$$\Rightarrow \frac{1}{2}\iint |v|^2\, dv = \frac{1}{2}\int_S (\omega \times v)\cdot n\, ds + \int_v v\cdot\omega\, dv$$

proved

(15)

volume integral, $= \int_v \nabla F \cdot dv$

$$= \int_v \left[\frac{d}{dx}(2x\cancel{y}+z) + \frac{d}{dy}(y)^2 + \frac{d}{dz}\{-(x+3y)\} \right] dv$$

$x=0, y=0, z=0$

$2x+2y+z=6$

$$= \int_v (2y + 2y + 0)\, dv$$

$$= \int_v 4y\, dz\, dy\, dx$$

$$= \int_{x=0}^{6/2} \int_{y=0}^{\frac{6-2x}{2}} \int_{z=0}^{6-2x-2y} 4y\, dz\, dy\, dx$$

$$= 4\int_{x=0}^{3} \int_{y=0}^{3-x} y\, [z]_0^{6-2x-2y}\, dy\, dx$$

$$= 8 \int^{3-x} y$$

$$= 8\int_{x=0}^{3}\int_{y=0}^{3-x} y\,(3-x-y)\,dy\,dx$$

$$= \int_{x=0}^{3} \cancel{\int_{y=0}} \frac{4}{3}\,(3-x)^{3}\,dx$$

$$= \frac{4}{3}\int_{x=0}^{3} (3-x)^{3}\,dx$$

$$= \frac{4}{3}\left[\left(-\frac{(3-x)^{4}}{4}\right)\right]_{0}^{3}$$

$$= \frac{4}{3}\left[\cancel{0}\;0 + \frac{3^{4}}{4}\right]$$

$$= \frac{4}{3} \times \frac{3^{4}}{4}$$

$$= 3^{3}$$

$$\int_{S} \nabla\cdot F\,dv = 27$$

calculate the surface integral;

the total surface S is composed of four faces. S1 (slanted), S_2 ($x=0$), S_3 ($y=0$) & S_4 ($z=0$)

now,

Face S2, plane $x=0$,

$$\hat{n}_2 = -i$$

$$\therefore F\cdot\hat{n}_2 = \{(0+z)i + y^2 j - (3y)k\}\cdot(-i)$$

$$= -z$$

$$\therefore \iint_{S_2} F\cdot\hat{n}\, ds = \int_{z=0}^{6}\int_{y=0}^{\frac{6-z}{2}} (-z)\, dy\, dz$$

$$= -\int_{z=0}^{6} [zy]_0^{(6-z)/2}\, dz$$

$$= -\int_0^6 z(6-z)/2\, dz$$

$$= -\frac{1}{2}\int_0^6 (6z - z^2)\, dz$$

$$= -\frac{1}{2}\left[6\frac{z^2}{2} - \frac{z^3}{3}\right]_0^6$$

$$= -\frac{1}{2}\left[3z^2 - \frac{z^3}{3}\right]_0^6$$

$$= -\frac{1}{2}\left[3(6)^2 - \frac{(6)^3}{3}\right]$$

$$= -\frac{1}{2}(108 - 72)$$

$$= -18$$

Face S_3, plane $y=0$ and $\hat{n}_3 = -j$

$\therefore$ $F.\hat{n}_3 = (zi - x k)(-j) = 0$

$\therefore$ $\iint_{S_3} F.\hat{n}_3 \, ds = 0$

Now, Face S_4, plane $z=0$, $\hat{n}_4 = -k$

$\therefore$ $F.\hat{n}_4 = \{2xyi + y^2 j - (x+3y)k\}.(-k)$

$= x+3y$

$$\therefore \iint_{S_4} F\cdot \hat{n}_4 \, ds = \int_{x=0}^{3}\int_{y=0}^{3-x} (x+3y)\, dy\, dx$$

