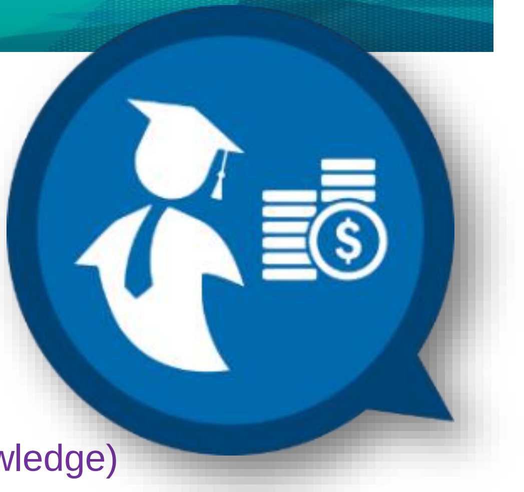
Computer Graphics

Two-Dimensional Transformations

Edited by
Dr. Md. Manjur Ahmed
Faculty of Computer Systems and Software Engineering
manjur@ump.edu.my

Chapter Description



- Aims
  - Basic of Computer Graphics.

- Expected Outcomes
  - Understand the basic concept of computer graphics. (CO1: Knowledge)
  - Ability to use the computer graphics technology. (CO1: Knowledge)

- References
  - Dr. Masudul Ahsan, Dept. Of CSE, Khulna University of Engineering and Technology (KUET), Bangladesh.
  - Computer Graphics by Zhigang Xiang, Schaum's Outlines.
  - Donald Hearn & M. Pauline Baker, Computer Graphics with OpenGL, 4th Edition, Boston : Addison Wesley, 2011.

Modeling Transformations

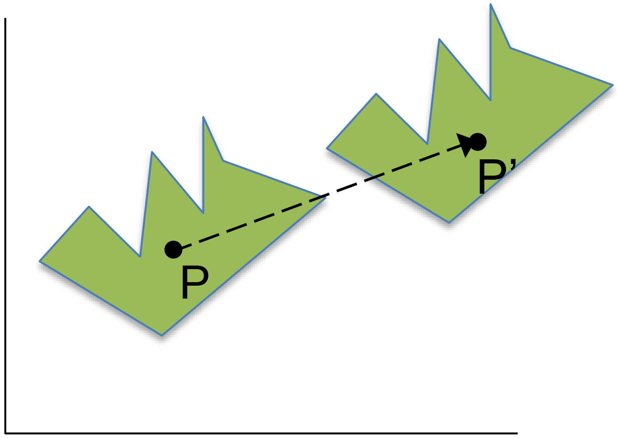
- Simulate the manipulation of objects in space
- Two contrary points of view for describing object
  - **Geometric transformation** –
    - Relative to a stationary coordinate system
    - Changes in orientation, size and shape
  - **Coordinate Transformation** – Keeping the object stationary while coordinate system is transformed with respect to the stationary object.

Geometric transformation

- Basic Transformations
  - Translation
  - Rotation
  - Scaling
  - Shear
  - Mirror reflection

Translation – 2D Transformation (about the Origin)

- Object is moved to a new position
  - ✓ Considering distance and direction



- Let, new point P′(x′,y′) is found by adding translation distance (t_x,t_y) to P(x,y). Then displacement vector is

$$x' = x + t_x \text{ and } y' = y + t_y \quad \text{----} \quad (1)$$

Let's column vector

$$P = \begin{bmatrix} x \\ y \end{bmatrix} \quad P' = \begin{bmatrix} x' \\ y' \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Therefore, Eqn (1) can be rewritten as $P' = P + T$

Rotation – 2D

- Object is rotated along a circular path using rotation angle (θ)
  - Rotation angle (θ)
    - Counter clockwise, + θ
    - Clockwise, - θ
  - Consider center (0,0) of rotation means that origin as pivot point

