p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

As, all the truth value of $(p \wedge q) \rightarrow (q \vee q)$ are true. So. we can say that $(p \wedge q) \rightarrow (q \vee q)$ is a tautology. (Showed)

a) Let $P(x)$ be the statement "$x = x^2$". If the domain consists of integer's, what are the truth values?

i. $P(0)$ ii. $\exists_x P(x)$ iii. $\forall_x P(x)$

The statement "$x = x^2$" can be rewritten as "$x - x^2 = 0$" or "$x(1-x) = 0$". Domain $= \{-\infty, \ldots, -2, -1, 0, 1, 2$

Construct the truth table of the compound proposition: $(p \vee q) \rightarrow (p \wedge q)$.

Truth table of the compound proposition: $(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tatology.

If all the truth value of a compound proposition is alaways true then it is called a tautology. so, The truth value of this compound pro-position $(p \wedge q) \rightarrow (p \vee q)$.

Truth table:

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

Difference between Implication and Biconditional

Implication	Biconditional
i. Let p and q be propositions. The proposition "if p then q" denoted by p→q is called implication or conditional statement.	i. Denotes the logical "if and only if" relationship. For propositions p and q ~~the~~ denoted by p→q is called ~~implication~~ or Biconditional state.

ii.

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

ii.

p	q	p⇔q
T	T	T
T	F	F
F	T	F
F	F	T

iv. Implication ($\rightarrow$): Indicates the logical "If...then" relationship. Given proposition p and q, the implication $p \rightarrow q$ is true unless p is true and q is false; Otherwise, it is false.

Truth table (Conditional):

p	q	$p \rightarrow q$
T	T	~~F~~ T
T	F	~~T~~ F
F	T	~~T~~ T
F	F	~~F~~ T

v. Equivalence / Bicondition ($\leftrightarrow$): Denotes the logical " if and only if" relationship. For propositions p and q, the equivalence $p \leftrightarrow q$ is true if and only if p and q have the same truth value.

iii. p is called hypothesis (or premise) and q is called conclusion (or consequence).

iv. Example:

"If you try hard for your exam, then you will succeed".

p = You tried hard for your exam

q = you succeed.

$p \to q$

iii. $(p \to q) \wedge (q \to p) \equiv p \leftrightarrow$

v. Ex:

2+2=4 if and only if 1+1=2

p: 2+2=4 (True)

q: 1+1=2 (True)

$p \leftrightarrow q$: T (Truth value)

The primary logical connectives in propositional logic are:

i. Negation ($\neg$): Represents the logical opposite of a proposition. If p is a proposition, the $\neg p$ is the negation of p.

Ex: p : Dhaka is the capital of Bangladesh.

The truth value of this proposition is T (True).

Negation of this proposition –

$\neg p$: Dhaka is not the capital of Bangladesh.

p		$\neg p$
T		F
F		T

ii. Conjunction ($\wedge$): Denotes the logical "and" operation. Given two propositions p and q, the conjunction $p \wedge q$ is true only when both p and q are true; otherwise, it is false. Truth table:

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

iii. Disjunction(∨): Represents the logical "or" operation. For propositions 'p' and 'q', the disjunction p∨q is true if at least one of p or q is true; it is false only when both p and q are false.

Truth table:

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

than, it also has a higher resolution camera.

Truth value of this proposition is:
F(False)

) Define propositional logic. Write down the difference between implication and biconditional with example.

A proposition is a declarative statement that is either true (T) or false (F), but not both.

A propositional logic, also known as propositional calculus or sentential logic, is a formal system that deals with the study of logical relationships and operations involving propositions. Propositional logic focuses on the analysis and manipulation of simple statements, reffered to as proposition i.

P: Smartphone B has the most RAM of these smartphones.

Truth value of this proposition is: T (True)

ii.

P: Smartphone C has more ROM or a higher resolution camera than smartphone B.

Truth value of this proposition is: True(T)

iii. P: Smartphone B has more RAM, more ROM and a higher resolution camera than Smartphone A.