$dx\,dy$

$$= \int_{x=0}^{3} \left[xy + 3\frac{y^2}{2} \right]_0^{3-x} dx$$

$$= \int_{x=0}^{3} \left[x(3-x) + 3/2\,(3-x)^2 \right]_0 dx$$

$$= \int_{x=0}^{3} \left[3x - x^2 + \frac{3}{2}(9 - 6x + x^2) \right] dx$$

$$= \left[3\cdot x^2/2 - x^3/3 + \frac{3}{2}\left(9x - 3x^2 + x^3/3\right) \right]_0^3$$

$$= \left[3\,\frac{3^2}{2} - \frac{(3)^3}{3} + \frac{3}{2}\left(9\times 3 - 3(3)^2 + (3)^3/3\right) \right] - \boxed{0+0}$$

$$= \frac{27}{2} - 9 + \frac{3}{2}(27 - 27 + 9)$$

$$= \frac{27-18}{2} + \frac{3}{2}\times 9$$

$$= \int_{x=0}^{3} \Big[2x(3-x)^2 + \frac{2}{3}(3-x)^3 - 5x(3-x) - \frac{7}{2}(3-x)^2 + 12(3-x) \Big]\, dx$$

$$= \int_{x=0}^{3} \Big[2x(9-6x+x^2) + \frac{2}{3}(9 - 3\cdot 3^2\cdot x + 3\cdot 3\cdot x^2 - x^3) - 15x + 5x^2 - \frac{7}{2}(9-6x+x^2) + 36 - 12x \Big]\, dx$$

$$= \cancel{2x^2/2} \int_{x=0}^{3} \Big[18x - 12x^2 + 2x^3 + \frac{2}{3}(9 - 27x + 9x^2 - x^3) - 15x + 5x^2 - \frac{7}{2}(9-6x+x^2) + 36 - 12x \Big]\, dx$$

$$9x^2 - 9x$$

$$= \Big[18\frac{x^2}{2} - 12\frac{x^3}{3} + 2x^4/4 + \frac{2}{3}\Big(9 - 27\frac{x^2}{2} + 9\frac{x^3}{3} - x^4/4\Big) - 15x^2/2 + 5x^3/3 - \frac{7}{2}\Big(9x - 6x^2/2 + x^3/3$$

$$+36x - 12\,\frac{x^2}{2}\Big]_0^3$$

11

$$= 27$$

$\therefore$ total surface integral value

$$\iint_S F\cdot\hat{n}\,ds = \cancel{27} \iint_{S_1} + \iint_{S_2} + \iint_{S_3} + \iint_{S_4}$$

$$= 27 + (-18) + 0 + 18$$

$$= 27$$

$$= \int \nabla\cdot F\,dv$$

Hence the theorem is verified

lecture - Page (5)

01)

$\vec{a} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 4\hat{k}$

let θ be the angle btw them,

then we know

$$\vec{a} \cdot \vec{b} = ab \cos\theta$$

$$\Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{ab}$$

$$= \frac{6 + 8 - 24}{\sqrt{3^2 + 2^2 + 6^2}\sqrt{2^2 + 4^2 + (-4)^2}}$$

$$= -\frac{5}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{5}{21}\right)$$

$$\therefore \theta = 103.774^\circ$$ Ans:

Q2

let, $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$.

$\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$

$a =$

$\therefore$ ~~schaar~~ scalar product of $\vec{a}, \vec{b}$

is ~~aa~~ $= (2\hat{i} + 3\hat{j} + \hat{k}) \cdot (3\hat{i} + \hat{j} - 2\hat{k})$

$= 6 + 3 - 2$

$= 7$

we know,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$\frac{a.b}{|a||b|}$ $\rightarrow$ $|a| =$

$|\vec{a}|\,|\vec{b}|$

$$= \frac{7}{\left(\sqrt{2^2 + 3^2 + 1^2}\right)\left(\sqrt{3^2 + 1^2 + (-2)^2}\right)}$$

$= 0.5$

$\therefore \theta = \cos^{-1}(0.5)$

$\therefore \theta = 60^\circ$

Ans.

04 Find the sine of the angle between the vector $\hat{i}+2\hat{j}+3\hat{k}$ & $3\hat{i}-4\hat{j}+2\hat{k}$

Let θ be the angle between them,

we know

$$\vec{a}\times\vec{b} = |\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$$

or, $$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}$$

$$= \frac{9\sqrt{5}}{\sqrt{14}\times\sqrt{29}}$$

$\therefore\ \theta = 87.155^\circ$ Ans.

