"Fixed Point Iterative Method"

FINAL-(2017-18)

1) (b) Find a real root of the equation $x^3-2x^2-4=0$ by using fixed point method with initial value 1 and 4.

Solution:

Given equation,

$$x^3-2x^2-4=0 \quad \ldots\ldots (1)$$

Let, $f(x) = x^3-2x^2-4$

Now, $f(1) = -5$ and $f(4) = 28$

As, $f(1) \cdot f(4) < 0$, so, there exist at least one root between 1 & 4

Let, initial approximate value of root is, $x_0 = \frac{1+4}{2} = 2.5$

From (1) $\Rightarrow$ $x^3 = 2x^2+4$

$$\Rightarrow x = (2x^2+4)^{1/3}$$

$$= \phi(x), \text{ whereas, } \phi(x) = (2x^2+4)^{1/3}$$

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$$\phi'(x) = \frac{1}{3}(2x^2+4)^{-2/3} \cdot 4x$$

$$= \frac{4x}{3}(2x^2+4)^{-2/3}$$

As, $|\phi'(x_0)| = 0.5143 < 1$

So, applying fixed point iteration method we get,

$x_{n+1} = \phi(x_n)$, $\phi(x) = (2x^2+4)^{1/3}$

x_{n+1}	$\phi(x_n)$
x_1	2.545821685
x_2	~~2.54582168~~ 2.569385314
x_3	2.58199928
x_4	2.587727357
x_5	2.590928233
x_6	2.592573394
x_7	2.59348938
x_8	2.593853508

From the iteration $\cancel{x_5, x_6}$, x_7 & x_8 we get our approximate root = 2.594

"Missing Value" (FINAL - (18-19))

Q(a) Why we use operatore in Numerical Method?

Solution:

In numerical methods, operators are used to:

1) Simplify expressions for differentiation, integration etc.
2) Develop numerical formulas, like finite differences.
3) Analyze algorithms for accuracy, stability & convergence.
4) Make complex processes compact & easier to manipulate.

3) (d)

Find the missing term in the following table:

x	1	2	3	4	5	6	7	8
f(x)	1	8	---	64	---	216	343	512

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Solution:

As, here, the number of given ~~value of~~ y values are 6

So, from the table we get 5th ~~value~~ degree polynomial of $f(x)$.

Let, $y_0 = 1$; $y_1 = 8$; $y_2 = ?$; $y_3 = 64$; $y_4 = ?$; $y_5 = 216$; $y_6 = 343$; $y_7 = 512$

Now, $\Delta^6 y_0 = 0$

$\Rightarrow (E-1)^6 y_0 = 0$

$\Rightarrow (E^6 - 6E^5 + 15E^4 - 20E^3 + 15E^2 - 6E + 1) y_0 = 0$

$\Rightarrow y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$

$\Rightarrow 343 - (6 \times 216) + 15y_4 - (20 \times 64) + 15y_2 - (6 \times 8) + 1 = 0$

^6C_r

$^6C_1 = \frac{6!}{5!\,1!}$

$^6C_0$

$^6C_2 = \frac{6.5.4!}{2!\,4!}$

$^6C_3$

$^6C_4 = \frac{6.5.4!}{4!\,2!}$

$^6C_5$

$^6C_6 = \frac{6!}{6!\,0!}$

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$x = \cancel{2.13.31}$
$y = \cancel{67.31}$

$\Rightarrow 15y_4 - 15y_2 = 2280$

$\therefore y_4 + y_2 = 152 \quad \cdots \quad \cdots (i)$

$-7y_4 = -875$
$-3y_2 + 3y_4 = 456$
$3y_2 + 10y_4 = 1331$
$\cancel{13y_4}$
$\Rightarrow \cancel{y_4 = \frac{1787}{13}}$
$\Rightarrow \cancel{y_2 = \frac{1787}{13} - 152}$
$= \frac{-189}{13}$

