FINAL-18-19
FINAL-(19-20)
SKIP → 2nd Part

Chapter – 4(A)

1. Using (i) Newton's forward difference formula and (ii) Newton's backward difference formula, derive expressions for the first & second derivatives of a function. Hence also prove that

$$y_0' = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + \cdots\right]y_0$$

$$y_0'' = \frac{1}{h^2}\left[\Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \frac{5}{6}\Delta^5 + \cdots\right]y_0$$

and
$$y_n'' = \frac{1}{h^2}\left[\nabla + \frac{1}{2}\nabla^2 + \frac{1}{3}\nabla^3 + \frac{1}{4}\nabla^4 + \cdots\right]y_n$$

$$y_n'' = \frac{1}{h^2}\left[\nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4 + \cdots\right]y_n$$

Solution:

(i) ~~We have~~ From the Newton's ~~formula~~ forward difference formula we get,

$$y(x) = y_0 + u\Delta y_0 + u(u-1)\frac{\Delta^2 y_0}{2!} + u(u-1)(u-2)\frac{\Delta^3 y_0}{3!} + u(u-1)(u-2)(u-3)\frac{\Delta^4 y_0}{4!} + \cdots\cdots$$

$$= y_0 + u\Delta y_0 + (u^2-u)\frac{\Delta^2 y_0}{2!} + (u^3-3u^2+2u)\frac{\Delta^3 y_0}{3!} ~~+ u^4 - 6u^3 + 11u^2 - 6u~~ + (u^4-6u^3+11u^2-6u)\frac{\Delta^4 y_0}{4!} + \cdots\cdots \quad (1)$$

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where, $u = \frac{x - x_0}{h} = \left(\frac{x}{h} - \frac{x_0}{h}\right)$

$\therefore \frac{du}{dx} = \frac{1}{h}$

Differentiating (1) with respect to x we get,

$$\frac{dy}{dx} = \frac{d}{dx}\left[y_0 + u\Delta y_0 + (u^2-u)\frac{\Delta^2 y_0}{2!} + (u^3-3u^2+2u)\frac{\Delta^3 y_0}{3!} + (u^4-6u^3+11u^2-6u)\frac{\Delta^4 y_0}{4!} + \cdots\right]$$

$$= \frac{d}{du}\left[y_0 + u\Delta y_0 + (u^2-u)\frac{\Delta^2 y_0}{2!} + (u^3-3u^2+2u)\frac{\Delta^3 y_0}{3!} + (u^4-6u^3+11u^2-6u)\frac{\Delta^4 y_0}{4!} + \cdots\right]\frac{du}{dx}$$

$$= \frac{1}{h}\left[\Delta y_0 + (2u-1)\frac{\Delta^2 y_0}{2!} + (3u^2-6u+2)\frac{\Delta^3 y_0}{3!} + (4u^3-18u^2+22u-6)\frac{\Delta^4 y_0}{4!} + \cdots\right] \quad \cdots\cdots (2)$$

$$\frac{d^2y}{dx^2} = \frac{1}{h}\cdot\frac{d}{du}\left[\Delta y_0 + (2u-1)\frac{\Delta^2 y_0}{2!} + (3u^2-6u+2)\frac{\Delta^3 y_0}{3!} + (4u^3-18u^2+22u-6)\frac{\Delta^4 y_0}{4!} + \cdots\right]\frac{du}{dx}$$

$$= \frac{1}{h^2}\left[2\cdot\frac{\Delta^2 y_0}{2!} + (6u-6)\frac{\Delta^3 y_0}{3!} + (12u^2-36u+22u)\frac{\Delta^4 y_0}{4!} + \cdots\right]$$

$$= \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + (12u^2-36u+22u)\frac{\Delta^4 y_0}{4!} + \cdots\right] \quad \cdots\cdots (3)$$

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Hence, (2) and (3) are the 1^{st} and 2^{nd} derivatives respectively of a function with regards to Newton's forward difference formula.

ii) From the Newton's backward difference formula we get,

$$\cancel{y(x) = y_n + u\Delta y_n + u(u+1)\,\Delta\nabla}$$

$$y(x) = y_n + u\nabla y_n + u(u+1)\frac{\nabla^2 y_n}{2!} + u(u+1)(u+2)\frac{\nabla^3 y_n}{3!} + u(u+1)(u+2)(u+3)\frac{\nabla^4 y_n}{4!} + \cdots\cdots$$

$$= y_n + u\nabla y_n + (u^2+u)\frac{\nabla^2 y_n}{2!} + (u^3+3u^2+2u)\frac{\nabla^3 y_n}{3!} + (u^4+6u^3+11u^2+6u)\frac{\nabla^4 y_n}{4!} + \cdots \quad \cdots\cdots (4)$$

