Kinematics -Coordinate Transformations Manipulator Kinematics

Presented By

Shakil Ahmed [20CSE027]

Contents:

- Kinematics
  - Object Location and Motion
  - Transformation Matrices
  - Homogeneous Transformations
- Forward Kinematics
- Inverse Kinematics .
- Differential Relationships

Object location

- Position of a point in space
- Location of a rigid body in space –Homogeneous
- transformation matrix

Object Motion

- ❑Translation (Transformation Matrix) and Inverse
- ❑Basic Rotation (Transformation Matrices) and Inverse.
- ❑General Rotation

Examples:

Properties of homogeneous transformation matrix.

Given an object in a physical world,

- how to describe its position and orientation
- how to describe its change of position and orientation due to motion

are two basic issues we need to address before talking about having a robot moving physical objects around.

The term location refers to the position and orientation of an object.

Coordinate frames

If 'p' is a vector in R, and X={x^1, x^2 x^3...x") be a complete orthonormal set of R, then the coordinates of p with respect to X are denoted as $[p]^x$ and are defined as

$$p = \sum_{k=1}^{n} [p]_k^X x^k$$

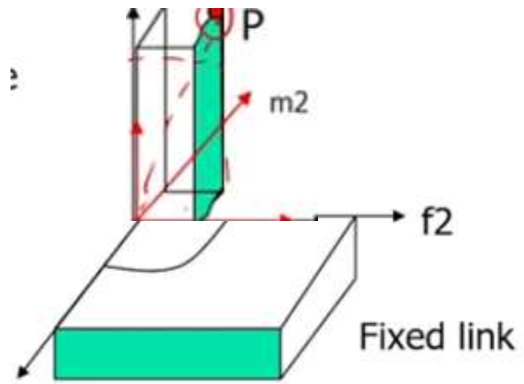
The complete orthonormal set X is sometimes called an orthonormal coordinate frame.

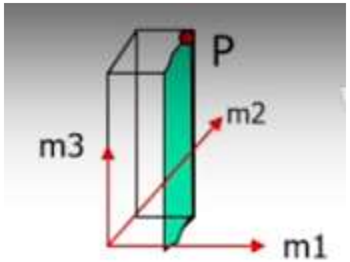
The 'kth' coordinate of p wrt X is

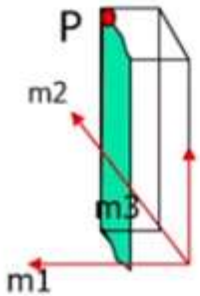
$$[p]_k^x = p.x^k$$

Coordinates Transformations

Represent position of p wrt fixed frame f={f1,f2,f3}







The two sets of coordinates of P are given by

$[p]^M$=[p.m1, p.m2, p.m3]

$[p]^F$=[p.f1, p.f2, p.f3]

The coordinate transformation problem is to find the coordinates of p wrt F, given the coordinates of p wrt M.

Coordinate Transformation Matrix

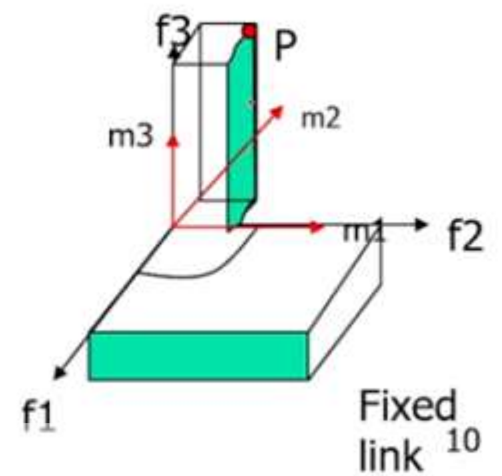
Let $F=\{f^1, f^2, f^3,..f^n\}$ and $M=\{m^1, m^2,m^3,.. m^n\}$ be coordinate frames of R^n with F being an orthonormal frame. Then for each point p in R^n,

$$[p]^F = A\,[p]^M$$

where A is an nxn matrix defined by $A_{kj} = f^k.m^j$ for $l \le k,\ j \le n$

The matrix A is known as Coordinate transformation matrix.

$$A = \begin{bmatrix} f^1.m^1 & f^1.m^2 & f^1.m^3 \\ f^2.m^1 & f^2.m^2 & f^2.m^3 \\ f^3.m^1 & f^3.m^2 & f^3.m^3 \end{bmatrix}.$$



Inverse Coordinate Transformation

Let F and M be two orthonormal coordinate frames in R^n, having the same origin, and let A be the coordinate Transformation matrix that maps M coordinates to F coordinates, then the transformation matrix which maps F coordinates into M coordinates is given by A^{-1} , where

