Chapter - 3(A)

'Interpolation With Equal Intervals'

Interpolation:

Interpolation is the process of calculating a value between two points on the curve ~~by~~ of a function from the given points which also lie on the same curve.

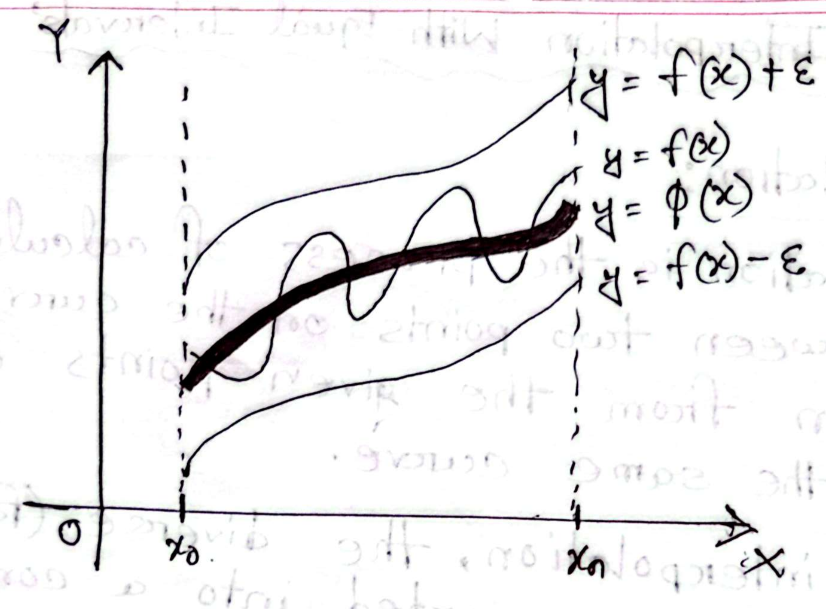
* Using interpolation, the diverse (বিভিন্ন) data can be converted into a concise (সংক্ষিপ্ত) function.
* It is used in geography to predict data points such as noise level, rainfall, elevation (উচ্চতা) & so on.

Extrapolation:

The method of estimating outer ~~or~~ unknown value with the help of given set ~~of~~ of observation is known as extrapolation.

* It involves using existing data to predict future value.
* Estimate ~~an~~ unknown values within a~~s~~ dataset.

* Deduce → তথ্য থেকে যুক্তির সাহায্যে কোন সিদ্ধান্তে উপনীত হওয়া।



Here, $\phi(x)$ is an interpolation function.

Deduce Newton-Gregory's formula for forward interpolation with equal intervals.

OR, Derive the Newton's forward difference interpolation formula.

<u>Solution:</u>

Let $y = f(x)$ denotes a function which takes the values $y_0, y_1, y_2, \dots, y_n$ for the equidistant values $x_0, x_1, x_2, \dots, x_n$ respectively of the independent variable x. Also, let $\phi(x)$

denotes a polynomial of nth degree where $f(x)$ and $\phi(x)$ agree at the tabulated (সারণীযুক্ত) points $(x_0, y_0), (x_1, y_1), (x_2, y_2), \cdots\cdots, (x_n, y_n)$. This polynomial may be written as:

$$\phi(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + \cdots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1}) \quad \cdots\cdots (1)$$

where, $a_0, a_1, a_2, \cdots, a_n$ are the coefficients.

Consider,

$x_1 - x_0 = h$; $x_2 - x_0 = 2h$; $x_3 - x_0 = 3h$ & so on.