where, $u = \frac{x-x_n}{h} = \left(\frac{x}{h} - \frac{x_n}{h}\right)$

$$\frac{du}{dx} = \frac{1}{h}$$

Differentiating (4) with respect to x we get,

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$$\frac{dy}{dx} = \frac{d}{du}\left[y_n + u\nabla y_n + (u^2+u)\frac{\nabla^2 y_n}{2!} + (u^3+3u^2+2u)\frac{\nabla^3 y_n}{3!} + (u^4+6u^3+11u^2+6u)\frac{\nabla^4 y_n}{4!} + \cdots\right]\frac{du}{dx}$$

$$= \frac{1}{h}\left[\nabla y_n + (2u+1)\frac{\nabla^2 y_n}{2!} + (3u^2+6u+2)\frac{\nabla^3 y_n}{3!} + (4u^3+18u^2+22u+6)\frac{\nabla^4 y_n}{4!} + \cdots\right] \quad \cdots\cdots(5)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{h}\,\frac{d}{du}\left[\nabla y_n + (2u+1)\frac{\nabla^2 y_n}{2!} + (3u^2+6u+2)\frac{\nabla^3 y_n}{3!} + (4u^3+18u^2+22u+6)\frac{\nabla^4 y_n}{4!} + \cdots\right]\frac{du}{dx}$$

$$= \frac{1}{h^2}\left[2\cdot\frac{\nabla^2 y_n}{2!} + (6u+6)\frac{\nabla^3 y_n}{3!} + (12u^2+36u+22)\frac{\nabla^4 y_n}{4!} + \cdots\right]$$

$$= \frac{1}{h^2}\left[\nabla^2 y_n + (u+1)\nabla^3 y_n + (12u^2+36u+22)\frac{\nabla^4 y_n}{4!} + \cdots\right] \quad \cdots\cdots(6)$$

Hence, (5) and (6) are the 1st and 2nd derivatives respectively of a function

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with regards to Newton's backward difference formula.

2nd Part:

At the point $x=x_0$ we get from forward formula $u=0$.
Hence on ~~substiatu~~ substituting this value of u we get,

From (2),

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\cdots\right]$$

$$\Rightarrow y_0' = \frac{1}{h}\left[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + \cdots\right] y_0$$

From (3),

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \cdots\right]$$

$$\Rightarrow y_0'' = \frac{1}{h^2}\left[\Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \cdots\right] y_0$$

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Similarly at point the point $x = x_n$ we get from the backward formula $u=0$

Substituting this value of u we get,

From (5),

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\cdots\right]$$

$$\Rightarrow y'_n = \frac{1}{h}\left[\nabla + \frac{1}{2}\nabla^2 + \frac{1}{3}\nabla^3 + \frac{1}{4}\nabla^4 + \cdots\right] y_n$$

From (6),

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\Rightarrow y''_n = \frac{1}{h^2}\left[\nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4 + \cdots\right] y_n$$

Find the 1st, 2nd & 3rd derivatives of (i) Stirling formula (ii) Bessel's formula.

Solution:

(i) We know, Stirling's formula is:

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$(u^3-u)(u^2-4)$
$= u^5-5u^3+4u$

$\cancel{y = y_0 + u\,\nabla y_{-1}}$

$$y = y_0 + u\left(\frac{\Delta y_{-1}+\Delta y_0}{2}\right) + \frac{u^2}{2}\Delta^2 y_{-1} + \frac{u(u^2-1)}{3!}\left(\frac{\Delta^3 y_{-2}+\Delta^3 y_{-1}}{2}\right) + \frac{u^2(u^2-1)}{4!}\Delta^4 y_{-2} + \frac{u(u^2-1)(u^2-4)}{5!}\left(\frac{\Delta^5 y_{-3}+\Delta^5 y_{-2}}{2}\right) + \cdots$$

$$= y_0 + \frac{u}{2}(\Delta y_{-1}+\Delta y_0) + \frac{u^2}{2}\Delta^2 y_{-1} + \frac{u^3-u}{12}(\Delta^3 y_{-2}+\Delta^3 y_{-1}) + \frac{u^4-u^2}{24}\Delta^4 y_{-2} + \frac{u^5-5u^3+4u}{240}(\Delta^5 y_{-3}+\Delta^5 y_{-2}) + \cdots$$

where, $u = \dfrac{x-x_0}{h}$

$\therefore \dfrac{du}{dx} = \dfrac{1}{h}$

Differentiating with respect to x, we get,

$$\frac{dy}{dx} = \frac{d}{du}\left[y_0 + \frac{u}{2}(\Delta y_{-1}+\Delta y_0) + \frac{u^2}{2}\Delta^2 y_{-1} + \frac{u^3-u}{12}(\Delta^3 y_{-2}+\Delta^3 y_{-1}) + \frac{u^4-u^2}{24}\Delta^4 y_{-2} + \frac{u^5-5u^3+4u}{240}(\Delta^5 y_{-3}+\Delta^5 y_{-2}) + \cdots\right]\frac{du}{dx}$$