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & -4 & 2 \end{vmatrix}$$

$$= \hat{i}(4+12) - \hat{j}(2-9) + \hat{k}(-4-6)$$

$$= 16\hat{i} + 7\hat{j} - 10\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{16^2+7^2+(-10)^2}$$

$$= 9\sqrt{5}$$

$$|\vec{a}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{3^2+(-4)^2+2^2} = \sqrt{29}$$

(5) Find the unit vector perpendicular to each $2\hat{i}+\hat{j}+\hat{k}$ & $\hat{i}-\hat{j}+2\hat{k}$

Soln

Let $v = \vec{a} \times \vec{b}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$= \hat{i}(2+1) - \hat{j}(4-1) + \hat{k}(-2-1)$

$= 3\hat{i} - 3\hat{j} - 3\hat{k}$

$\therefore$ ~~unit~~ vector

$$\hat{\eta} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$$= \frac{3\hat{i}-3\hat{j}-3\hat{k}}{\sqrt{3^2+(-3)^2+(-3)^2}}$$

$$= \frac{1}{3\sqrt{3}}(3\hat{i}-3\hat{j}-3\hat{k})$$

$$= \frac{1}{\sqrt{3}}\hat{i} - \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$$

Ans

(6) Find angle, $3\hat{i}-6\hat{j}+2\hat{k}$, makes with the co-ordinate axes.

Soln Let $\hat{a} = 3\hat{i}-6\hat{j}+2\hat{k}$ & α, β, γ be the angles which a makes with the +ve direction of x, y, z axes ~~repecti~~ respectively

$|a| = \sqrt{3^2 + (-6)^2 + 2^2} = 7$ $\hat{i}\cdot\hat{i} = 1$

$\hat{a}\cdot\hat{i} = a\cos\alpha$

$\Rightarrow \hat{a}\cdot\hat{i} = 7\cos\alpha$

$\Rightarrow (3\hat{i} - 6\hat{j} + 2\hat{k})\cdot\hat{i} = 7\cos\alpha$

$\Rightarrow 7\cos\alpha = 3$

$\Rightarrow \cos\alpha = \frac{3}{7}$

$\therefore \alpha = \cos^{-1}\left(\frac{3}{7}\right)$

Ans.

similarly

$\hat{a}\cdot\hat{j} = a\cos\beta$

$\Rightarrow -6 = 7\cos\beta$

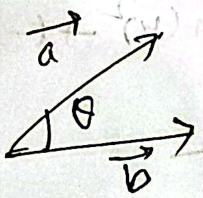
$\therefore \beta = \cos^{-1}\left(\frac{-6}{7}\right)$ A

$\hat{a}\cdot\hat{k} = a\cos\gamma$

$\Rightarrow 2 = 7\cos\gamma$

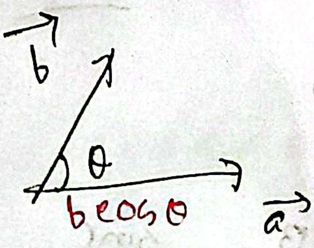
$\therefore \gamma = \cos^{-1}\left(\frac{2}{7}\right)$ A

$\vec{a}\cdot\vec{b} = ab\cos\theta$



$$b\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$b\cos\theta$



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projection

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$b\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$$

উপাংশ

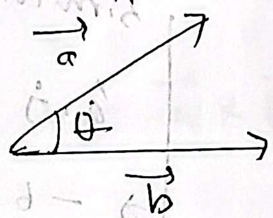
component

$$b\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\,\hat{a}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$\hat{a} = \frac{1}{|\vec{a}|}$

(7) Find the projection vector $a = 2i - 3j - k$ on the vector $b = i + 4j - 2k$



projection vector $\vec{a}$ on the vector $\vec{b}$ is

$$a\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

$$= \frac{2 - 12 + 2}{\sqrt{1^2 + (4)^2 + (-2)^2}}$$

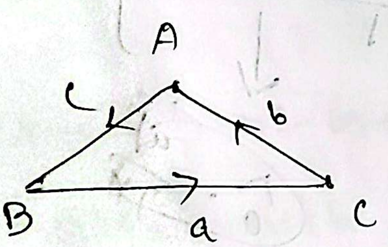
$$= \frac{-8}{\sqrt{21}}$$

Ans.