$\therefore \phi(x_0) = y_0$

$\Rightarrow y_0 = \phi(x_0)$

$\Rightarrow y_0 = a_0$

$\Rightarrow a_0 = y_0$

$\cancel{\phi(x_1) = a_0 + a_1(x-x_0)}$

$\phi(x_1) = y_1$

$\Rightarrow y_1 = \phi(x_1)$

$\Rightarrow y_1 = a_0 + a_1(x_1 - x_0)$

$\Rightarrow y_1 = y_0 + a_1 h$

$\therefore a_1 = \dfrac{y_1 - y_0}{h} = \dfrac{\Delta y}{h}$

$\phi(x_2) = y_2$

$\Rightarrow y_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$

$= \cancel{y_0 \pm}\ a_0 + a_1 \cdot 2h + a_2 \cdot 2h \cdot h$

$= y_0 + \left(\dfrac{y_1 - y_0}{h}\right) \cdot 2h + a_2 \, 2h^2$

$\Delta y_0 = y_1 - y_0$

$\Delta y_1 = y_2 - y_1$

$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$

$\Delta y_0 = y_1 - y_0$

$(y_2 - y_1) - (y_1 - y_0)$

$\Delta^2 y_0 = (y_2 - y_1) - (y_1 - y_0)$

$= y_0 + 2y_1 - 2y_0 + a_2 2h^2$

$\Rightarrow a_2 = (y_2 - 2y_1 + y_0) / 2h^2$

$\Rightarrow a_2 = \dfrac{y_2 - 2y_1 + y_0}{2.1.h^2}$

$\therefore a_2 = \dfrac{\Delta^2 y_0}{2!h^2}$

$\phi(x_3) = y_3$

$\Rightarrow y_3 = a_0 + a_1(x_3 - x_0) + a_2(x_3 - x_0)(x_3 - x_1) + a_3(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)$

$= y_0 + \left(\dfrac{y_1 - y_0}{h}\right) \cdot 3h + \dfrac{y_2 - 2y_1 + y_0}{2h^2} \cdot 3h \cdot 2h + a_3 \cdot 3h \cdot 2h \cdot h$

$= y_0 + 3y_1 - 3y_0 + 3y_2 - 6y_1 + 3y_0 + 6h^3 a_3$

$\Rightarrow \cancel{a_0} = y_0 - 3y_1 + 3y_2 + 6h^3 a_3$

$\Rightarrow a_3 = \dfrac{y_3 - y_0 + 3y_1 - 3y_2}{3 \cdot 2 \cdot 1\, h^3} = \dfrac{\Delta^3 y_0}{3!h^3}$

Similarly we get,

$$a_4 = \frac{\Delta^4 y_0}{4!\,h^4} \;;\; a_5 = \frac{\Delta^5 y_0}{5!\,h^5}, \dots, a_n = \frac{\Delta^n y_0}{n!\,h^n}$$

Now from (1) we get,

$$\phi(x) = y_0 + \frac{\Delta y}{h}(x-x_0) + \frac{\Delta^2 y}{2!\,h^2}(x-x_0)(x-x_1) + \frac{\Delta^3 y}{3!\,h^3}(x-x_0)(x-x_1)(x-x_2) + \dots\dots + \frac{\Delta^n y_0}{n!\,h^n}(x-x_0)(x-x_1)\dots\dots(x-x_{n-1}) \dots\dots (2)$$

Let, $\frac{x-x_0}{h} = u$ whereas $x = x_0 + hu$ we get,

$$\frac{x-x_1}{h} = \frac{x-(x_0+h)}{h} = \frac{x-x_0}{h} - \frac{h}{h} = \frac{x-x_0}{h} - 1 = u-1$$

$$\boxed{\because x_1 - x_0 = h \;\&\; x-x_0/h = u}$$

$$\frac{x-x_2}{h} = \frac{x-(x_0+2h)}{h} = \frac{x-x_0}{h} - 2 = u-2$$

$$\dots \quad \dots \quad \dots \quad \dots$$

$$\dots \quad \dots \quad \dots$$

$$\frac{x-x_{n-1}}{h} = \frac{x-(x_0+(n-1)h)}{h} = \frac{x-x_0}{h} - (n-1) = u-n+1$$

From (2) we get,

$$\phi(x) = g(u) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots + \frac{u(u-1)(u-2)\dots(u-n+1)}{n!}\Delta^n y_0 \quad \dots\dots (3)$$

Both (2) and (3) are known as Newton-Gregory formula for forward interpolation.

