

to 1

(iii) $\sum_{n=1}^{\infty} \frac{1}{2^n-1}$

This series converges because it behaves similarly to a geometric series $\sum \frac{1}{2^n}$, which converges. for a large value of n, 2^n-1 behaves similarly to 2^n and the series conver

(iv) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$; $\frac{n^2-1}{n^2+n} = \frac{(n+1)(n-1)}{n(n+1)} = 1-\frac{1}{n}$;

which doesn't approaches to '0' and goes to infinity. Therefore series is divergent.

ⓥ $\sum_{n=1}^{\infty} (-1)^{n\mp 1} \frac{n^2}{n^3+1}$; $\frac{n^2}{n^3+1} = \frac{\frac{n^2}{n^3}}{1+\frac{1}{n^3}} \quad \frac{1/n}{1+\frac{1}{n^3}}$

This series converges as the general term $\frac{n^2}{n^3+1}$ decreases as $n \to \infty$ and approches zero.

Josephus Binary property:

we know the formula for Josephus problem, which is, $J(n) = J(2^m + l) = 2l + 1$

for example,

$$J(100) = J(2^6 + 36) = 2 \times 36 + 1 = 73.$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\underline{1100100} \qquad 0\underbrace{100100} \qquad \underbrace{100100}1$$

if we observe, we can see that the binary value of the total numbers of people and the value of l is same except the MSB bit. If we doubled the value of l we just do left shift operation and then add 1 to the number. So, we can find the survivor number by changing the total number of people's MSB to the last

$$n = (b_m b_{m-1} b_{m-2} \cdots b_2 b_1 b_0)_2$$

- b_m is always 1.

$$\text{So, } l = (0\, b_{m-1} b_{m-2} \cdots b_2 b_1 b_0)_2$$

$$\therefore 2l = (b_{m-1} b_{m-2} \cdots b_1 b_0 0)_2$$

$$2l+1 = (b_{m-1} b_{m-2} \cdots b_1 b_0 1)_2$$

As b_m is always 1. we can replace it at the last digits of $(2l+1)$.

$$\therefore 2l+1 = J(n) = (b_{m-1} b_{m-2} \cdots b_1 b_0 b_m)_2$$

$\downarrow$ $\downarrow$ $\downarrow$ proved

⊞ Kendall's notation for queuing model:-

(i) Kendall's notation for queuing model :-

(A|B|C) : (D|E)

A → is the arrival time distribution

B → is the service time distribution

C → is the number of servers

D → queue length (i.e., no of customers allowed in the queue)

E → queue discipline

(ii) Kendall's notation for question (ii)

m m m

(iii) m|G|∞

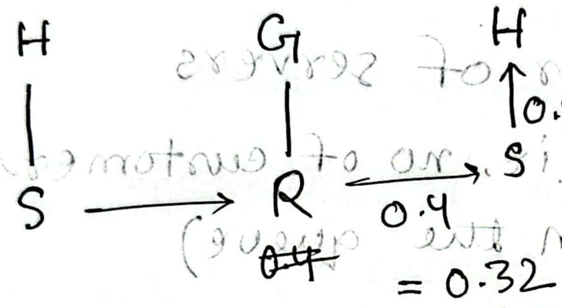
(iv) (M M m) :(K N)

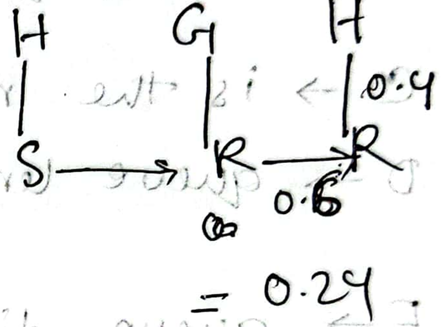
17-18

2(b) (ii)

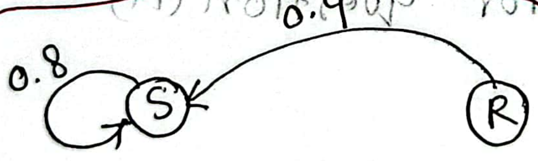
H G H → First day S and second d
R are fixed.

So, all possible probabilities —





∴ So the most probable outcome for the third day
is Rainy.



$$S = 0.8S + 0.4R = \frac{2}{3} = 0.67$$

$$\therefore R = \frac{1}{3} = 0.33$$

(1) Draw the transition matrix with the probability if emission of 3 days are G H G given the SRS respectively.