(8) $a \cdot b = 0$ (perpendicular)

(9) Prove that $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Sol^n



let ~~A,B,C~~ a, b, c represent the sides of ΔABC.

then we have, $a + b + c = 0$ —— (1)

Take there cross product of a, b, c separately with ① . Then,

$$\cancel{a \times (a+b+c) = 0}$$
$$\cancel{b \times (a+b+c) = 0}$$
$$\cancel{c \times (a+b+c) = 0}$$

$a \times (a+b+c) = 0$	$b \times (a+b+c) = 0$	$c \times (a+b+c) = 0$
$\Rightarrow a \times (b+c) = 0$	$b \times (a+c) = 0$	$c \times (a+b) = 0$
$\Rightarrow a \times b + a \times c = 0$	or, $a \times b = b \times c$	or, $c \times a = b \times c$
$\Rightarrow a \times b = -a \times c$		
$\therefore a \times b = c \times a$		

$\therefore \quad a \times b = b \times c = c \times a$

i.e., $ab \sin(\pi - C) = bc \sin(\pi - B) = ca \sin(\pi - C)$

$\Rightarrow ab \sin C = bc \sin B = ca \sin C$

or, $\dfrac{\sin C}{c} = \dfrac{\sin B}{b} = \dfrac{\sin A}{a}$

proved

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(12) $\text{Area (triangle)} = \frac{1}{2}\left|\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right|$

$\overrightarrow{AB} = \hat{i} - 4\hat{j} - \hat{k}$, $\overrightarrow{AC} = -2\hat{i} - \hat{j} + \hat{k}$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 1 & -4 & -1 \\ -2 & -1 & 1 \end{vmatrix}$

$= i(-4-1) - j(1-2) + k(-1-8)$

$= -5\hat{i} + \hat{j} - 9\hat{k}$

$= \sqrt{107}$

$\text{Area (triangle)} = \frac{1}{2}\sqrt{107}$

Ans.

(14) Parallelopiped $\longrightarrow \vec{a}\cdot(\vec{b}\times\vec{c}) = 0$

$\text{volume} = [\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$

Ex: 1 $\phi = xy + 2yz + 3xz$ find grad ϕ

we know, $\text{grad}\,\phi = \nabla \phi$

$= \nabla (xy + 2yz + 3xz)$

$= \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)(xy + 2yz + 3xz)$

$= \cancel{y\hat{i} + 2z\hat{j} + 3x\hat{k}}$

$= \hat{i}(y + 3z) + \hat{j}(x + 2z) + \hat{k}(2y + 3x)$

Ans

2. $\vec{V} = 2x^2z\hat{i} - xy^2z\hat{j} + 3yz^2\hat{k}$ find div(v) at $(2, -1, 3)$

Sol^n $\text{div}\, v = \nabla \cdot \vec{V}$

$= \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(2x^2z\hat{i} - xy^2z\hat{j} + 3yz^2\hat{k})$

$= 4xz - 2xyz + 6yz$

$\therefore$ at $(2,-1,3)$ value $= 4\times 2\times 3 - 2\times 2\times(-1)\times 3 + 6\times(-1)\times 3$

$= 24 + 12 - 18$

$= 36 - 18$

$= 18$ **Ans:**

③ $\vec{V} = xy^2\hat{i} - 2xyz\hat{j} + x^2z^2\hat{k}$ find curl v

$$\text{curl } V = \nabla \times \vec{V}$$

$$= \left(\frac{d}{dx}\hat{i} + \frac{d}{dy}\hat{j} + \frac{d}{dz}\hat{k}\right) \times v$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ xy^2 & -2xyz & x^2z^2 \end{vmatrix}$$

$$= \hat{i}(0 + 2xy) - \hat{j}(2xz^2 - 0) + \hat{k}(-2yz - 2xy)$$

Ans.

Slide - 02

(3) prove $\text{curl}(r+s) = \text{curl}\, r + \text{curl}\, s$

or, $\nabla \times (r+s) = \nabla \times r + \nabla \times s$

Now $\text{curl}(r+s) = \nabla \times (r+s)$

$$= \frac{d}{dx} i\,(r+s) + \frac{d}{dy} j\,(r+s) + \frac{d}{dz} k\,(r+s)$$

$$= \Sigma i \frac{\delta}{\delta x} \times (r+s)$$

$$= \Sigma i \times \left(\frac{\delta r}{\delta x} + \frac{\delta s}{\delta x}\right)$$