*** Different form the above formula → From Book.

Deduce Newton-Gregory's formula for backword interpolation with equal intervals.

OR, Derive the Newton's backward difference interpolation formula.

<u>Solution:</u> Let $y=f(x)$ denotes a function which takes the values $y_0, y_1, y_2, \dots, y_n$ for the equidistant

values $x_0, x_1, x_2, \ldots, x_n$ respectively of the independents variable x.

Also, let $\phi(x)$ denotes a nth degree polynomial where $f(x)$ and $\phi(x)$ agree at the tabulated points $(x_0,y_0), (x_1,y_1), (x_2,y_2), \ldots, (x_n,y_n)$

This polynomial may be written as:

$$\phi(x) = a_0 + a_1(x-x_n) + a_1(x-x_n)(x-x_{n-1}) + a_3(x-x_n)(x-x_{n-1})(x-x_{n-2}) + \ldots\ldots + a_n(x-x_n)(x-x_{n-1})\ldots(x-x_1) \quad \ldots\ \ldots\ \ldots\ (1)$$

where, $a_0, a_1, \ldots, a_n$ are the coefficients.

Cosider, $x_n - x_{n-1} = h$, $x_n - x_{n-2} = 2h$, $x_n - x_{n-3} = 3h$ and so on.

$\therefore \phi(x_n) = y_n$

$\Rightarrow y_n = \phi(x_n) = a_0$

$\phi(x_{n-1}) = y_{n-1}$

$\Rightarrow y_{n-1} = \phi(x_{n-1})$

$\Rightarrow y_{n-1} = a_0 + a_1(x_{n-1} - x_n)$

$\Rightarrow y_{n-1} = y_n + a_1(-h)$

$\Rightarrow a_1 = \dfrac{y_n - y_{n-1}}{h} = \dfrac{\nabla y_n}{h}$

$$\phi(x_{n-2}) = y_{n-2}$$

$$\Rightarrow y_{n-2} = \phi(x_{n-2})$$

$$\cancel{= a_0 + a_1(x - x_n) + a_2(x - x_n)(x - x_{n-1})}$$

$$= a_0 + a_1(x_{n-2} - x_n) + a_2(x_{n-2} - x_n)(x_{n-2} - x_{n-1})$$

$$= y_n + \frac{y_n - y_{n-1}}{h}(-2h) + a_2(-2h)(-h)$$

$$= y_n - 2y_n + 2y_{n-1} + a_2 2h^2$$

$$\Rightarrow a_2 = \frac{\cancel{y_n}\; y_n - 2y_{n-1} + y_{n-2}}{2.1.h^2}$$

$$\Rightarrow a_2 = \frac{\nabla^2 y_n}{2!h^2}$$

$$\phi(x_{n-3}) = y_{n-3}$$

$$\Rightarrow y_{n-3} = a_0 + a_1(x_{n-3} - x_n) + a_2(x_{n-3} - x_n)(x_{n-3} - x_{n-1})$$
$$+ a_3(x_{n-3} - x_n)(x_{n-3} - x_{n-1})(x_{n-3} - x_{n-2})$$

$$\Rightarrow y_{n-3} = y_n + \frac{y_n - y_{n-1}}{h}(-3h) + \cancel{a_2(-3h)(-2h)}$$
$$\frac{y_n - 2y_{n-1} + y_{n-2}}{2h^2}(-3h)(-2h) + a_3(-3h)(-2h)(-h)$$

$$\Rightarrow y_{n-3} = y_n - 3y_n + 3y_{n-1} + 3y_n - 6y_{n-1} + 3y_{n-2}$$

$$\cancel{\phi} - a_3 6h^3$$

$$= \cancel{y}y_n - 3y_{n-1} + 3y_{n-2} \;\cancel{\phi} - a_3 6h^3$$

$$\Rightarrow a_3 = \frac{y_n - 3y_{n-1} + 3y_{n-2} - y_{n-3}}{3.2.1\,h^3}$$

$$= \frac{\nabla^3 y_n}{3!\,h^3}$$

Similarly we get,

$$a_4 = \frac{\nabla^4 y_n}{4!\,h^4}\;;\; a_5 = \frac{\nabla^5 y_n}{5!\,h^5}\;;\;\ldots\;;\; a_n = \frac{\nabla^n y_n}{n!\,h^n}$$