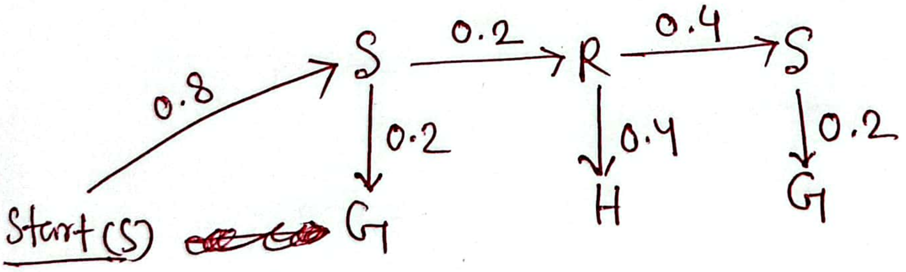
Transition matrix:—

$$\begin{array}{ccc} & S & R \\ S & 0.8 & 0.2 \\ R & 0.4 & 0.6 \end{array}$$

Emission Matrix:—

$$\begin{array}{ccc} & H & G \\ S & 0.8 & 0.2 \\ R & 0.4 & 0.6 \end{array}$$

∴ probability =



So, total probability = $0.8 \times 0.2 \times 0.2 \times 0.4 \times 0.4 \times 0.2$

$= 0.001024$

$P(GHG \mid SRS) = 0.001024$

47. Confusion Matrix:-

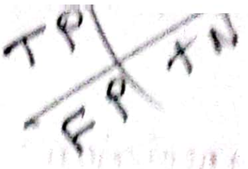
Truth	Dog	Car	Dog	Dog	Dog	phone	Home	Dog	Dog	Bird
prediction	Dog	Dog	Dog	No dog	Dog	No dog	Dog	No dog	Dog	Dog

	predicted dog	predicted No dog
Actual dog	TP = 4	FN = 2
Actual Not dog	FP = 3	~~FN~~= TN = 1

	predicted NO	predicted Yes
Actual NO	TN = 1	FP = 3
Actual Yes	FN = 2	TP = 4

	predicted dog	predicted No dog
Actual dog	TP = 4	FN = 2
Actual No dog	FP = 3	TN = 1

~~precision =~~



$$\text{Accuracy} = \frac{TN+TP}{TP+FN+FP+TN} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$\text{(*) precision} = \frac{\text{সঠিক Yes prediction}}{\text{কতগুলো Yes prediction}} = \frac{4}{7}$$

$$= \frac{TP}{TP+FP} = \frac{4}{4+3} = \frac{4}{7}$$

~~Recall = Actual positive / predicted positive = Actual dog / predicted dog = f~~

$$\text{(*) Recall} = \frac{\text{predicted dog}}{\text{Actual dog}} = \frac{4}{6} = \frac{2}{3}$$

$$= \frac{TP}{TP+FN} = \frac{4}{6} = \frac{2}{3}$$

$$\text{(*) f1 Score} = 2 * \frac{\text{precision} * \text{Recall}}{\text{precision} + \text{Recall}} = 2 * \frac{\frac{4}{7} \times \frac{2}{3}}{\frac{4}{7} + \frac{2}{3}}$$

$$= \frac{2 \times \frac{8}{21}}{\frac{12+14}{21}}$$

$$= \frac{16}{\cancel{21}} \times \frac{\cancel{21}}{26} = \frac{8}{13}$$

TP	FN
FP	TN

$$\text{precision} = \frac{TP}{TP+FP}$$

$$\text{Recall} = \frac{TP}{TP+FN}$$

18-19

1(b) A Smoke detector system uses two devices A, B.

probability that smoke is detected by device A

$$P(A) = 0.95$$

probability smoke is detected by device B

$$P(B) = 0.90.$$

" " " " Both device,

$$P(A \cap B) = 0.88.$$

(i) probability smoke will be detected by either A, B or both:

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$= 0.95 + 0.90 - 0.88$$

$$= 0.97$$

(II) probability that the smoke will go unditected:

$$p(\text{unditeeted}) = 1 - P(A \cup B)$$
$$= 1 - 0.97 = 0.03$$

Am

18-19
8(d)

$$\lambda = 6 \times \frac{30}{60} = 3 / 30 \text{ min period}$$

$K = 4$ (No of events)

$$P(X=4) = \frac{e^{-\lambda} \lambda^{k}}{k!}$$

$$= \frac{e^{-3} \cdot 3^{4}}{4!}$$

$$= 0.16804$$

Am

☐ Queuing networks:
1. open queuing networks
2. closed queuing "
3. Mixed " "

Queuing networks is a system that consists of multiple interconnected queues where entities are move from one queue to another.

☐ Conditional probability is the probability of occuring some event which is related to any condition.

$$P(A|B) = \frac{P(A)\ P(B|A)}{P(B)}$$

☐ Expectation is a concept in probability that represents the long term average or mean of a random variable

$$E(X) = \sum_{i=1}^{n} X_i \cdot P_i$$

wher, $x_1, x_2, x_3 \ldots$ are random variable

and, $P_1, P_2, P_3 - \ldots$ are probabilities.

