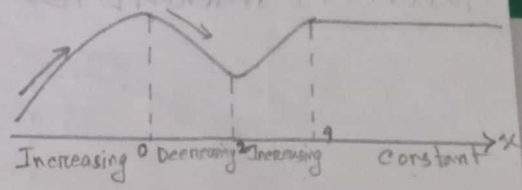
Increasing and Decreasing Functions



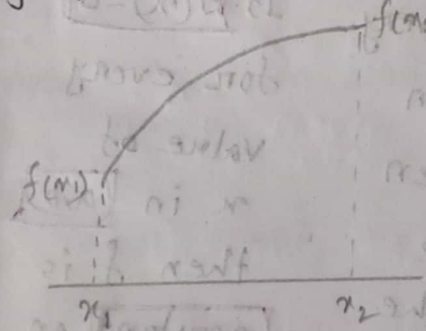
Increasing and Decreasing functions defination:

Let, f be defined on an interval, and let x_1 and x_2 denote points in that interval.

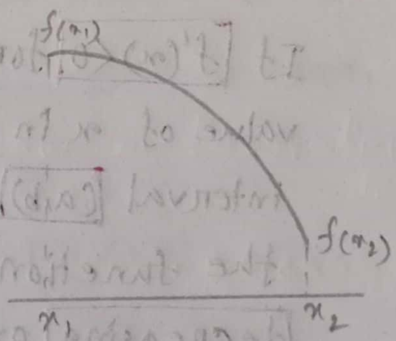
ⓐ f is increasing on the interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$

ⓑ f is decreasing on the interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

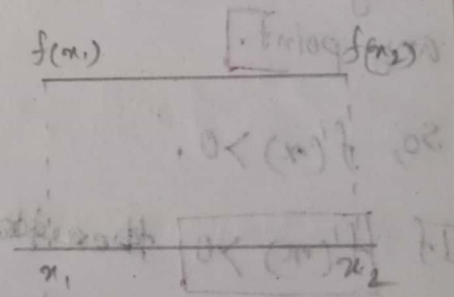
ⓒ f is constant on the interval if $f(x_1) = f(x_2)$ for all points x_1 and x_2.



$f(x_1) < f(x_2)$ if $x_1 < x_2$

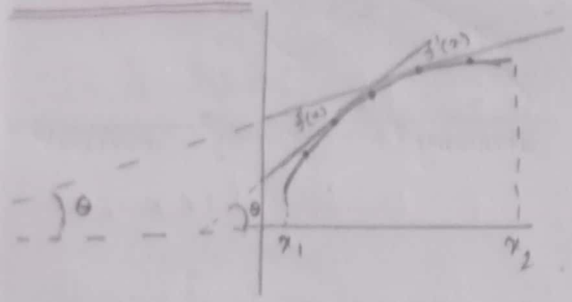


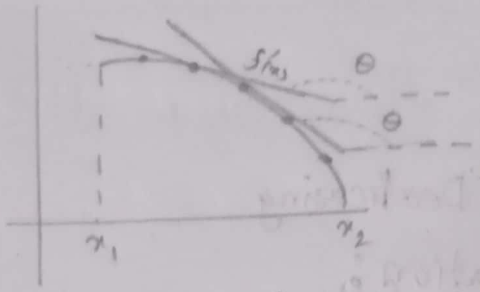
$f(x_1) > f(x_2)$ if $x_1 < x_2$

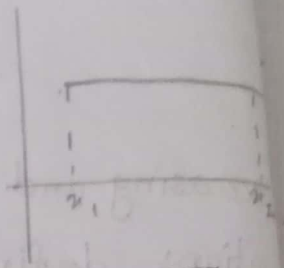


$f(x_1) = f(x_2)$ for all x_1 and x_2

Theorem: Let f be a function that is continuous on a closed interval $[a,b]$ and differentiable on the open interval (a,b).