From (1) we get,

$$\phi(x) = y_n + \frac{\nabla y_n}{h}(x-x_n) + \frac{\nabla^2 y_n}{2!\,h^2}(x-x_n)(x-x_{n-1})$$

$$+ \frac{\nabla^3 y_n}{3!\,h^3}(x-x_n)(x-x_{n-1})(x-x_{n-2}) + \cdots\cdots$$

$$+ \frac{\nabla^n y_n}{n!\,h^n}(x-x_n)(x-x_{n-1})\cdots(x-x_1)$$

$$\cdots\cdots\;\cdots(2)$$

Let, $\frac{x-x_n}{h} = u$; whereas $x = x_n + hu$

$$\frac{x-x_{n-1}}{h} = \frac{x-(x_n-h)}{h} = \frac{x-x_n}{h} + 1 = u+1$$

$$\frac{x-x_{n-2}}{h} = \frac{x-(x_n-2h)}{h} = \frac{x-x_n}{h} + 2 = u+2$$

$$\cdots \qquad \cdots \qquad \cdots \qquad \cdots$$

$$\cdots \qquad \cdots \qquad \cdots \qquad \cdots$$

$$\frac{x-x_1}{h} = \frac{x-(x_n-(n-1)h)}{h} = \frac{x-x_n}{h} + (n-1)$$
$$= u + (n-1)$$

From (2) we get

$$\phi(x) = \psi(u) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots + \frac{u(u+1)(u+2)\cdots(u+n-1)}{n!}\nabla^n y_n \cdots (3)$$

Here, both (2) & (3) are known as Newton Gregory formula for backward interpolation.

Problem – 1

Using Newton's formula for interpolation, estimate the population for the year 1905.

Year	1891	1901	1911	1921	1931
Population	98,752	1,32,285	1,68,076	1,95,690	2,46,050

Solution:

First we form a difference table of the given data:

Year (x)	Population (y)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	98,752	33533	2,258	-10,435	41,358
1901	1,32,285	35,791	-8,177	30,923	
1911	1,68,076	27,614	22,746		
1921	1,95,690	50,360			
1931	2,46,050				

From the Newton-Gregory formula for forward interpolation we get,

$$y(x) = y_0 + u\Delta y_0 + \cancel{u(u+1)}\frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \dots \dots \dots (1)$$

where, $u = \frac{x - x_0}{h}$

Here,

$x = 1905,\ x_0 = 1891$ & $h = 10$

$\therefore u = \dfrac{1905-1891}{10}$

$= 1.4$

and, $y_0 = 98,752$

$\Delta y_0 = 33,533$

$\Delta^2 y_0 = 2,258$

$\Delta^3 y_0 = -10,435$

$\Delta^4 y_0 = 41,358$

From (1) we get,

$$y(1905) = 98,752 + (1.4)(33,533) + \frac{(1.4)(1.4-1)}{2!}(2258) + \frac{(1.4)(1.4-1)(1.4-2)}{3!}(-10,435) + \frac{(1.4)(1.4-1)(1.4-2)(1.4-3)}{4!}(41,358)$$

$= 98,752 + 46,946.2 + 632.24 + 584.36 + 926.42$

$= 1,47,841$ (almost)

$\therefore$ Population of the 1905 $= 1,47,841$ (almost)

Problem-2

Find the cubic polynomial which takes the following values; $y(0)=1, y(1)=0, y(2)=1$ and $y(3)=10$; Hence obtain $y(4)$.