Again, $\Delta^6 y_1 = 0$

$(E-1)^6 y_1 = 0$

$\Rightarrow y_7 - 6y_6 + 15y_5 - 20y_4 + 15y_3 - 6y_2 + y_1 = 0$

$\Rightarrow 512 - (6\times343) + (15\times216) - 20y_4 + (15\times64) - 6y_2 + 8 = 0$

$\Rightarrow 3y_2 + 10y_4 = 1331 \quad \cdots \quad \cdots (ii)$

From (i) & (ii) we get,

$y_2 = \cancel{-189/13} \; 27$

& $y_4 = \cancel{1787/13} \; 125$

So, our desired table:

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	$\cancel{-189/13}$ 27	64	$\cancel{1787/13}$ 125	216	343	512

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FINAL (19-20)

3] (d) Find the missing term in the following table:

x	10	15	20	25	30	35
f(x)	1	8	---	64	---	216

Solution:

As, here the number of given ~~values of y are~~ values are 4.

So, from the table we get 3rd degree polynomial f(x).

Let, $y_0 = 1$

$y_1 = 8$

$y_2 = ?$

$y_3 = 64$

$y_4 = ?$

$y_5 = 216$

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1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Now, $\Delta^4 y_0 = 0$

$\Rightarrow (E-1)^4 y_0 = 0$

$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1) y_0 = 0$

$\Rightarrow y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$\Rightarrow y_4 - (4\times 64) + 6y_2 - (4\times 8) + 1 = 0$

$\Rightarrow y_4 - 256 + 6y_2 - 32 + 1 = 0$

$\Rightarrow 6y_2 + y_4 = 287 \quad \ldots \ldots (i)$

Again, $\Delta^4 y_1 = 0$

$\Rightarrow (E-1)^4 y_1 = 0$

$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1) y_1 = 0$

$\Rightarrow y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$

$\Rightarrow 216 - 4y_4 + (6\times 64) - 4y_2 + 1 = 0$

$\Rightarrow -4y_2 - 4y_4 = -601 \quad \ldots \ldots (ii)$

$\Rightarrow 4y_2 + 4y_4 = 601$

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From (I) & (II) we get,

$y_2 = \frac{547}{20}$

& $y_4 = \frac{1229}{10}$

$29y_2 + 4y_4 = 1148$
$4y_2 + 4y_4 = 601$
(−) (−) (−)

$20y_2 = 547$
$y_2 = \frac{547}{20}$

$y_4 = 287 - 6y_2$
$= 287 - 6\left(\frac{547}{20}\right)$
$= \frac{2458}{20}$
$= \frac{1229}{10}$

So, the desired table is:

x	10	15	20	25	30	35
$f(x)$	1	8	547/20	64	$\frac{1229}{10}$	216

FINAL (16-17)

3) (b) Consider the following table which contains the data of number of studens (approx) apply from year 2012 to 2019. In addition, it also shows the value in 2016 is missing. Now your task is to find the approximate

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value for the year:

x	2012	2013	2014	2015	2016	2017
f(x)	17.1	13	14	9.6	...	12.4

Solution:

As, the number of y-given values are 5. So, from the table we get 4th degree polynomial f(x).

Let, $y_0 = 17.1$

$y_1 = 13$

$y_2 = 14$

$y_3 = 9.6$

$y_4 = ?$

$y_5 = 12.4$

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5

Now, $\Delta^5 y_0 = 0$

$$\Rightarrow (E-1)^5 y_0 = 0$$

$$\Rightarrow (E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1) y_0 = 0$$

$$\Rightarrow y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$$

$$\Rightarrow 12.4 - 5y_4 + 10(9.6) - 10(14) + 5(13) - (17.1) = 0$$

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$$\Rightarrow -5y_4 = -16.3$$

$$\therefore y_4 = 3.26$$

So, the approximate value of the year 2016 is $= 3.26$

Repeat

(c) Find the missing term in the following table:

x		
f(x)		

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