এই বৃত্তচাপটির যেকোনো point এ tangent line draw করলে তার slope অর্থাৎ θ এর মান $90° > \theta$ হয়, এবং ~~positive~~ $f'(x)$ ~~হয়~~ এর value positive হয়। ~~so~~

$\boxed{f'(x)}$ means slope of the tangent to a curve at any point.

So, $f'(x) > 0$.

If $\boxed{f'(x) > 0}$ ~~therefore~~ for ~~function~~ every value of x in ~~[a,b]~~ open interval $\boxed{(a,b)}$, then the function is $\boxed{\text{increasing}}$ on the closed interval $[a,b]$.

→ if the interval is closed $[a,b]$ then the condition is $f'(x) \geq 0$.

এই বৃত্তচাপটির যেকোনো point এ tangent line draw করলে তার slope অর্থাৎ θ এর মান $90° < \theta$ হয় এবং $f'(x)$ এর value negative হয়।

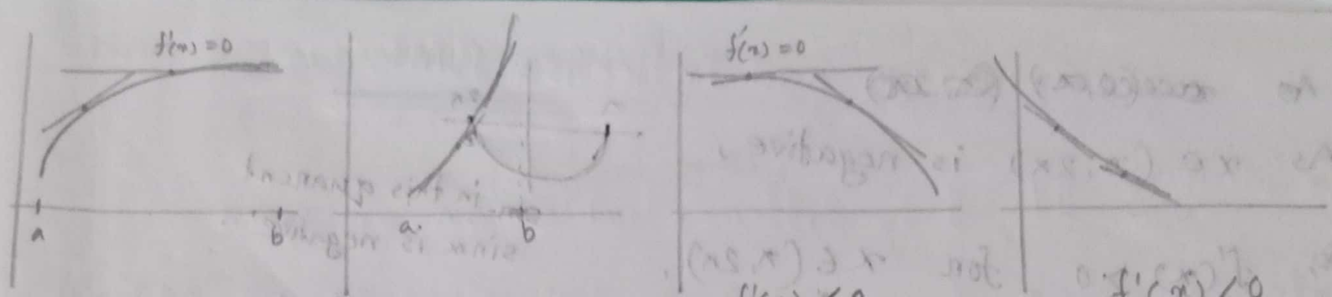
So, $f'(x) < 0$

If $\boxed{f'(x) < 0}$ for every value of x in open interval $\boxed{(a,b)}$ then the function is $\boxed{\text{decreasing}}$ on the closed interval $[a,b]$

এই অবস্থায় রেখার যেকোনো বিন্দুতে tangent line draw করলে তার slope অর্থাৎ $\theta = 0$ হবে।

So, $f'(x) = 0$

If $\boxed{f'(x) = 0}$ for every value of x in $\boxed{(a,b)}$ then f is $\boxed{\text{constant}}$ on $[a,b]$.

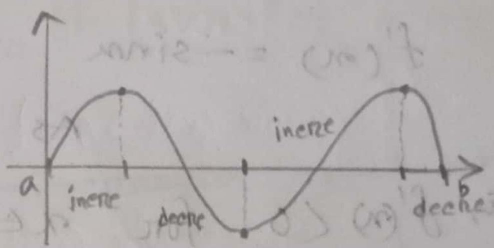


$f'(x) \geq 0$
$x \in (a,b)$
increasing function

$f'(x) > 0$
$x \in (a,b)$
strickly increasing

$f'(x) \leq 0$
$x \in (a,b)$
decreasing function

$f'(x) < 0$
$x \in (a,b)$
strickly decreasing



neither increasing nor decreasing. $x \in (a,b)$

Quesion type:

Type-1 Prove that $f(x)$ is increasing or decreasing in a given interval $[a,b]$.