Solution:

First we form a difference table with the given data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1	-1	2	6
1	0	1	8	
2	1	9		
3	10			

From Newton's forward difference ~~interpolati~~ interpolation we get,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \cdots (1)$$

where, $u = \frac{x - x_0}{h}$

Here, $x_0 = 0, h = 1$

$\therefore u = \frac{x-0}{1} = x$

$y_0 = 1$
$\Delta y_0 = -1$
$\Delta^2 y_0 = 2$
$\Delta^3 y_0 = 6$

From (1) we get,

$$y(x) = 1 + x(-1) + \frac{x(x-1)}{2!}(2) + \frac{x(x-1)(x-2)}{3!}6$$

$$= 1 - x + x^2 - x + x(x^2 - 3x + 2)$$

$$= x^2 - 2x + 1 + x^3 - 3x^2 + 2x$$

$= x^3 - 2x^2 + 1$; which is the cubic polynomial.

$$\therefore \cancel{f(4)}\, y(4) = 4^3 - 2(4)^2 + 1$$

$$= 64 - 32 + 1$$

$$= 33$$

Problem -03

The table below gives the value of $\tan x$ for $0.10 \le x \le 0.30$

x	0.10	0.15	0.20	0.25	0.30
$y = \tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

Find $\tan(0.26)$ by Newton-Gregory interpolation formula and compare the result obtained with exact value of $\tan(0.26)$.

Solution:

First we form a backward difference table:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0.10	0.1003				
0.15	0.1511	0.0508			
0.20	0.2027	0.0516	0.0008		
0.25	0.2553	0.0526	0.001	0.0002	
0.30	0.3093	0.054	0.0014	0.0004	0.0002

From the Newton's backward interpolation we get,

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_n + \cdots \quad (1)$$

where, $u = \frac{x - x_n}{h}$

Here, $x = 0.26$, $x_n = 0.30$, $h = 0.05$

$$\therefore u = \frac{0.26 - 0.30}{0.05} = -0.8$$

~~From (1) we get,~~

~~$y(0.26) = \tan(0.26) + 0.0$~~

~~$x_n = 0.30$~~

$y_n = 0.3093$

$\nabla y_n = 0.0540$

$\nabla^2 y_n = 0.0014$

$\nabla^3 y_n = 0.0004$

$\nabla^4 y_n = 0.0002$

From (1) we get,

$y(0.26) = \tan(0.26)$

$$= 0.3093 + (-0.8)(0.0540) + \frac{(-0.8)(-0.8+1)}{2!}(0.0014)$$

$$+ \frac{(-0.8)(-0.8+1)(-0.8+2)}{3!}(0.0004) +$$

$$\frac{(-0.8)(-0.8+1)(-0.8+2)(-0.8+3)}{4!}(0.0002)$$

$$\cancel{= 0.3093} - 0.0432 \cancel{- 0.000112}$$

$$= 0.3093 - 0.0432 - (1.12\times10^{-4}) - (1.28\times10^{-5}) - (3.52\times10^{-6})$$

$$= 0.266$$

$\therefore \tan(0.26) = 0.2662$

exact ~~Real~~ value $= 0.26602154$

Hence, the comparison of the computed & exact values give the error ~~2.1541×10^{-5}~~ 1.78459×10^{-4} which is very small.

Problem-04

Given tabulated values of polynomial; obtain the polynomial & find $f(7.5)$.

x	1	2	3	4	5	6	7	8
$f(x)$	1	8	27	64	125	216	343	512

Solution:

First, we form a backward difference table:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1	1				
2	8	7			
3	27	19	12		
4	64	37	18	6	
5	125	61	24	6	0
6	216	91	30	6	0
7	343	127	36	6	0
8	512	169	42	6	0

From the Newton's backward interpolation formula we get,

$$y(x) = \cancel{f(x)}\, y_n + u\nabla y + \frac{u(u+1)}{2!}\nabla^2 y + \frac{u(u+1)(u+2)}{3!}\nabla^3 y$$

$$+ \cancel{u(u+}\ \ldots\ldots \quad (1)$$

where, $u = \frac{x - x_n}{h}$ $[x_n = 8, h = 1]$

$= \frac{x-8}{1}$

$\therefore u = x - 8$

~~From~~ $y_n = 512$

$\nabla y_n = 169$

$\nabla^2 y_n = 42$

$\nabla^3 y_n = 6$

From (1) we get,

$$y(x) = f(x) = 512 + (x-8)169 + \frac{(x-8)(x-8+1)}{2!}42 + \frac{(x-8)(x-8+1)(x-8+2)}{3!}6$$