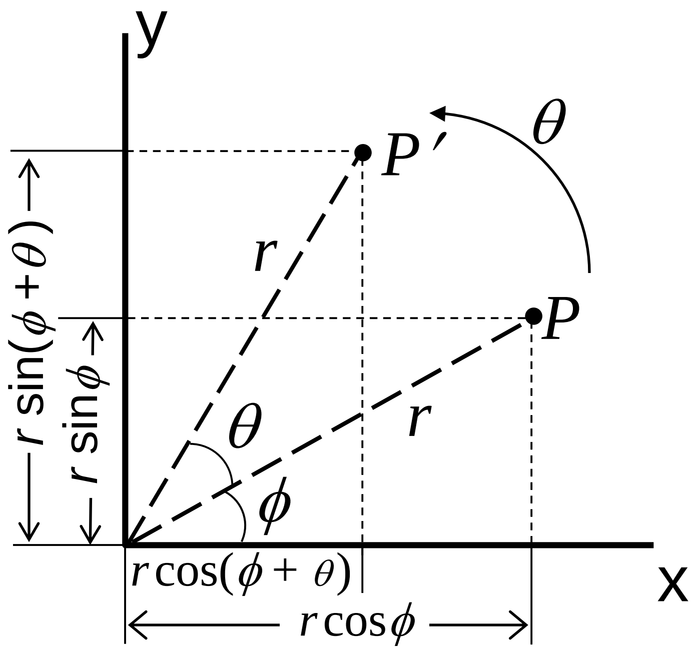
Rotation – 2D

$$P = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r\cos(\phi+\theta) \\ r\sin(\phi+\theta) \end{bmatrix}$$

$$= \begin{bmatrix} r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$



Rotation – 2D

Matrix form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \cdot \cos\theta - y \cdot \sin\theta \\ x \cdot \sin\theta + y \cdot \cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Therefore,

$$P' = R \cdot P$$

R is called Rotation Matrix

Scaling – 2D

- **Consider the dimensions of an object**, either Expand or compress
- **Scaling factors** for XY Plane (2D) are defined as s_x and s_y.

  magnification if s_x (or s_y) > 1

  reduction if s_x (or s_y) < 1

$$x' = s_x \cdot x \quad , \quad y' = s_y \cdot y$$

In matrix form where *S* is scaling matrix.

$$P' = S \cdot P$$

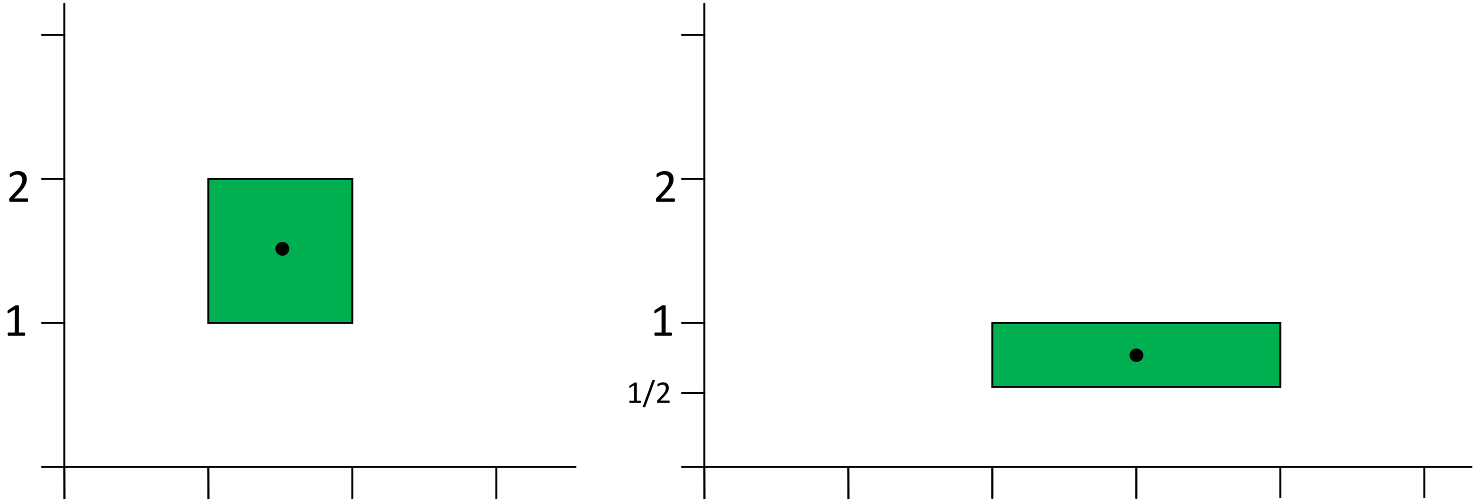
$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$\begin{bmatrix} x \cdot s_x \\ y \cdot s_y \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling – 2D