$$= \Sigma i \frac{dr}{dx} + \Sigma i \frac{ds}{dx}$$

$$= \text{curl}(r) + \text{curl}(s)$$

$$= \nabla \times r + \nabla \times s$$

prove,

(4) $\text{grad}(uv) = u\,\text{grad}(v) + v\,\text{grad}(u)$

or $\nabla (uv) = u \nabla (v) + v \nabla (u)$

Soln

$$\text{grad}(uv) = \nabla (uv)\ \Sigma i \frac{d}{dx}(uv)$$

$$= \Sigma i \left(u \frac{dv}{dx} + v \frac{du}{dx}\right)$$

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$$= \sum i u \frac{dv}{dx} + \sum i v \frac{du}{dx}$$

$$= u \sum i \frac{dv}{dx} + v \sum i \frac{du}{dx}$$

$$\cancel{= u \nabla + v \nabla}$$

$$= u \nabla v + v \nabla u$$

proved

vector-03

Md Azad

(1) $I = \int_0^2 (x\,dy - y\,dx) = \cancel{\int_0^2 (x - x)\,dx}$

$= \int_0^2 (x$

$y = x$

$dy = dx$

01

line integral

(i) $I = \int (x\,dy - y\,dx)$

since, $y = x$, let $x = t$, $y = t$

$dx = dt$, $dy = dt$

Now, $I = \int_0^2 (\cancel{x}^{t}\,dt - \cancel{x}^{t}\,dt)$

$= \int_0^2 (\cancel{t} - \cancel{t})\,dt$

$= 0$

Ans:

(ii) $y = x^2$,

let $x = t$, $y = t^2$

$\Rightarrow dx = dt$, $dy = 2t\,dt$

~~$I = \int_0^2 (\cancel{x}^{t} \cdot 2t\,dt - \cancel{x^2}\,dt)$~~

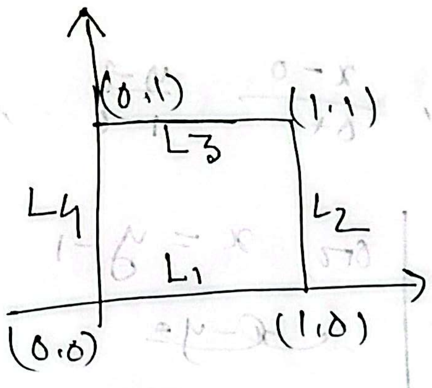
~~$= \int_0^2 \quad 2 \quad dt$~~

$I = \int_0^2 (t \cdot 2t\,dt - t^2 \cdot dt)$

$= \int_0^2 (2t^2 - t^2)\,dt = \left[\frac{t^3}{3}\right]_0^2 = 8/3$

Ans.

(iii)



There are Four line integrals L1 (0,0) to (1,0) ~~with~~ which along $y=0$, $dy=0$; L_2 along (1,0) to (1,1) which is $x=1$, $dx=0$, L_3 along (1,1) to (0,1) which is along $y=1$, $dy=0$ and L_4 along (0,1) to (0,0) which is along $x=0$, $dx=0$

$$\therefore I = \int (x\,dy - y\,dx)$$

we have $I = L_1 + L_2 + L_3 + L_4$

$$= 0 + \int_0^1 dy - \int_1^0 dx + 0$$

$$= 2$$

(3)

$$\frac{x-1}{1-0} = \frac{y-0}{0-1}, \quad \frac{x-0}{0+1} = \frac{y-1}{1-0}, \quad \frac{x+1}{-1-1} = \frac{y-0}{0-0}$$