$$= 512 + 169x - 1352 + (x-8)(x-7)21 + (x-8)(x-7)(x-6)$$

$$= 512 + 169x - 1352 + (x^2-15x+56)21 + (x^2-15x+56)(x-6)$$

$$= 512 + 169x - 1352 + 21x^2 - 315x + 1176 + x^3 - 15x^2 + 56x - 6x^2 + 90x - 336$$

$\therefore \cancel{f(x) = x^3}$

$\therefore f(x) = x^3$

Thus the required polynomial be $f(x)=x^3$

$\therefore \cancel{y(x}\cdot f(7.5) = (7.5)^3$

$\Rightarrow 421.875$

Problem-05

The no. of candidates who secured marks between certain limits in an examination as follows Estimate the no. of candidates getting marks less than 70:

No. of candidates	Marks
41	0-19
62	20-39
65	40-59
50	60-79
17	80-99

Solution:

First, we form a total difference table:

Marks	No. of Candidates
~~Not more~~ Less than 20	41
Less than 40	$41+62=103$
Less than 60	$103+65=168$
Less than 80	$168+50=218$
Less than 100	$218+17=235$

Now we form a difference table for above data:

Marks less than x	Candidates number y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
20	41				
40	103	62			
60	168	65	3		
80	218	50	-15	-18	
100	235	17	-33	-18	0

From the Newton's backward interpolation formula we get,

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots \cdots \quad (1)$$

where, $u = \frac{x - x_n}{h}$

Here, $x = 70$, $x_n = 100$ & $h = 20$; $u = \frac{70-100}{20} = -1.5$

$y_n = 235$

$\nabla y_n = 17$

$\nabla^2 y_n = -33$

$\nabla^3 y_n = -18$

From (1) we get,

$$y(70) = 235 + (-1.5)(17) + \frac{(-1.5)(-1.5+1)}{2!}(-33) + \frac{(1.5)(-1.5+1)(-1.5+2)}{3!}(-18)$$

$$= 235 - 25.5 - 12.375 - 1.125$$

$$= 196$$

$\therefore$ No. of candidates less than 70 marks

$= 196.$

Problem-5

The population of a town in the decennial census was as given below:

Year (x)	1891	1901	1911	1921	1931
Population: y (thousands)	46	66	81	93	101

Construct difference tables and estimate the population for the year (i) 1895 (ii) 1925

Solution:

To find the population for the year 1895, Newton's forward formula is more suitable. Similarly for the year 1925, Newton's backward formula is more suitable.

Now, we construct a difference table:

x	y	Δy / ∇y	$\Delta^2 y$ / $\nabla^2 y$	$\Delta^3 y$ / $\nabla^3 y$	$\Delta^4 y$ / $\nabla^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

x	y	Δy / ∇y	$\Delta^2 y$ / $\nabla^2 y$	$\Delta^3 y$ / $\nabla^3 y$	$\Delta^4 y$ / $\nabla^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

Forward

Backward

1) From the Newton's forward interpolation we get,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \dots \quad \dots (1)$$

where, $u = \frac{x - x_0}{h}$

Here, $x = 1895$, $x_0 = 1891$, $h = 10$

$\therefore u = \frac{1895 - 1891}{10}$

$= \frac{4}{10} = 0.4$

$$\therefore y(1895) = 46 + (0.4)(20) + \frac{(0.4)(0.4-1)}{2!}(-5) + \frac{(0.4)(0.4-1)(0.4-2)}{3!}(2) + \frac{(0.4)(0.4-1)(0.4-2)(0.4-3)}{4!}(-3)$$

$= 54.85$

$-0.8064 \quad 0.1608 \quad 0.056 \quad 0.48$

$\therefore$ The estimated population of the year 1895 is 54.85 thousands.

ii) From the Newton's backward interpolation we get,

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 y_n + \cdots \cdots (2)$$

~~Here~~, where, $u = \frac{x - x_n}{h}$

Here, $x = 1925$, $x_n = 1931$, $h = 10$

$$\therefore u = \frac{1925 - 1931}{10} = -0.6$$

From (2) we get,

$$y(1925) = 101 + (-0.6)(8) + \frac{(-0.6)(-0.6+1)}{2!}(-4) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(-1) + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4!}(3)$$

$= 96.68$

$\therefore$ The estimated population of the year 1925 is 96.68 thousands.