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$$= \frac{1}{h}\Big[\tfrac{1}{2}(\Delta y_{-1} + \Delta y_0) + u\Delta^2 y_{-1} + \frac{3u^2-1}{12}\cancel{(\Delta^3 y_{-2} + \Delta^3 y_{-1})}(\Delta^3 y_{-2} + \Delta^3 y_{-1}) + \frac{4u^3-2u}{24}\Delta^4 y_{-2} + \frac{5u^4-15u^2+4}{240}(\Delta^5 y_{-3} + \Delta^5 y_{-2}) + \cdots\Big]$$

$$\cancel{\therefore \frac{d^2y}{dx^2} = \frac{1}{h}\frac{d}{du}\Big[\tfrac{1}{2}(\Delta y_-}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{h^2}\Big[\Delta^2 y_{-1} + \frac{u}{2}(\Delta^3 y_{-2} + \Delta^3 y_{-1}) + \frac{6u^2-1}{12}\Delta^4 y_{-2} + \frac{2u^3-3u}{24}(\Delta^5 y_{-3} + \Delta^5 y_{-2}) + \cdots\cdots\Big]$$

$$\therefore \frac{d^3y}{dx^3} = \frac{1}{h^3}\Big[\tfrac{1}{2}(\Delta^3 y_{-2} + \Delta^3 y_{-1}) + \cancel{\frac{3u}{2}\Delta^4 y_{-2}}\ u\Delta^4 y_{-2} + \frac{(2u^2-1)}{8}(\Delta^5 y_{-3} + \Delta^5 y_{-2}) + \cdots\cdots\Big]$$

(ii) We know, Bessel's formula is

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2}\,\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \frac{u(u-\frac{1}{2})(u-1)}{3!}\Delta^3 y_{-1} + \frac{u(u^2-1)(u-2)}{4!}\,\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2} + \cdots\cdots$$

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$$= y_0 + u\Delta y_0 + \frac{u^2-u}{4}\left(\Delta^2 y_{-1} + \Delta^2 y_0\right) + \frac{u^3 - 3u^2/2 + u/2}{6}\Delta^3 y_{-1}$$

$$+ \frac{u^4 - 2u^3 - u^2 + 2u}{48} \cdot \left(\Delta^4 y_{-2} + \Delta^4 y_{-1}\right) + \cdots \cdots$$

where, $u = \frac{x - x_0}{h} \quad \therefore \frac{du}{dx} = \frac{1}{h}$

$$\therefore \frac{dy}{dx} = \frac{d}{du}\Big[y_0 + u\Delta y_0 + \frac{u^2-u}{4}\left(\Delta^2 y_{-1} + \Delta^2 y_0\right) + \frac{u^3 - 3u^2/2 + u/2}{6}\Delta^3 y_{-1} + \frac{u^4 - 2u^3 - u^2 + 2u}{48}\left(\Delta^4 y_{-2} + \Delta^4 y_{-1}\right) + \cdots\Big]\frac{du}{dx}$$

$$= \frac{1}{h}\Big[\Delta y_0 + \frac{2u-1}{4}\left(\Delta^2 y_{-1} + \Delta^2 y_0\right) + \frac{3u^2 - 3u + 1/2}{6}\Delta^3 y_{-1} + \frac{4u^3 - 6u^2 - 2u + 2}{48}\left(\Delta^4 y_{-2} + \Delta^4 y_{-1}\right) + \cdots\Big]$$

$$= \frac{1}{h}\Big[\Delta y_0 + \frac{2u-1}{4}\left(\Delta^2 y_{-1} + \Delta^2 y_0\right) + \frac{6u^2 - 6u + 1}{12}\Delta^3 y_{-1} + \frac{2u^3 - 3u^2 - u + 1}{24}\left(\Delta^4 y_{-2} + \Delta^4 y_{-1}\right) + \cdots\cdots\Big]$$

$$\therefore \frac{d^2y}{dx^2} = \frac{1}{h^2}\Big[\frac{1}{2}\left(\Delta^2 y_{-1} + \Delta^2 y_0\right) + \frac{2u-1}{2}\Delta^3 y_{-1} + \frac{6u^2 - 6u - 1}{24}\left(\Delta^4 y_{-2} + \Delta^4 y_{-1}\right) + \cdots\cdots\Big]$$

$$\therefore \frac{d^3y}{dx^3} = \frac{1}{h^3}\Big[\Delta^3 y_{-1} + \frac{2u-1}{4}\left(\Delta^4 y_{-2} + \Delta^4 y_{-1}\right) + \cdots\Big]$$

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Q. Derive expression of first derivative of Newton's general interpolation formula.