Truth value of this proposition is: F(False)

iv. P: If smartphone B has more RAM and more ROM than Smartphone C, 1/a) Suppose that Smartphone A has 256 MB RAM and 32GB ROM, and the resolution of its camera is 8MP. Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4mp. and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 Mp. Determine the truth value of each of these propositions:

i. Smartphone B has the most RAM of these three smartphone

ii. Smartphone C has more ROM or a higher resolution camera than Smartphone B.

iii. Smartphone B has more RAM, more Rom, and a higher resolution camera than Smartphone A.

iv. If Smartphone B has more RoM than Smartphone C, then it also has a higher resolution camera.

9. (a) Let $p(n) = a_0 + a_1 n + a_2 n^2 \cdots + a_n n^m$; that is, suppose degree $p(n) = m$, prove that $p(n) = O(n^m)$.

Let, $b_0 = |a_0|$, $b_1 = |a_1|$, $\ldots$ $b_m = |a_m|$

than for $n \geq 1$

$$P(n) \leq b_0 + b_1 n + b_2 n^2 \cdots + b_n n^m = \left(\frac{b_0}{n^n} + \frac{b_1}{n^{m-n}} + \cdots + b_m\right) n^m.$$

$\Rightarrow P(n) \leq (b_0 + b_1 + \cdots + b_m) = M n^m$

Where, $M = |a_0| + |a_1| + \cdots + |a_m|$.

hence, $P(n) = O(n^m)$

1) Define set with example. What is the power set of {a, b, c, 2} and Cartesian product of {1, 2, 3} and {x, y, z}.

A set is an unordered collection of distinct objects, called elements or members of the set. A set is said to contain its elements. (A set is the collection of well defined and distinct object)..

We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

Ex: The set V of all vowles in the English alphabet can be written as: V = {a, e, i, o, u}.

Suppose that, $f(a) = f(b)$, where a and b are in the domain of f

$$f(a) = a+1$$
$$f(b) = b+1$$

If $f(a) = f(b)$ then

$$a+1 = b+1$$

Subtracting 1 from both sides, we get $a=b$.

Therefore, we have shown that if $f(a) = f(b)$, then $a=b$, which means that the function $f(x) = x+1$ is one to one. $\forall_a \forall_b ((a \neq b) \to (f(a) \neq f(b))$

note: $\forall_a \forall_b ((f(a) = f(b)) \to (a=b))$

Onto function (surjective) - $\forall_x \exists_x (f(x) = y)$

$$f(x) = x+1$$
$$f(x) = y$$
$$y = x+1 \quad \therefore x = y-1$$

To find the power set, we can list all possible combinations of elements:

$2^n = 2^4 = 16$

$P(a, b, c, 2) = \{\{\cdot\}, \{a\}, \{b\}, \{c\}, \{2\}, \{a, b\},$
$\{a, c\}, \{a, 2\}, \{b, c\}, \{b, 2\}, \{c, 2\}, \{a, b, c\}, \{a,$
$\{a, b, 2\}, \{b, c, 2\}, \{a, b, c, 2\}\}$

The cartesian product of $\{1, 2, 3\}$ and $\{x, y, z\}$ is : $\{1, 2, 3\} \times \{x, y, z\}$

$\{(1, x), (1, y), (1, z), (2, x), (2, y), (2, z),$
$(3, x), (3, y), (3, z)\}$

d) Determine wheter the function $f(x) = x+1$ from the set of real numbers to itself — one to one (or Injective)

6. Show that, $A\times A \neq B\times A$, when A and B are nonempty, unless $A=B$.

Let's assume $A=B$

let us consider, $(x,y)\in(A\times B)$

That means, $x\in A$ and $y\in B$

As given in the problem $A=B$, we can write,

$$x\in B \text{ and } y\in A$$

$$(x,y)\in(B\times A)$$

$\Rightarrow$ That means, $(A\times B)\subseteq(B\times A)$ — (i)