or, $x-1 = -y$

$\therefore x+y=1$

or, $x = y-1$

$\therefore$ ~~$x-y$~~

$\therefore y-x=1$

$\therefore y=0$

Hence the integral are

$I_1 = \int_1^0 \left[(1-x)^2 dx + x^2 dx\right]$ ~~for~~ $x+y=1$

or, $y = 1-x$

$dy = -dx$

for,

$x+y=1$

$\Rightarrow y = 1-x$

$\Rightarrow dy = -dx$

$\Rightarrow$ ~~$I_1 = \int_1^0 (1-x)^2 dx + x^2 dx$~~

$\Rightarrow I_1 = \left[-\frac{1}{3}(1-x)^3 - \frac{1}{8}x^3\right]$

$$\int_1^0 (1-2x+x^2)\,dx + x^2\,dx$$

$$= \left[x - \frac{2x^2}{2} + \frac{x^3}{3} + \frac{x^3}{3}\right]_1^0$$

$\sin 2\theta - \sin^4\theta +$

$\cos^2\theta \sin\theta$

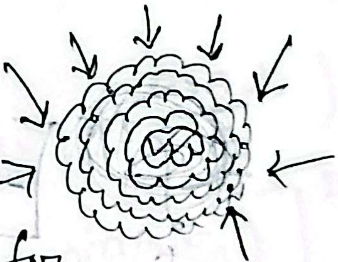
$(1-\sin^2\theta)\sin\theta$

$\sin\theta - \sin^3\theta$

$$= 0 - 0 + 0 + 0 - \left(1 - \frac{2}{2} + \frac{1}{3} + \frac{1}{3}\right)$$

$$= -\left(\frac{2}{3}\right)$$

$$= -\frac{2}{3}$$



for,

$y - x = 1$

$y = 1 + x,$

$dy = dx$

Now,

$$I_2 = \int_0^1 \left[(1+x)^2\,dx - x^2\,dx\right]$$

$$= \int_0^1 (1 + 2x + x^2\,dx - x^2\,dx)$$

$$= \left[x + \cancel{2}\frac{x^2}{\cancel{2}} + \frac{x^3}{3} - \frac{x^3}{3}\right]_0^1$$

$$= 1 + 1 + \frac{1}{3} - \frac{1}{3} - 0$$

$$= 2$$

$$\therefore I = I_1 + I_2 = -\frac{2}{3} + 2 = \frac{-2+6}{3} = \frac{4}{3}.$$

$\frac{4}{3}$

$$= \int_0^{2\pi} \left(-\sin t - 2\sin^2 t - \sin^3 t - \cos^3 t\right) dt$$

$$= -\int_0^{2\pi} \left[\left(\sin t + \frac{1-\cos 2t}{} + \frac{3}{4}(\sin t - \sin 3t) + \frac{1}{4}(3\cos t + \cos 3t) \right) \right] dt$$

$$= -\left[-\cos t + t - \frac{\sin 2t}{2} + \frac{3}{4}\left(-\cos t + \frac{\cos 3t}{3}\right) + \frac{1}{4}\left(3\sin t + \frac{\sin 3t}{3}\right) \right]_0^{2\pi}$$

