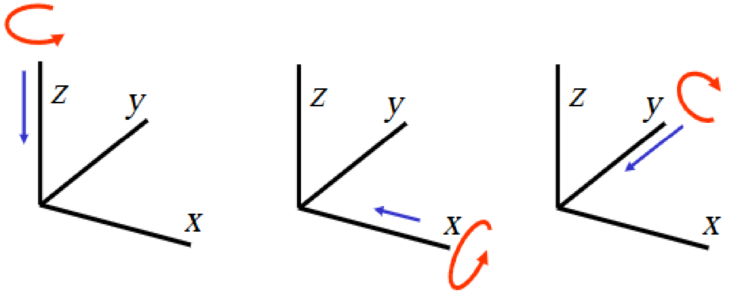
Homogeneous Transformation Matrices in Robotics

Lec-4

Presented By-
Nure Hafsa Shefa (20CSE018)

Rotation in 3D



Rotate over angle θ *around x-as:*

$x' = x$

$y' = y \cos\theta - z \sin\theta$

$z' = y \sin\theta + z \cos\theta$

Rotate over angle θ *around y-as:*

$x' = x \cos\theta + z \sin\theta$

$y' = y$

$z' = z \cos\theta - x \sin\theta$

Rotate over angle θ *around z-as:*

$x' = x \cos\theta - y \sin\theta$

$y' = x \sin\theta + y \cos\theta$

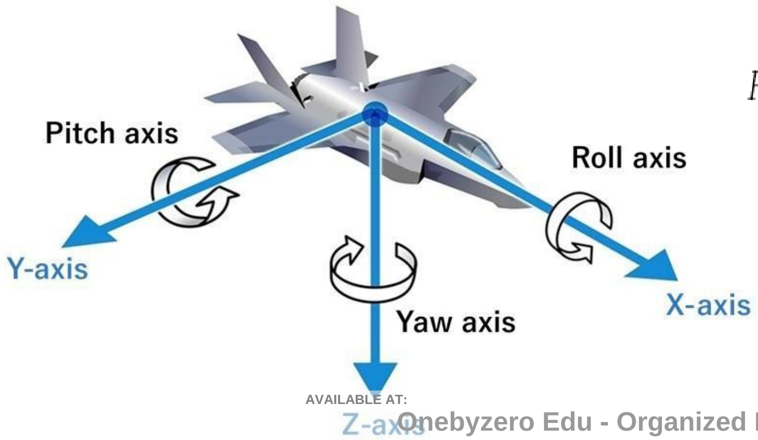
$z' = z$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Composite Rotations: Yaw-Pitch-Roll

Imagine three lines running through an airplane and intersecting at right angles at the airplane's center of gravity.

- Rotation around the front-to-back axis is called roll.
- Rotation around the side-to-side axis is called pitch.
- Rotation around the vertical axis is called yaw.



$$R_z(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_y(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}, \quad R_x(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{pmatrix}.$$

$$R(\alpha,\beta,\gamma) = R_z(\alpha)\, R_y(\beta)\, R_x(\gamma) =$$

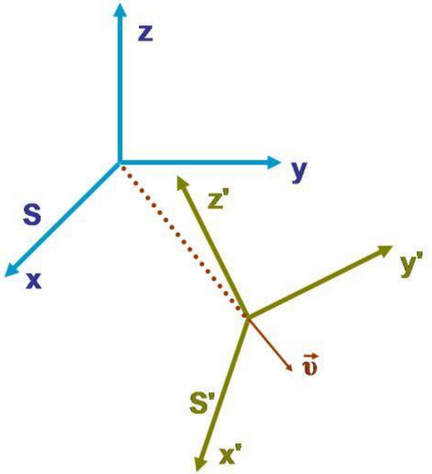
$$\begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{pmatrix}.$$

Homogeneous Coordinates (Observation)

- We need **pure rotations and translations** to characterize the **position and orientation** of a point relative to the coordinate frame attached to the base.

- While a rotation can be represented by a **3x3** matrix, it is not possible to represent translation by the same dimension.

- So, we need to move to a higher dimensional space, like- here we need **four** dimensional space of homogeneous coordinates.