Similarly we can prove,

$$(B\times A)\subseteq(A\times B) \text{ — (ii)}$$

So, by the defination of set we can say from (i) and (ii) $A\times B = B\times A$ is only possible if $A=B$ (Showed)

a) Use set builder notation to give a description of each of these sets:

i. $\{-3,-2,-1,0,1,2,3\}$. ii. $\{m, n, o, p\}$

i. $\{-3,-2,-1,0,1,2,3\} = \{x \mid x \in \mathbb{Z} \text{ and } -3 \leq 3 \leq 3\}$

This set builder notation reads as " the set of all x such that x belongs to the integers and x is between -3 and 3 (inclusive).

ii. $\{m, n, o, p\}$ can be described using set builder notation as:

$\{m, n, o, p\} = \{x \mid x$ is a letter in the English alphabet and $x \in \{m, n, o, p\}\}$

This set builder notation reads as ' the set of all x such that x a letter in the English alphabet and x belongs to the set $\{m, n, o, p\}$ "

ii. Indirect proof: An implication $p \to q$ is equivalent to its contra-positive $\neg q \to \neg p$. Therefore we can prove $p \to q$ by showing that whenever q is false, then p is also false.

Example: "If $3n+2$ is odd, then n is odd"

n is even.

Then $n = 2k$ where k is an integer.

$$3n+2 = 3(2k)+2$$
$$= 6k+2 = 2(3k+1) = 2m$$

iii. Proof by ~~Contrapositive~~ Contradiction: We assume that q is false ($\neg q$ is true). Then by logical argument we arrive at situation where $\neg q$ implies a contradiction. This can happen only when $\neg q$ is false, which implies that q must be true.

iv. Proof by Mathematical induction:

and $\forall_x P(x)$ is false (Truth value)

b) Let f_1 and f_2 be functions from R to R such that $f'(x) = x_2$ and $f_2(x) = x - x^2$. What are the function $f_1 + f_2$ and $f_1 f_2$?

Ans: The functions $f_1(x) = x^2$ and $f_2(x) = x - x^2$ are given. To find $f_1 + f_2$, we simply add the two functions together:

$$f_1(x) + f_2(x) = x^2 + x - x^2$$
$$= x$$

Therefore, $f_1 + f_2 = x$

To find $f_1 f_2$, we simply the two functions together: $f_1 f_2 = f_1(x) \times f_2(x) = x^2(x - x^2)$

$$= x^3 - x^4$$

Therefore $f_1 f_2(x) = x^8 - x^9$

Q: Mention some methods of proving theorems. Prove that if $m+n$ and $n+p$ are even integers, where m, n and p are integers then $m+p$ is even. What kind of proof did you use?

There are various methods and techiniques used in mathematics and logic to prove theorems. Some commonly employed methods are:

i. Direct Method: An implication $p \to q$ can be proved by showing that if p is true, then q is also true.

Example: "If n is odd, then n^2 is odd".

Proof: n is odd.

Then $n = 2k+1$, where k is an integer.

Consequently, $n^2 = (2k+1)^2$
$= 4k^2+4k+1 = 2(2k^2+2k)+1$
$= 2m+1$ [Set, $2k^2+2k = m$]

Since n^2 can be written in this form, it is odd.

$P(0)$ if we subtitute 0 for x in the statement "$x=x^2$", we get $0=0^2$, which is true. There-fore the truth value of $P(0)$ is true.

ii. $\exists_x P(x)$ means "there exists an x such that $x=x^2$"

If we solve the equation $x(1-x)=0$, we get $x=0$ or $x=1$. Therefore, there exist integers such that $x=x^2$.

The truth value of $\exists_x P(x)$ is true.

iii. $\forall_x P(x)$ means "for all x, $x=x^2$".

If we try to solve the equ $x=x^2$, we get $x(x-1)=0$. This means that $x=0$ or $x=1$ satisfy the equ. but other integeres do not. Therefore the statement "for all x, $x=x^2$" is false

Applying inclusion-exclusion :

Probability of selecting a number divisible by either 2 or 5 = $P(A \cup B) = P(A) + P(B) - P(A \cap$

$$= \frac{1}{2} + \frac{1}{5} - \frac{1}{10}$$

$$= \frac{5+2-1}{10}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

Therefore, the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5 is 3/5.