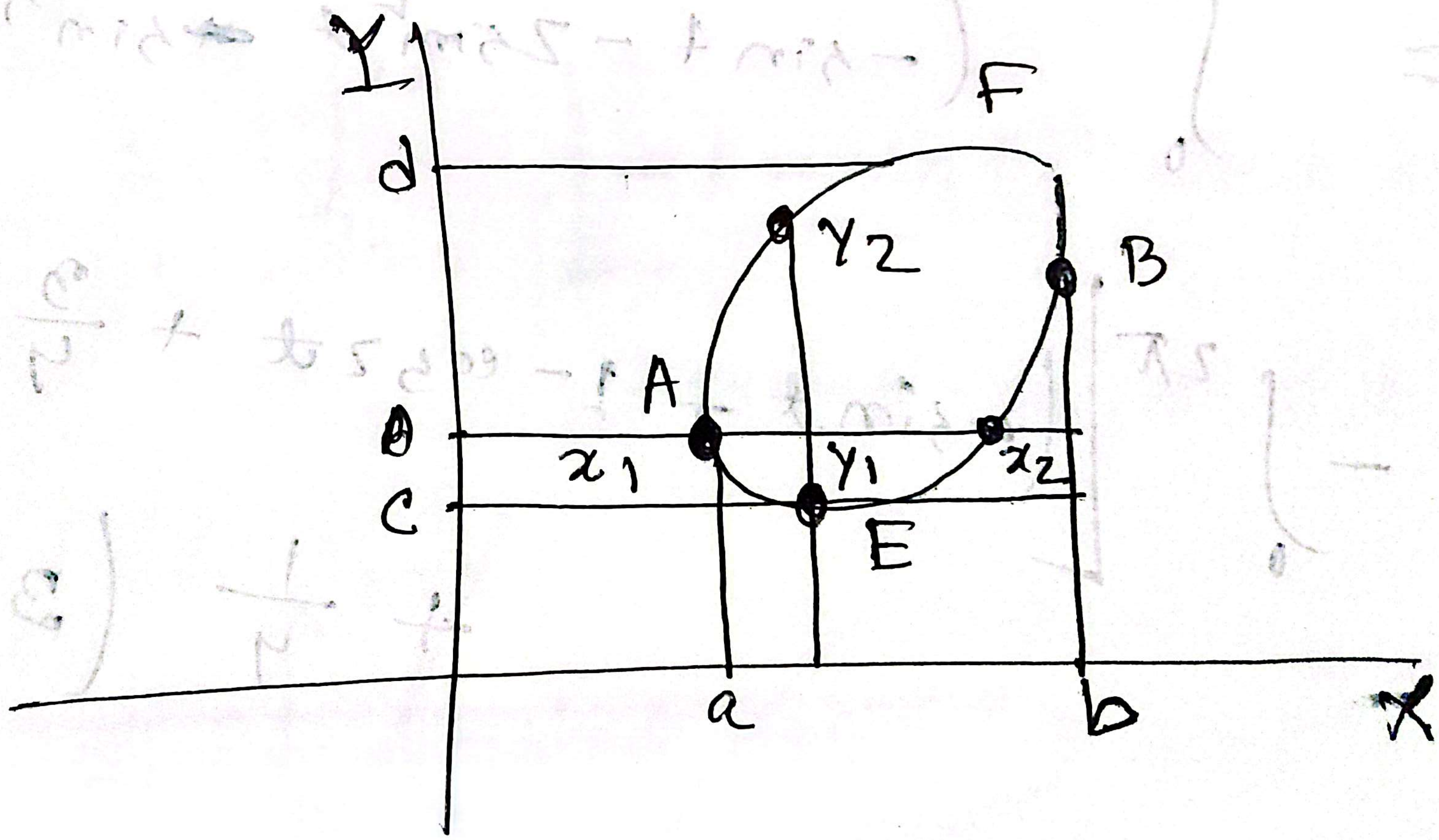
$$= -\left[\left\{ -1 + 2\pi - 0 + \frac{3}{4}\left(-1 + \frac{1}{3}\right) + \frac{1}{4}\left(3\times 0 + \frac{0}{3}\right) \right\} - \left\{ -1 + 0 - 0 + \frac{3}{4}\left(-1 + \frac{1}{3}\right) + \frac{1}{4}(3\times 0 + 0) \right\} \right]$$

$$= -\left[-1 + 2\pi + \frac{3}{4}\left(-\frac{2}{3}\right) + 0 - \left(-1 - \frac{3}{4}\times\frac{2}{3} + 0\right) \right]$$

$$= -\left[-1 + 2\pi - \frac{1}{2} + 1 + \frac{1}{2} \right] = -2\pi$$

Ans:

Green's theorem :-



$\int_C \quad\quad\quad\quad \int_R$

proof:

let, $y_1 = y_1(x), \; y_2 = y_2(x)$

then,

$$\iint_R \frac{dm}{dy}\, dx\, dy = \int_a^b dx \int_{y_1(x)}^{y_2(x)} \frac{dm}{dy}\, dy$$

$$= \int_a^b \Big[m(x, y) \Big]_{y_1(x)}^{y_2(x)} dx$$

$$= \int_a^b \Big[m\{x, y_2(x)\} - m\{x, y_1(x)\} \Big] dx$$

$$= - \int_a^b m(x, y_1(x))\, dx - \int_b^a m(x, y_2(x))\, dx$$

$$= - \oint_C m\, dx$$

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$$\therefore \oint_C M\,dx = -\iint_R \frac{dM}{dy}\,dx\,dy \quad \cdots (1)$$

for x- axis cut x_1 and x_2 point

then eqn of the curve EAF and EBF are $x_1 = x_1(y)$ & $x_2 = x_2(y)$

& limit $y = c$ to $y = d$

$$\therefore \iint_R \frac{dN}{dx}\,dx\,dy = \int_c^d dy \int_{x_1(y)}^{x_2(y)} \frac{dN}{dx}\,dx$$