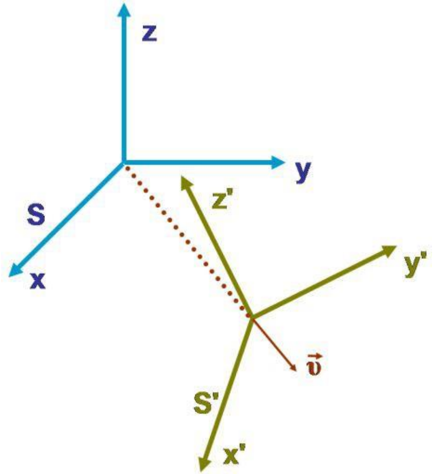
Homogeneous Coordinates (Definition)

Let q be a point in R^3, and let F be an orthonormal coordinate frame of R^3. If σ is any non zero scale factor, then the homogeneous coordinates of q with respect to F are denoted as [q]F and defined:

$$[q] \cdot F = [\sigma \cdot q1, \; \sigma \cdot q2, \; \sigma \cdot q3, \; \sigma]$$

In robotics we use **σ = 1** for convenience So,

$$[q]F = [q1, \; q2, \; q3, \; 1]$$

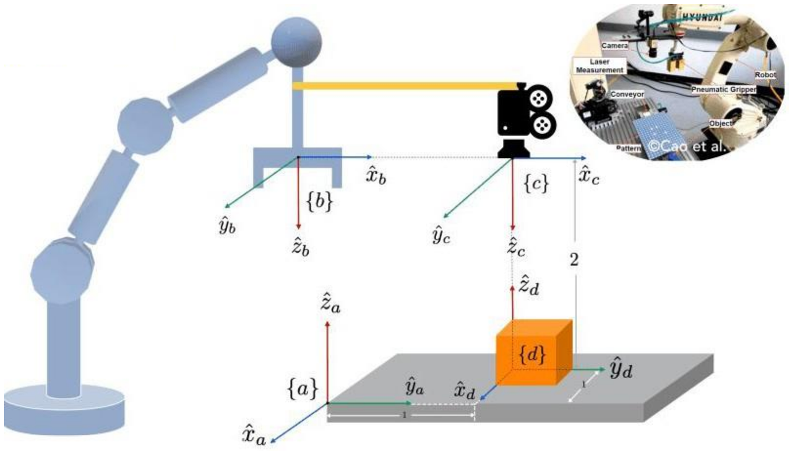


Homogeneous Transformation Matrix

If a physical point in **three** dimensional space is expressed in terms of its homogeneous coordinates and we want to change from one coordinate frame to another, we use a **4x4** homogeneous transformation matrix. In general **T** is —

$$T = \left[\begin{array}{c|c} R & p \\ \hline \eta^T & \sigma \end{array}\right]$$

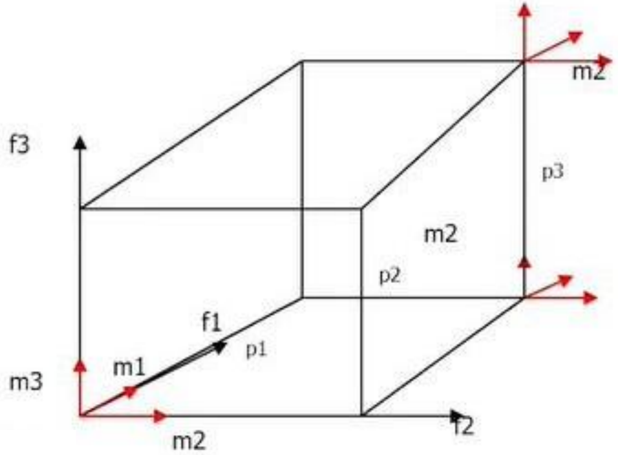
- R is the 3x3 matrix rotation matrix
- P is a 3x1 translation vector
- η is a perspective vector, set to zero
- σ is 1 for robotics



Homogeneous Transformation Matrix

Using homogeneous coordinates, translations also can be represented by **4x4** matrices. In terms of homogeneous coordinate frames, the translation of **M** can be represented by a **4x4** matrix, denoted **Tran(p)**, where —

$$Tran(p) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & p_1 \\ 0 & 1 & 0 & p_2 \\ 0 & 0 & 1 & p_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$