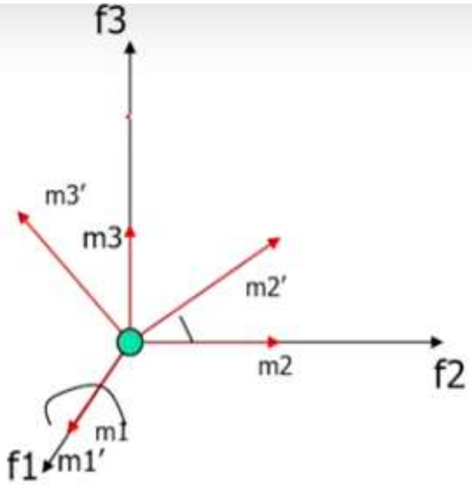
$$A^{-1}=A^{T}$$

Rotations

In order to specify the position and orientation of the mobile tool in terms of a coordinate frame attached to the fixed base, coordinate transformations involving both rotations and translations are required.

Fundamental Rotations

If the mobile coordinate frame is obtained from the fixed coordinate frame F by rotating M about one of the unit vectors of F, then the resulting coordinate transformation matrix is called a fundamental rotation matrix.



In the space R3, there are 3 possibilities.

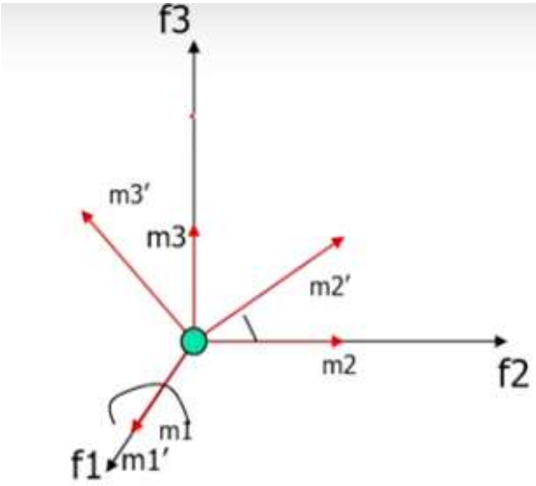
$$R_1(\phi) = \begin{bmatrix} f^1.m^{1'} & f^1.m^{2'} & f^1.m^{3'} \\ f^2.m^{1'} & f^2.m^{2'} & f^2.m^{3'} \\ f^3.m^{1'} & f^3.m^{2'} & f^3.m^{3'} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & f^2.m^{2'} & f^2.m^{3'} \\ 0 & f^3.m^{2'} & f^3.m^{3'} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix}$$

Fundamental Rotations

$$R_1(\phi) \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$R_2(\phi) \Rightarrow \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{bmatrix}$$

$$R_3(\phi) \Rightarrow \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



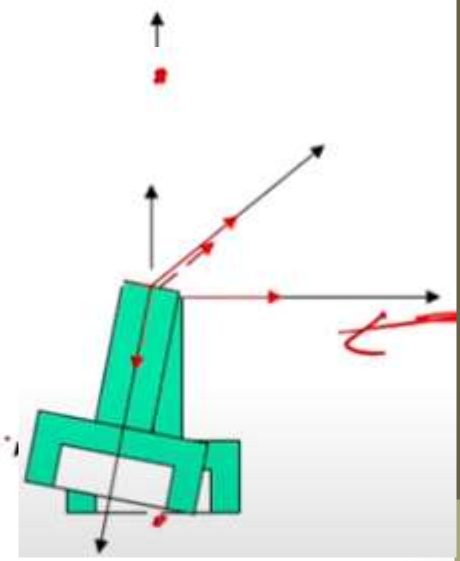
The k^{th} row and the k^{th} column of $R_K(4)$ are identical to the kth row and the kth column of identity matrix. In the remaining 2x2 matrix, the diagonal terms are cos() while the off diagonal terms are +-sin(). The sign of the off diagonal term above the diagonal is $(-1)^k$.

Composite Rotations

A Sequence of fundamental rotations about the unit vectors cause composite rotations.

Algorithm for composite rotation:

- Initialise rotation matrix to R=I, which corresponds to F and M being coincident
- If the mobile frame M is rotated by an amount about the kth unit vector of F, then pre-multiply R
- If the mobile frame M is rotated by an amount, about it's own kth vector, then post-multiply R .
- If there are more rotations go back to 2. The resulting matrix maps M to F



Thank You all For Your Attention…….