$$= \cancel{\int_c^d \frac{N(x,y)}{dx}}\,dy$$

$$= \int_c^d \left[N(x,y)\right]_{x_1(y)}^{x_2(y)} dy$$

$$= \int_c^d N\{x_2(y), y\} - N\{x_1(y), y\}\,dy$$

$$\therefore \int_c^d N(x_2(y), y)\,dx + \int_d^c N(x_1(y), y)\,dx$$

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$$= \oint_c N\,dy$$

$$\therefore \oint_c N\,dy = \iint_R \frac{dN}{dx}\,dx\,dy \quad \text{(ii)}$$

add (i) & (ii)

$$\oint_c M\,dx + \oint_c N\,dy = -\iint_R \frac{dM}{dy}\,dx\,dy + \iint_R \frac{dN}{dx}\,dx\,dy$$

$$\therefore \oint_c M\,dx + \oint_c N\,dy = \iint_R \left(\frac{dN}{dx} - \frac{dM}{dy}\right) dx\,dy$$

Proved

Divergence theorem (Gauss's theorem)

The normal surface integral of the vector function F over the boundary of a closed region R is equal to the volume integral of divergence of F taken throughout the enclosed space.

$$\int_V \operatorname{div} F \, dv = \int_S F \cdot n \, ds$$

$$\int_V \nabla \cdot F \, dv = \int_V F \cdot n \, ds$$

where V is the volume

(13)

$\phi \quad \phi\phi \qquad \phi$

$$\int_S F\cdot n\, ds = \int_V \operatorname{div} F\, dv$$

let. $F = a\phi$,

$$\int_S a\phi\, ds = \int_V \operatorname{div}(a\phi)\, dv$$

$$\nabla\cdot(a\phi) = \nabla\phi\cdot a + \phi(\nabla\cdot a) = a\cdot\nabla\phi$$

a

$$\int_S F\cdot n\, ds = \int_V \operatorname{div} F\, dv$$

$$\int_S a\phi\cdot n\, ds = \int_V \operatorname{div}(a\phi)\, dv$$

$$\nabla\cdot(a\phi) = \nabla\phi\cdot a + a\cdot\nabla\phi$$

$$\int_S a\phi\cdot n\, ds = \int_V \operatorname{div}(a\phi)\, dv$$

$$\nabla\cdot(a\phi) = \nabla\phi\cdot a +$$

Foto

foto

$$a\int_s \phi n\, ds = \int_v a\cdot\nabla\phi\, dv$$

$$= a\int_v \nabla\phi\, dv$$

$$\int_s \phi n\, ds = \int_v \nabla\phi\, dv$$

proved

$$\cancel{w = \frac{1}{2}\text{curl } v}\quad \cancel{\int_s F\cdot n\, ds}$$

(14)

$$\int_s F\cdot n\, ds = \int_s \text{div } F\, dv$$

let, $F = u\times v$

then $$\int (u\times v)\, n\, ds = \int \nabla(u\times v)\, dv$$

$$= \int v\cdot(\nabla\times u)\, dv - \int u\cdot(\nabla\times v)\, dv$$

$$= \int_v \text{curl } u\, dv$$

$$= \int \text{curl } v\, dv - \int u\cdot 2w\, dv$$

$$\cancel{\text{or, } \frac{1}{2}v^2\, dv = \frac{1}{2}\int u\times v\cdot n\, ds + \int u\cdot w\, dv}$$

proved

(14) Given,

$$v = \nabla \times u$$

$$w = \frac{1}{2} \nabla \times v$$

Now, vector identity,

$$\nabla \cdot (u \times v) = v \cdot (\nabla \times u) - u \cdot (\nabla \times v)$$

$$= v \cdot v - u \cdot (\nabla \times v)$$

$$\nabla \cdot (u \times v) = |v|^2 - u \cdot (\nabla \times v)$$

integrate over the volume v

$$\int_v \nabla \cdot (u \times v)\, dV = \int_v |v|^2 dv - \int_v u \cdot (\nabla \times v)\, d$$

apply the divergence theorem,

$$\int_S (u \times v) \cdot n\, ds = \int_v |v|^2 dv - \int_v u \cdot (\nabla \times v)\, dv$$

since, $w = \frac{1}{2} \nabla \times v$

$$\Rightarrow \nabla \times v = 2w$$

$$\int_v |v|^2 dv = \int_S (u \times v) \cdot n\, ds + \int_v u \cdot 2w\, dv$$