B = All the number divisible by 5.

$n(A) = 20$

there are 20 positive integers (multiples of 5) between 1 and 100 that are divisible by 5.

So, the probability of selecting a number divisible by 5 is : $P(B) = \frac{n(B)}{n(S)}$

$$= \frac{20}{100} = \frac{1}{5}$$

There are 10 positive integers (multiples of 10) between 1 and 100 that are divisible by both 2 and 5. $n(A \cap B) = 10$

So, the Probability of selecting a number divisible by both 2 and 5 is:

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{10}{100} = \frac{1}{10}$$

Password = $(10 \times 36^5) + (10 \times 36^6) + (10 \times 36^7)$

2) Proved that, if $k+1$ or more pigeons are distributed among k pigeonholes, 1 then at least one pigeonhole contains two or more pigeons.

Suppose that A and B are events from a sample space S such that $P(A) \neq 0$ and $P(B) \neq 0$. Then proof the following:

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)\,P(B) + P(A|\bar{B})\,P(\bar{B})}$$

Answer: To prove the equation $P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)P(B) + P(A|\bar{B})\,P(\bar{B})}$ we can use Bayes' theorem.

Bayes' theorem states:

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

Now, let's express $P(A)$ using the law of total probability:

$$P(A) = P(A|B)\,P(B) + P(A|\bar{B})\,P(\bar{B})$$

Substituting this expression for $P(A)$ in Bayes' theorem:

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A|B)\,P(B) + P(A|\bar{B})\,P(\bar{B})}$$

Where A and B are not independent. Additionally, we assume that both $P(A)$ and $P(B)$ are non zero probabilities.

b) What is the probability that a positive integer selected at random from the set of positive integer not exceeding 100 is divisible by either 2 or 5?

$S = \{1, 2, 3, \ldots\ldots 100\}$ S∈ positive integer

$n(S) = 100$

A = All the number divisible by 2.

$n(A) = 50$

Here, 50 positive integers (even numbers) between 1 and 100 that are divisible by 2.

So, the probability of selecting ~~a~~ a number divisible by 2 is : $P(A) = \frac{n(A)}{n(S)}$

$= \frac{50}{100} = \frac{1}{2}$

b) Each user on a computer system has a password, which is six to eight characters long, where each char in an upper case leter or a digit. Each password must contain at least one digit. How many possible password are there?

Case 1 (~~7~~ 6 char Password): There are 26 Upper case and 10 possible digits, so total are 36 possible characters for each position in the password 10. Choise for the digit and 36 Choise for each of remaining 5 positions:

Case 1: 10×36^5

Case 2: 10×36^6

Case 3: 10×36^7

So, the total number of possible

5. a) Show that if n is an integer greater than 1, then n can be written as prime or product prime.

b. Use mathematical induction to prove that $2^n < n!$ for every integer n with $n \geq 4$. (Note that this inequality is false for $n = 1, 2$ and 3.)

This statement is true for the base case $n = 4$.

$$2^4 = 16 < 4! = 24$$

so the statement is true for $n = 4$

next, statement is true for some integer k, where $k >= 4$; $2^k < k!$

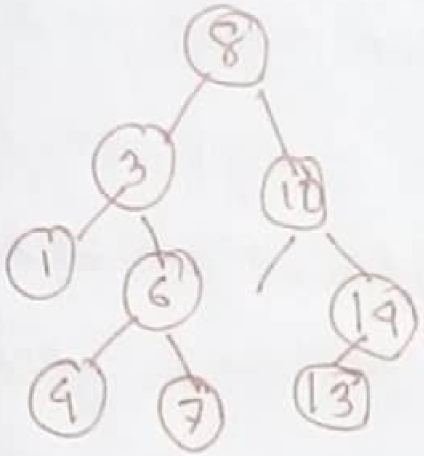
we now need to prove that the statement is true for $k+1$:

$$2^{(k+1)} < (k+1)!$$

$\Rightarrow$ 2^{k+1} as $2^2 k$ and $(k+1)!$ as $k!(k+1)$

$\Rightarrow 2 \times 2^k < k!\,(k+1)$

(1) Analysis the Complexity of the following binary Search tree:



The time complexity for all BST operations be it search operation or Insert operation or Delete operation is O(h) where h is the height of a BST.