~~Prob~~

"Question Solve"

Final (18-19)

2) (d) Construct a finite difference table for the function $f(x) = x^3 - 3x + 4$ in the interval $2 \le x \le 3$ with $h = 0.1$

Solution:

~~Accor~~ We form a finite difference table based on the given data:

x	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
2	6	0.961	0.126	6×10^{-3}	0
2.1	6.961	1.087	0.132	6×10^{-3}	0
2.2	8.048	1.219	0.138	6×10^{-3}	0
2.3	9.267	1.357	0.144	6×10^{-3}	0
2.4	10.624	1.501	0.15	6×10^{-3}	0
2.5	12.125	1.651	0.156	6×10^{-3}	0
2.6	13.776	1.807	0.162	6×10^{-3}	0
2.7	15.583	1.969	0.168	6×10^{-3}	
2.8	17.552	2.137	0.174		
2.9	19.689	2.311			
3	22				

3] (c) Find a cubic polynomial in x which takes on the values $-3, 3, 11, 27, 57$ and 107 when $x = 0, 1, 2, 3, 4$ and 5 respectively.

Solution:

First we form a difference table for above data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	-3	6	2	6	0
1	3	8	8	6	0
2	11	16	14	6	
3	27	30	20		
4	57	50			
5	107				

From the Newton's forward interpolation we get,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \cdots \text{---} (1)$$

where, $u = \frac{x - x_0}{h}$

here, $x_0 = 0$, $h = 1$; $u = \frac{x-0}{1} = x$

From (1) we get,

$$y(x) = -3 + x(2) + \frac{x(x-1)}{2}(2) + \frac{x(x-1)(x-2)}{6}(6)$$

$$= -3 + 2x + x^2 - x + x(x^2 - 3x + 2)$$

$$= -3 + x + x^2 + x^3 - 3x^2 + 2x$$

$$= x^3 - 2x^2 + 3x - 3$$; this is our desired cubic polynomial.

4| (b) Write down Newton-Gregory's formula for forward and backward interpolation with equal intervals.

Solution: Previous note

FINAL-(16-17)

4| (b) Write down the Newton-Gregory's formula for forward and backward interpolation with equal intervals $\rightarrow$ Repeat

(c) For the following table find the value of $f(29)$ along with the name of interpolation formula with justification.

x	20	22	24	26	28	30
$f(x)$	165	166	168	169	170	171

Solution:

~~fi~~ To find the value of $f(29)$, Newton's backward formula is more suitable.:

~~Fist~~ First, we form a difference table for above data,

x	y	$\Delta\nabla y$
20	165	1
22	166	2
24	168	1
26	169	1
28	170	1
30	171	

x	$y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
20	165					
22	166	1				
24	168	2	1			
26	169	1	-1	-2		
28	170	1	0	1	3	
30	171	1	0	0	-1	-4

-3.29125

$0.103\cancel{375}$
0.0390625

From the Newton's backward interpolation formula we get,

$\cancel{y\ f(x) =}$

$$y(x) = f(x)$$
$$= y_n + u \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n$$
$$+ \frac{u(u+1)(u+2)(u+3)}{4!} \nabla^4 y_n + \cdots \cdots \quad (1)$$

whereas, $u = \frac{x - x_n}{h}$

Here, $x = 29$, $x_n = 30$, $h = 2$

$$\therefore u = \frac{29-30}{2} = -\frac{1}{2} = -0.5$$

From (1) we get,

$$\cancel{y}\ f(29) = 171 + (-0.5)(1) + \frac{(-0.5)(-0.5+1)}{2}(0)$$
$$+ \frac{(-0.5)(0.5+1)(-0.5+2)}{3!}(0)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(\cancel{1})$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{5!}(-4)$$
$$\cancel{+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)}{6!}(-4)}$$