<u>Solution:</u>

We know Newton's general interpolation formula is:

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)(x-x_2)(x-x_3) f(x_0,x_1,x_2,x_3,x_4) + \cdots$$

Differentiating w.r.t x we get,

$$f'(x) = f(x_0,x_1) + \{(x-x_0)+(x-x_1)\} f(x_0,x_1,x_2) + \{(x-x_0)(x-x_1) + (x-x_0)(x-x_2) + (x-x_1)(x-x_2)\} f(x_0,x_1,x_2,x_3) + \{(x-x_0)(x-x_1)(x-x_2) + (x-x_0)(x-x_1)(x-x_3) + (x-x_0)(x-x_2)(x-x_3) + (x-x_1)(x-x_2)(x-x_3)\} f(x_0,x_1,x_2,x_3,x_4) + \cdots$$

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Example-1

Evaluate the first and second derivatives of $y=\sqrt{x}$ function at $x=15$ from the following data:

x	15	17	19	21	23
$y=\sqrt{x}$	3.873	4.123	4.354	4.583	4.796

Solution:

We form a difference table of the given data:

x	$y=\sqrt{x}$	Δy	$\Delta^2 y$	$\Delta^3 y$
15	3.873	0.25	-0.019	0.017
17	4.123	0.231	-0.002	-0.014
19	4.354	0.229	-0.016	
21	4.583	0.213		
23	4.796			

Newton's forward interpolation formula is

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \quad \cdots (1)$$

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where, $u = \frac{x - x_0}{h}$

Here, $x = 15$ and $x_0 = 15$, $h = 2 \Rightarrow u = \frac{15-15}{2} = 0$

Now differentiating (1) w.r.t x and putting $u=0$ we get,

$$\left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{u=0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \cdots\right]$$

Putting the value of certain difference we get,

$$\left(\frac{dy}{dx}\right)_{u=0} = \left(\frac{dy}{dx}\right)_{x=15} = \frac{1}{2}\left[0.25 - \frac{1}{2}(-0.019) + \frac{1}{3}(0.017)\right]$$

$$= 0.13258$$

$$\left(\frac{d^2y}{dx^2}\right)_{u=0} = \left(\frac{d^2y}{dx^2}\right)_{x=15} = \frac{1}{4}\left[(-0.019) - 0.017\right]$$

$$= -0.009$$

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Question Solve (16-17)

8] (a) Find the first, second and third derivatives of the function tabulated below at the point $x = 1.5$

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.000	1.625	24.000	38.875	59.000

Solution:

Form a forward difference table of the given data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	~~$\Delta^4 y$~~	~~$\Delta^5 y$~~
1.5	3.375	3.625	−9	36.75	~~−72~~	~~120~~
2.0	7.000	−5.375	27.75	−35.25	~~48~~	
2.5	1.625	22.375	−7.5	12.75		
3.0	24.000	19.875	5.25			
3.5	38.875	20.125				
4.0	59.000					

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Newton's forward interpolation formula is:

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \cdots \quad (1)$$

where, $u = \frac{x - x_0}{h}$ $\therefore du/dx = \frac{1}{h}$.

Here, $x = 1.5 = x_0$, $h = 0.5$, $u = 0$

Now differentiating (1) w.r.t x ~~we get~~ ~~& putting $u=0$ we get~~.

$$\cancel{\left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \cdots\right]}.$$

$$\cancel{\left(\frac{d^2y}{dx^2}\right)_{u=0} = \frac{1}{h^2}\left[\Delta\right.}$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{3u^2 - 6u + 2}{6}\Delta^3 y_0 + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \cdots\right]$$

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Now $x = 1.5$ and $h = 0.5$, $u = 0$.

$$\left(\frac{dy}{dx}\right)_{x=1.5} = \frac{1}{0.5}\left[\right.$$

$$\left(\frac{dy}{dx}\right)_{x=1.5} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0\right]$$

$$= \frac{1}{(0.5)}\left[3.625 - \frac{1}{2}(-9) + \frac{1}{3}(36.75)\right]$$

$$= 40.75$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=1.5} = \frac{1}{(0.5)^2}\left[(-9) - (36.75)\right]$$

$$= -183$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=1.5} = \frac{1}{(0.5)^3}\left[36.75\right]$$

$$= 294$$

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