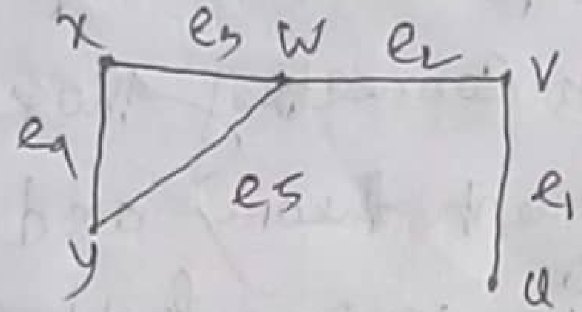
Thus, In general
Time Complexity of BST operations
$= O(\text{height})$

$\Rightarrow 2^k < 2\,k! < k!\,(k+1)$ [$\because 2 < k+1, k \geq$

$\Rightarrow 2 \times k! < (k+1)\,k!$

So, $2^{(k+1)} < (k+1)!$ and is true for $k+1$

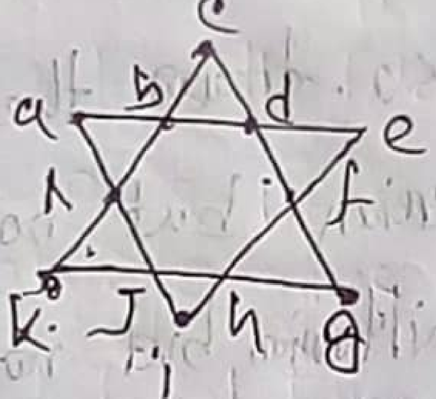
Example



then $u e_1 v e_2 w e_3 x e_4 y e_5 w$

Eulerian Circuit: A circuit in a graph is said to be an Eulerian circuit if it traverses each edge in the graph once & only once

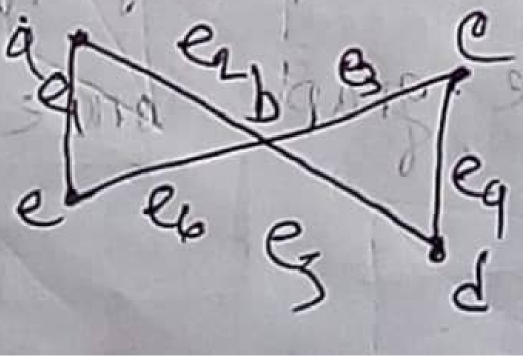
Example:



Start l and end l

Eulerian graph: A connected graph which contain an Eulerian circuit is called Eulerian graph

Example:



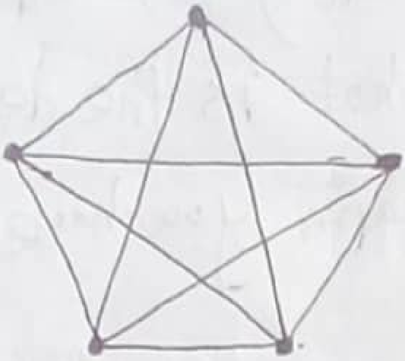
points on the plane and edges as curves or straight lines connecting those points. It is important in graph theory and has numerous applications in various fields as CSE, network design, and map representation.

b) Explain Eulerian and Hamiltonian graph with examples. also. draw the graphs of the following: i. Eulerian but not hamilton

ii. Hamiltonian but not Eulerian.

Answer: Ⓐ

Eulerian path: A path in a graph is said to be an Eulerian path if it traverses each edge in the graph once and only once.