Tran(p) is known as the fundamental homogeneous translation matrix

Inverse Homogeneous Transformation

If **T** be a homogeneous transformation matrix with rotation **R** and translation **P** between two orthonormal coordinate frames and if **η = 0**, **σ = 1**, then the inverse transformation is:

$$T^{-1} = \left[\begin{array}{ccc|c} & R^T & & -R^T p \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

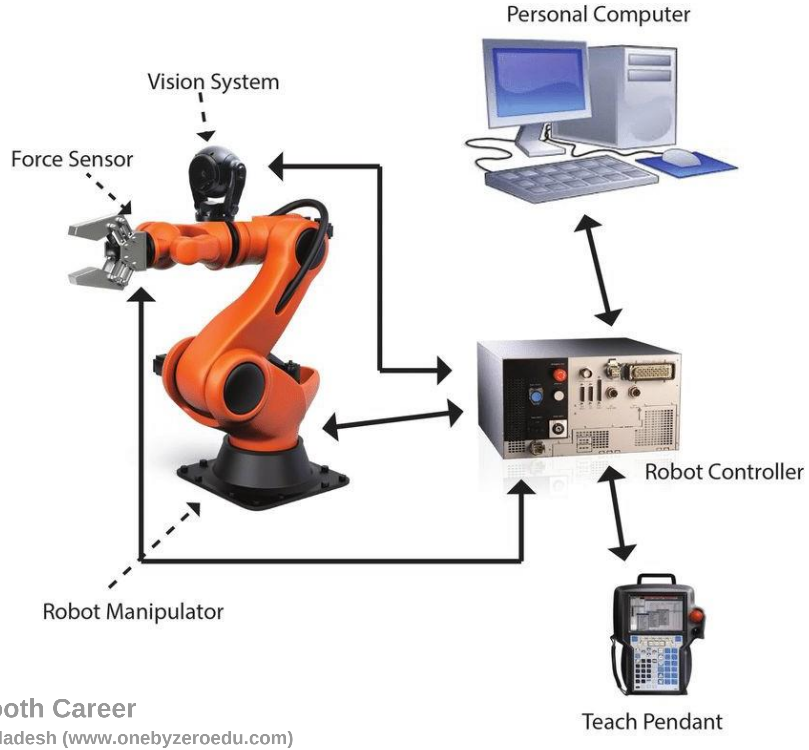
Why Transformation Matrices are important?

- Solve forward and inverse kinematics problems.
- Facilitate coordinate transformations between different components of a robot.
- Enable accurate manipulator control and trajectory planning.
- Support workspace analysis and collision avoidance.
- Integrate sensor data into the robot's coordinate system.
- Assist in robot calibration for improved accuracy.
- Aid in robot simulation and visualization.

Components of an Industrial Robot

The four main parts of an industrial robot are —

- Manipulator,
- Controller,
- Human interface device, and
- Power supply.