17-18

1 (c)

if 2 chips are selected from 6 chips total

outcome = $^6C_2$ = 15

chose 1 black chip from 4 black chip = $^4C_1$ = 4

" 1 blue " " 2 blue " = $^2C_1$ = 2

∴ p(different color) = $\frac{\text{favorable outcome}}{\text{Total outcome}}$

$= \frac{4\times 2}{15} = \frac{8}{15}$

∴ The probability that the two chips are of different color = $\frac{8}{15}$

5

Suppose $a_0 = 0$, $a_1 = 2$ and $a_{n+2} = 4a_{n+1} - 4a_n + n^2 - 5n + 2$

where n devides & for all $n \geq [3]$

a. find value for n where values is 5 and 6.

$a_0 = 0$

$a_2 = 2$

$a_{n+2} = 4a_{n+1} - 4a_n + n^2 - 5n + 2$

$n = 1$, $a_3 = 4a_2 - 4a_1 + 1 - 5 + 2$

$= 4a_2 - 4a_1 - 2$

$n = 2$, $a_4 = 4a_3 - 4a_2 + 4 - 10 + 2$

$= 4a_3 - 4a_2 - 4$

$n = 3$, $a_5 = 4a_4 - 4a_3 + 9 - 15 + 2$

$= 4a_4 - 4a_3 - 4$

$n = 4$, $a_6 = 4a_5 - 4a_4 + 16 - 20 + 2$

$= 4a_5 - 4a_4 - 2$

ⓐ verify the following relations for single server queu

① $L_s = L_q + 1 - P_0$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} + 1 - \left(1 - \frac{\lambda}{\mu}\right)$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} + \cancel{1} - \cancel{1} + \frac{\lambda}{\mu}$$

$$= \frac{\lambda^2 + \lambda(\mu-\lambda)}{\mu(\mu-\lambda)}$$

$$= \frac{\lambda(\lambda+\mu-\lambda)}{\mu(\mu-\lambda)}$$

$$= \frac{\lambda\cancel{\mu}}{\cancel{\mu}(\mu-\lambda)} = \frac{\lambda}{\mu-\lambda} = L_s$$

(proved)

⑪ $L = L_q + \rho$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} + \frac{\lambda}{\mu}$$

$$= \frac{\lambda^2 + \lambda(\mu-\lambda)}{\mu(\mu-\lambda)}$$

$$= \frac{\lambda(\cancel{\lambda}+\mu-\cancel{\lambda})}{\mu(\mu-\lambda)}$$

$$= \frac{\lambda\cancel{\mu}}{\cancel{\mu}(\mu-\lambda)} = \frac{\lambda}{\mu-\lambda} = L_s$$

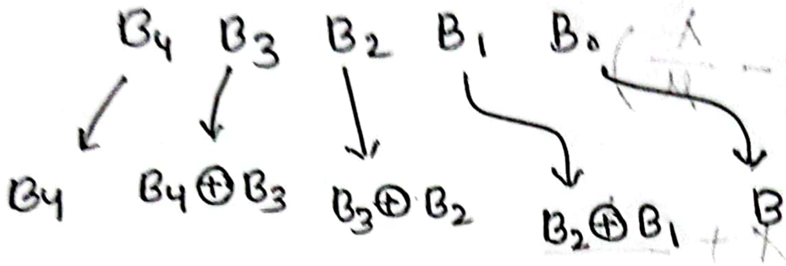
$P_0 = 1 - \rho$

$$= 1 - \frac{\lambda}{\mu} = P_0 = LHS$$

(proved)

Binary to gray code:

0 0	—	0
0 1	—	1
1 0	—	1
1 1	—	0

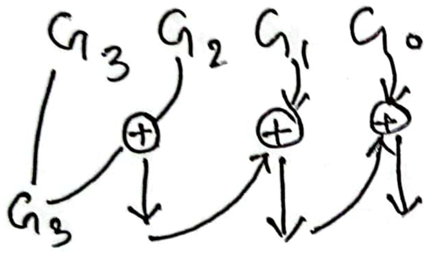


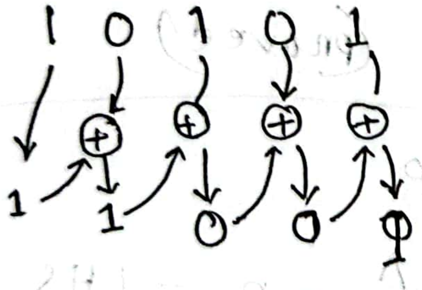
$\bar{A}B + A\bar{B}$

$(1\ 1\ 0\ 0\ 1)_B$

$(1\ 0\ 1\ 0\ 1)_G$

Gray to binary:





$(1\ 1\ 0\ 0\ 1)_B$

* De bruijn graphs and eularin walks:-

de druijn sequence: 0111101100101000