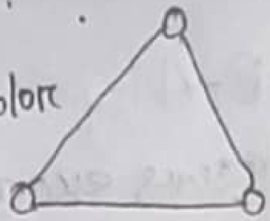


five vertices and 10 edges

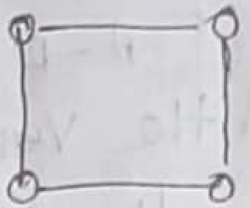
Fig: K5 graph

In this K_5 graph, each vertex is connected to every other vertex by an edge. Therefore, the degree of each vertex is 4 because each vertex is adjacent to 4 other vertices

Example:



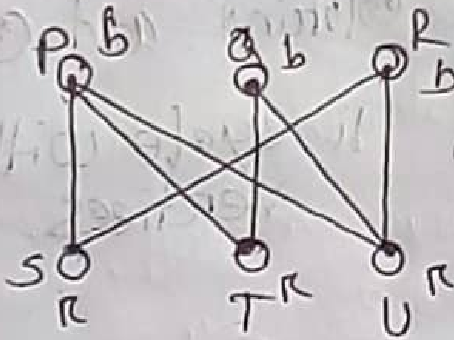
3 color



4 color graph

Chromatic number: It is defined as the least no. of colors needed for coloring the graph- denoted by $\chi(G)$ or K - chromatic Graph.

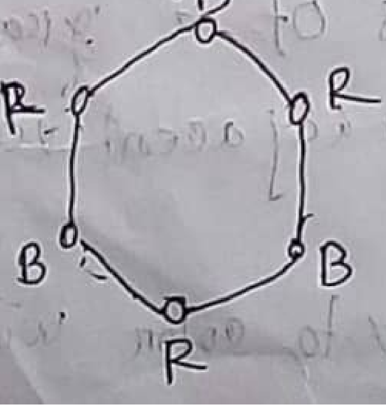
i.



$K_{3,3}$ Bipartite graph.

number of $K_{3,3}$ bipartite graph is 2

iii.

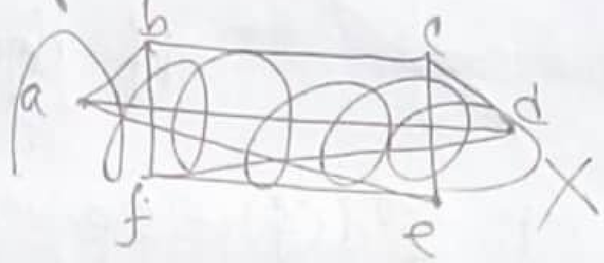


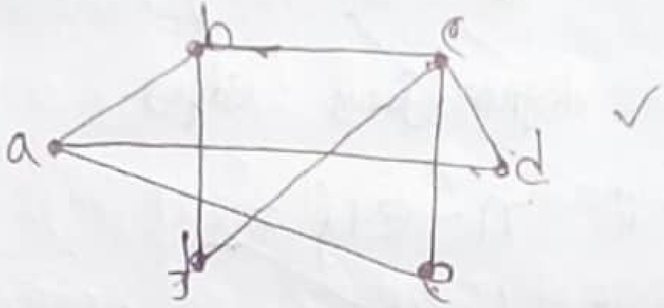
Chromatic number = 2

R = Red

B = Blue

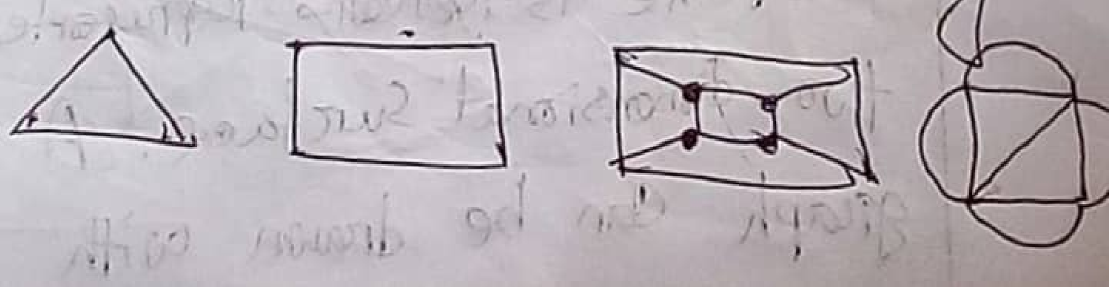
a) What is planner graph? Draw the planner graph of given simple graph.