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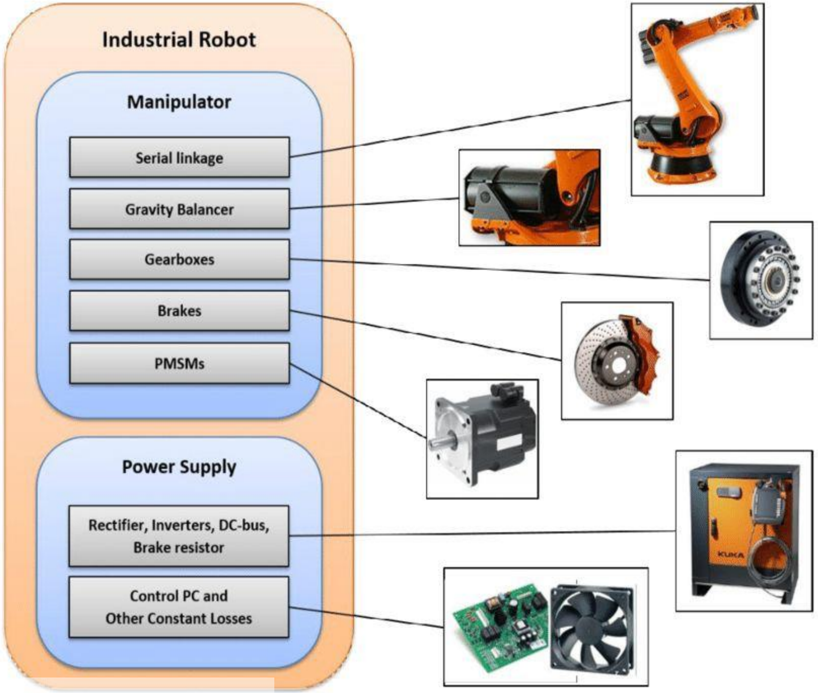
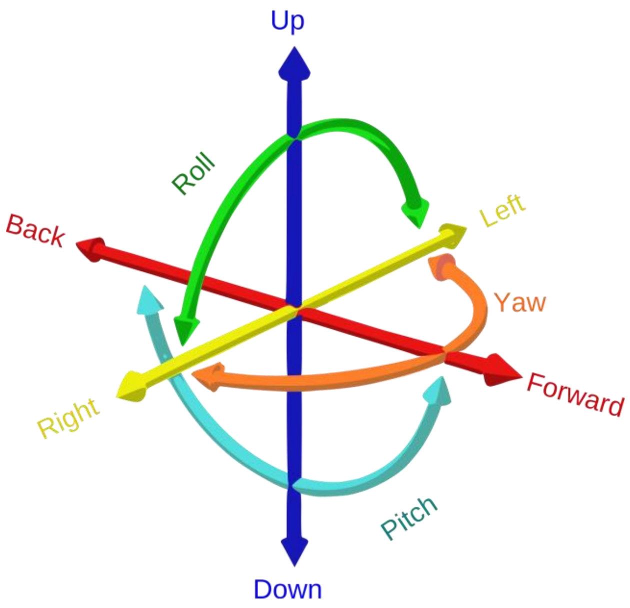


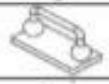
Degrees of Freedom (DoF)

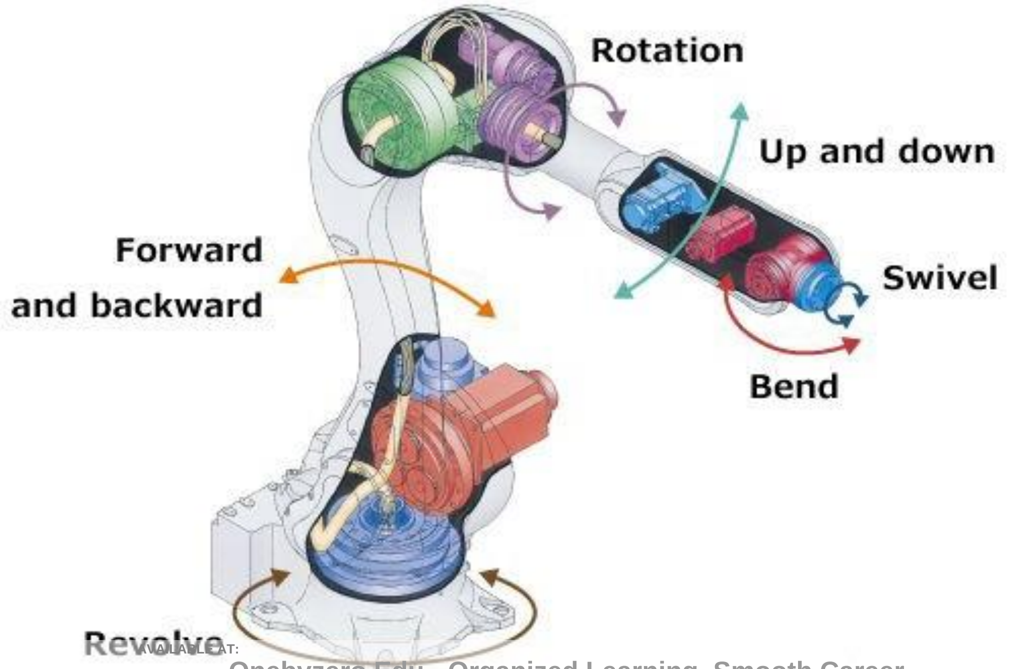
- In general, degrees of freedom (DOF) are the set of independent displacements that specify completely the displaced or deformed position of the body or system.
- In robotics, degrees of freedom is often used to describe the number of directions that a robot can move a joint.