অর্থাৎ function and interval দেওয়া থাকবে।

Q: Prove that $f(x) = \cos x$ is

(i) strickly increasing ~~increasing~~ in $(\pi, 2\pi)$

(ii) strickly decreasing in $(0, \pi)$

(iii) neither increasing nor decreasing $(0, 2\pi)$

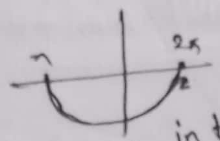
Solⁿ: (i)

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

~~As~~ ~~$(0, 2\pi)$~~ ~~$(\pi, 2\pi)$~~

As $x \in (\pi, 2\pi)$ is negative,



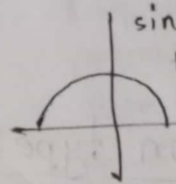
~~sin~~ in this quarent sinx is negative'

So, $f'(x) > 0$ for $x \in (\pi, 2\pi)$.

~~So~~ f^n is strickly increasing $(\pi, 2\pi)$.

(ii) $f(x) = \cos x$

$f'(x) = -\sin x$



sinx is Positive

As $x \in (0, \pi)$ is positive.

So, $f'(x) < 0$ for $x \in (0, \pi)$.

f^n is decreasing.

Type 2: Find an interval in which given f^n is increasing or decreasing.

Q: Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing

Sol^n: Given that,

$$f(x) = x^2 - 4x + 3$$

$$f'(x) = 2x - 4$$

Now, $f'(x) = 0$

$2x - 4 = 0$

$x = 2$ (critical point)

Alocate critical point,

$$-\infty \quad 0 \quad 2 \quad 4 \quad +\infty$$

$f'(x) = 2x - 4$

$f'(4) = 4.4 - 4 = 4$

$\therefore f'(4) > 0$

and $f'(2) = 2.2 - 4 = 0$

$f'(2) = 0$

again, $f'(x) = 2x - 4$

$f'(0) = 2.0 - 4 = -4$

$f'(0) < 0$

~~So, the increasin point out interval is $[2, +\infty)$~~

~~and~~

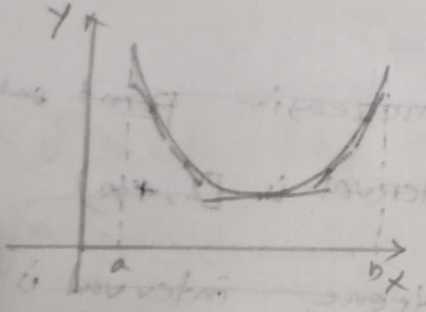
the ~~decre interval is $(-\infty, 2]$~~

So, f is increasing on $[2, +\infty)$.

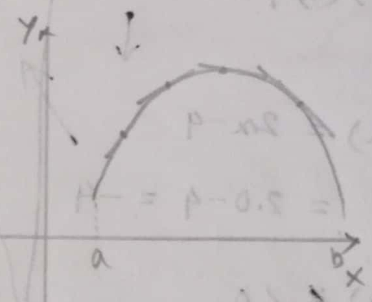
f " " on $(-\infty, 2]$.

Concavity and Inflection Points

Definition: If f is differentiable on an open interval, then f is said to be concave up on the open interval if f' is increasing on that interval and f is said to be concave down on the open interval if f' is decreasing on that interval.



Increasing slope



decreasing slope

যখন কোনো curveture এর কোনো বিন্দুতে tangent line draw করলে তা ঐ curve এর নিচে অবস্থান করে তখন তাকে Convex or concave up বলা হয়, এবং interval [a,b] point এ এটি increasing অবস্থায় থাকে।

যখন কোনো curveture এর কোনো বিন্দুতে tangent line draw করলে তা ঐ curve এর উপরে অবস্থান করে তখন তাকে Concave or Concave down বলা হয়, এবং interval [a,b] point এ এটি decreasing অবস্থায় থাকে।

Theorem: Let f be twice differentiable on an open interval

ⓐ If $f'' > 0$ for every value of x in the open interval, then f is concave up on that interval.

ⓑ If $f'' < 0$ for every value of x in the open interval, then f is concave down on that interval.