Ans: A graph G is said to be planer if G can be drawn in a plano so that no edges cross.

A graph G is said to be non-planar if it can not be drawn on a plane without crossing its edges. The geometric representation of a graph on any surface, such that no edges intersect is called embedding.

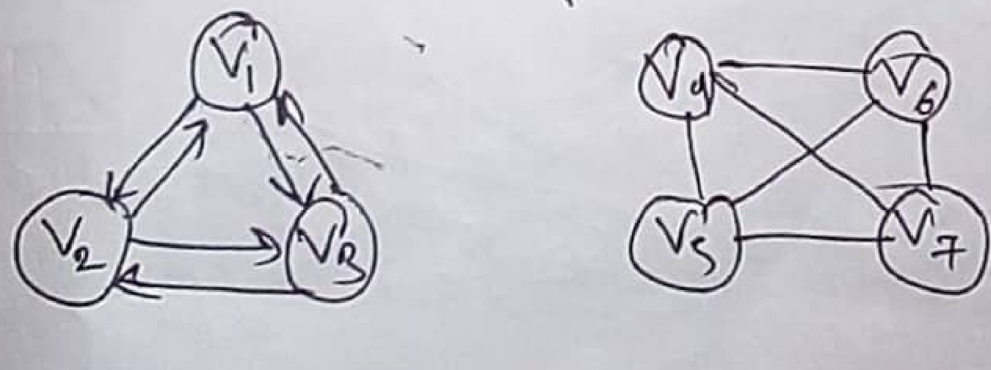


) What do you mean by a complete graph? Draw a K_B graph. What is the degree of a vertex of the K_B graph you have drawn?

Ans: A graph is said to be complete if and only if every pair of vertices is connected through an edge. (K_n)

That is, all vertices are connected with all other vertices. In an undirected graph with n vertices has exactly $\frac{n(n-1)}{2}$ edges.

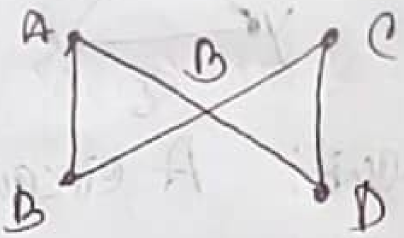
But in directed graph with n vertices has exactly $n(n-1)$ edges.



Hamiltonian Graph: A connected graph which contains Hamiltonian circuit is called Hamiltonian Graph

Example: Same

i. Eulerian but not hamiltonian:



we cannot find Hamiltonian cycle as the vertex B is repeated twice

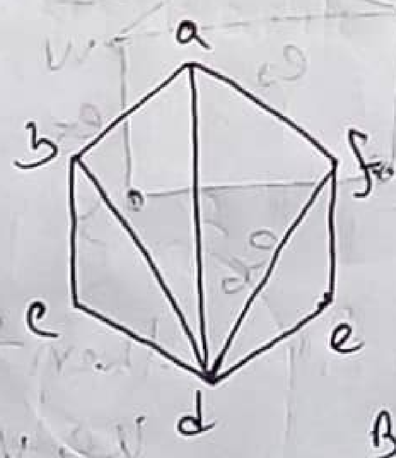
Therefore, Graph is Eulerian but not Hamiltonian.

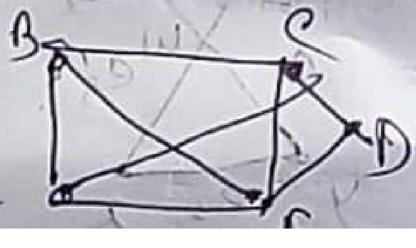
Graph contains the Eulerian cycle is

A—B—C—D—B—E—A

(all the edges occures exactly ones)

ii. Hamiltonian but not Eulerian:





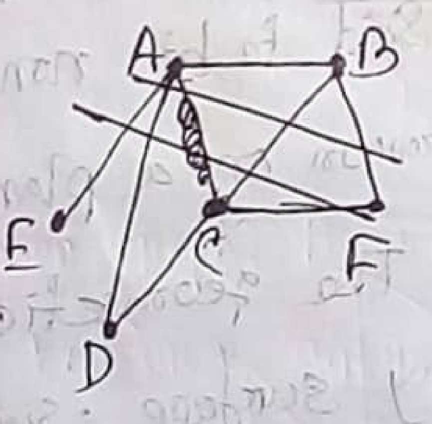
The max no. of faces that are possible for a Simple Connected planar graph with n vertic[es] are $\rightarrow$ if degree (f).