$= 170.65$ ~~(almost)~~ (approx)

FINAL (17-18)

~~2](d) The figure included a table depicts the population of the USA. Use language interpolation~~

4] (b) Write down the Newton-Gregory's formula for forward and backward interpolation with equal intervals. (Repeat)

Final-(19-20)

3] (b) Population was recorded as follows in a village as follows. Estimate the population for the year 2001.

Year (X)	1941	1951	1961	1971	1981	1991
Population (Y)	2500	2800	3200	3700	4350	5225

Solution:

~~Fist~~ First, we form a difference table for above data:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	2500	300	100	0	50	25
1951	2800	400	100	50	25	
1961	3200	500	150	75		
1971	3700	650	225			
1981	4350	875				
1991	5225					

From the Newton's Forward interpolation we get,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y_0 \quad \text{-----} (1)$$

where, $u = \frac{x - x_0}{h}$

Here, $x = 2001$, $x_0 = 1941$, $h = 10$

$$u = \frac{2001-1941}{10} = 6$$

From (1) we get,

$$y(2001) = 2500 + 6(300) + \frac{6\times(5)}{2}(100) + \frac{6.5.4}{6}(0) + \frac{6.5.4.3}{24}(50) + \frac{6.5.4.3.2}{120}(25)$$

$$= 2500 + 1800 + 1500 + 750 + 150$$

$$= 6,700$$

The population for the year 2001 is $= 6,700$

Problem

From the following table, estimate the number of students who obtain marks between 40 to 45.

Marks	Number of students
30-40	31
40-50	42
50-60	51
60-70	35
70-80	31

Solution:

Let $f(x)$ be the number of students who obtain marks above 30 but less than x.

x	40	50	60	70	80
$f(x)$	31	$31+42 = 73$	$73+51 = 124$	$124+35 = 159$	$159+31 = 190$

To estimate the number of the students who obtained marks between 40 to 45, we are find to $f(45) - f(40)$ i.e

$$f(45) - 31$$

To find $f(45)$ we form a difference table:

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31	42	9	-25	37
50	73	51	-16	12	
60	124	35	-4		
70	159	31			
80	190				

From, Newton's forward formula we get,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \dots \quad (1)$$

whereas, $u = \frac{x-x_0}{h}$

$x = 45,\ x_0 = 40,\ h = 10$

$u = \frac{45-40}{10} = 0.5$

$$\therefore f(45) = 31 + (0.5)(42) + \frac{(0.5)(0.5-1)}{2}(9) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(-25) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(37)$$

$= 47.8671875$

~~$\simeq 48$~~ $= 48$ (approx)

Hence the number of students who obtained marks between 40 to 45 is

$f(45)-31$

$= 48-31$

$= 17$

Chapter- 3(B)

"Interpolation with unequal Intervals"

Divided difference:

Let $f(x_0), f(x_1), f(x_2), \dots, f(x_n)$ are the functional values corresponding to any values (may or may not equidistant) $x_0, x_1, x_2, \dots, x_n$ of the argument. Then the divided difference of $f(x)$ in ascending order are defined as follows:

First order divided differences:

$$f(x_0,x_1) = \frac{f(x_0)-f(x_1)}{x_0-x_1} = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

$$f(x_1,x_2) = \frac{f(x_1)-f(x_2)}{x_1-x_2} = \frac{f(x_2)-f(x_1)}{x_2-x_1} \text{ etc}$$

Second order divided differences:

$$f(x_0,x_1,x_2) = \frac{f(x_0,x_1)-f(x_1,x_2)}{x_0-x_2}; \quad f(x_1,x_2,x_3) = \frac{f(x_1,x_2)-f(x_2,x_3)}{x_1-x_3} \text{ etc.}$$