"Mid Question, Solve"

Q) Use Newton Raphson's method, to find root of $x^2-x-2=0$ with $x_0=3$ where root lies between 0.5 and 0.7.

Solution:

Given, equation $x^2-x-2=0$

& $x_0 = 3$

Let,

$f(x) = x^2-x-2 \Rightarrow f(x_0) = f(3) = 4$

$f'(x) = 2x-1 \Rightarrow f'(x_0) = f'(3) = 5$

Using Newton Raphson's formula we get following table:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

Iteration	$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$	$f(x_n)$	$f'(x_n)$
1	2.2	0.64	3.4
2	2.012	0.036	3.024
3	2	0	3
4	2	0	3
5	2	0	3

So, the approximation root is 2.

Q] Write down the code for finding a real root of ~~equivalent~~ the equation $x^3+x-1=0$ using Newton Raphson method with initial value 0.6 & 0.8

Solution

Newton Raphson code for $x^3+x-1=0$ implementing through c++ is given below:

```
# include <bits/stdc++.h>
using namespace std
# define EPSILON 0.001

double func (double x)
{
    return x*x*x + x -1;
}
double derivfunc (double x)
{
    return 3*x*x +1;
}
```

$3 - \frac{y}{5}$

```
void newtonRaphson (double x)
{
    double h = func(x) / derivFunc(x);
    while (abs(h) >= EPSILON)
    {
        h = func(x) / derivFunc(x);
        x = x-h;
    }
    cout << "The value of the root
            is: " << x;
}

int main ()
{
    double x0 = -20;
    newtonRaphson(x0);
    return 0;
}
```

Output: The value of the root is: 0.6823278

0.360421

Q) Use Newton-Raphson Method to find a real root of the equation $3x + \sin x = e^x$ correct up to 5 decimal places.

Solution:

Given equation,

$$3x + \sin x - e^x = 0$$

Let, $f(x) = 3x + \sin x - e^x$, $f'(x) = 3 + \cos x - e^x$

Now, $f(0.3) = -0.1543386$

and $f(0.5) = 0.330704267$

As, $f(0.3) \cdot f(0.5) < 0$, so, there exists at least one root between 0.3 & 0.5.

So, initial approximation value, $x_0 = \frac{0.3 + 0.5}{2}$
$= 0.4$

Applying Newton Raphson's method we get the following table:

Iteration	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
1	0.35982538
2	0.360421576
3	0.360421703
4	0.360421703

From 2, 3 & 4 iteration, we get the approximation root correct up to 5 decimal placess of the given equation is = 0.36042

Fixed Point Iteration Method

2) (a) Find a real root of the equation $e^x - 3x = 0$ by using the fixed point iterative method.

Solution:

Given equation, $e^x - 3x = 0$ . . . (1)

Let, $f(x) = e^x - 3x$

Now, $f(1.5) = -0.018310929$

& $f(1.6) = 0.153032424$

As, $f(1.5).f(1.6) < 0$, so, there exists at least one root between 1.5 & 1.6.

~~Let,~~ So, initial approximation root is, $x_0 = \frac{1.5+1.6}{2}$

$= 1.55$

From (1) we get,

$e^x - 3x = 0$

$\Rightarrow e^x = 3x \Rightarrow \ln e^x = \ln 3x \Rightarrow x \ln e = \ln 3 + \ln x$

$\Rightarrow x = \ln 3 + \ln x$

$= \phi(x)$; whereas, $\phi(x) = \ln 3 + \ln x$

$\therefore \phi'(x) = \frac{1}{x}$

As, $|\phi'(x_0)| = 0.64516129 < 1$

So, applying fixed point iteration method, ($x_{n+1} = \phi(x_n)$; $\phi(x) = \ln 3 + \ln x$) we get the following table:

x_{n+1}	$\phi(x_n)$
x_1	1.53686722
x_2	1.52835836
x_3	1.522806481
x_4	1.51916729
x_5	1.516774638
x_6	1.51519842
x_7	1.51415869
x_8	1.513472253
x_9	1.513018805
x_{10}	1.512719152
x_{11}	1.512521083
x_{12}	1.512390139
x_{13}	1.512303561
x_{14}	1.512246319
x_{15}	1.51220846

From the iteration 14 & 15, we get our approximation root is = 1.5122