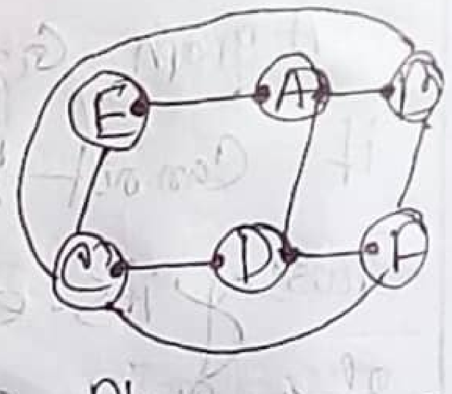
i. $\sum d(f) = 2e$

degree faces edges

ii. $n - e + f = 2$

iii. $f \leq 2n - 4$





Planar graph

The plane is typically represented as a flat, two dimensional Surface. A planar graph can be drawn with vertices a[nd]

Graph Contains the Hamiltonian cycle is
A-B-C-D-E-A
(All the vertics occurs exactly once)
Also the degree of B is 3 (it is an odd)
It is not an Eulerian
Graph is Hamiltonian graph.

) ~~What do you mean by~~.

2) Define graph coloring and chromatic number of i. $K_{3,3}$ ii. Cycle with even number of vertices.

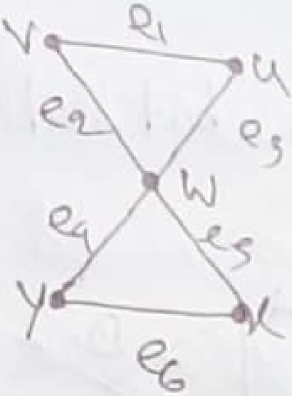
Ans:

Graph Coloring: Coloring of Graph Constitutes Coloring verikes edges of the graph. Coloring all the vertices of a graph is the property that no two adjacent vertices have same color.

(Always try to color with minimum colors)

Hamiltonian Path: A path which contain every vertex of a graph G exactly once is called Hamiltonian path

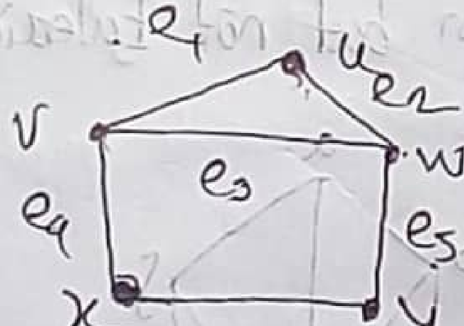
Example:



then $ve_1\, ve_2 we_5 xe_6 y$

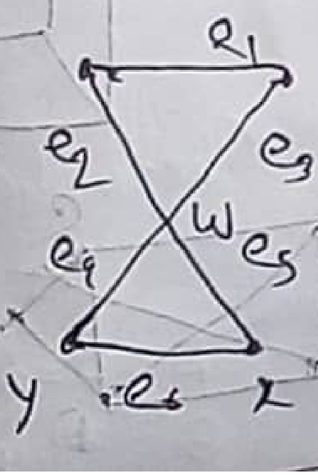
Hamiltonian Circuit: A circuit that passes through each of the vertices in a graph G exactly one except the starting vertex & end vertex is called Hamiltonian circuit.

Ex:



then,
$ue_1\, ve_4 xe_6 ye_5 we_2 u$

but



then
$ue_1\, ve_2 we_5 xe_6 ye_4 we_3 u$
is not hamiltonian circuit because w vertex repeat.