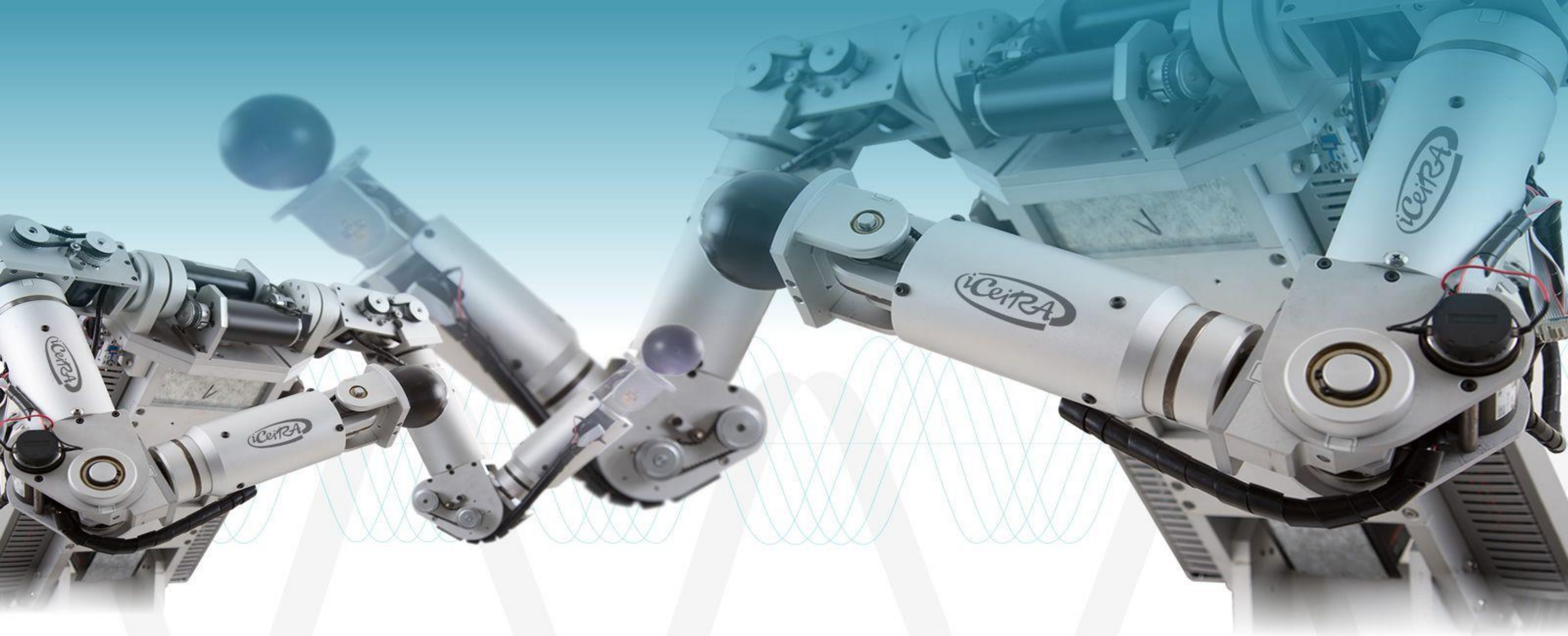


Degrees of Freedom (DoF)

- A human arm is considered to have 7 DOF. A shoulder gives pitch, yaw and roll, an elbow allows for pitch, and a wrist allows necessary to move the hand to any point in space, but people would lack the ability to grasp things from different angles or directions.
- A robot (or object) that has mechanisms to control all 6 physical DOF is said to be holonomic. An object with fewer controllable DOF than total DOF is to be non- holonomic and an object with more controllable DOF than total DOF (such as human arm) is said to be redundant.

Degrees of freedom	Free rotations	Free translations	Name	Kinematic pair: Form closure	Kinematic pair: Force closure
5	3	2	Sphere-plane		
4	3	1	Sphere-groove		
	2	2	Cylinder-plane		
3	3	0	Spheric		
	2	1	Sphere-slotted cylinder		
	1	2	Planar		
2	2	0	Slotted spheric		
	2	0	Toric		(shared with row above)
	1	1	Cylindric		
	1	1	Slotted cylinder		
1	1	0	Revolute		
	0	1	Prismatic		


Rotation
Up and down
Forward
and backward
Swivel
Bend
Revolve



Any Question???