- After scaling,
  - Centroid can be changed and new object will be located at a different position relative to origin

$$\text{scale factors, } s_x = 2,\ s_y = \tfrac{1}{2}$$



Types of Scaling:

Differential (sx != sy)

Uniform (sx = sy)

Homogeneous Co-ordinates

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- **Non-homogeneously:** In matrix form, translation, scaling and rotation are defined as:
  - translation: P′ = P + T, → 2×2 Matrix form
  - Scale: P′ = S · P → 3×3 Matrix form
  - Rotate: P′ = R · P → 3×3 Matrix form
- **Composition Transformations (??)----** more than one transformations at a time
  - But translation not expressed as a matrix multiplication method, thus difficult to determine
- **Homogeneously:** Allow all three transformation by the **multiplication of 3×3 matrices**
- Each Cartesian position (x,y) is represented by a triple (x_h, y_h, h), where,

$$x = \frac{x_h}{h} \qquad y = \frac{y_h}{h}$$

Homogeneous Co-ordinates

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- h can have any non zero value, better to use h = 1
- allows all transformation eq as matrix multiplication, and

1)Transformation matrices for translation

$$x' = x + t_x \quad , \quad y' = y + t_y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

↓

$$T_{(t_x, t_y)}$$

Homogeneous Co-ordinates

2) Transformation matrices for rotation

$$x' = x \cdot \cos\theta - y \cdot \sin\theta \quad , \quad y' = x \cdot \sin\theta + y \cdot \cos\theta$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Downarrow$$

$$R_{(\theta)}$$

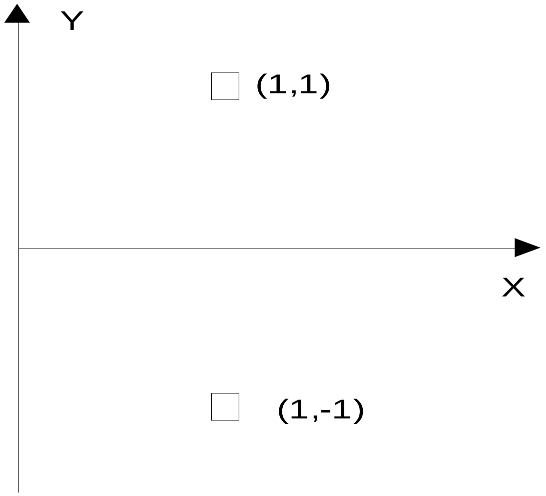
3) Transformation matrices for Scaling

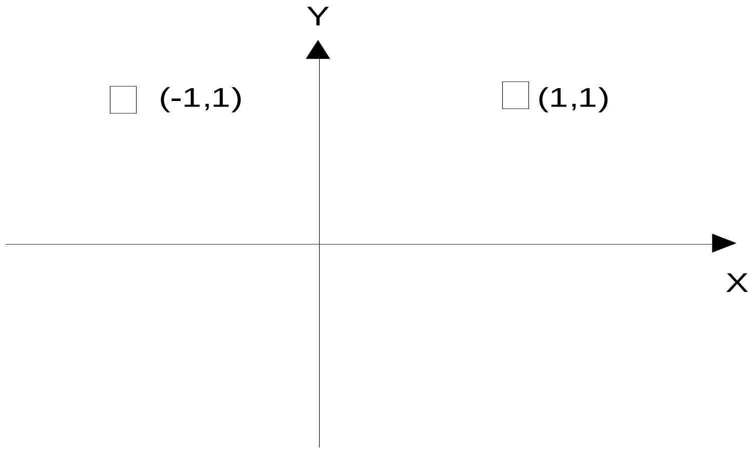
$$x' = s_x \cdot x \quad , \quad y' = s_y \cdot y$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_x & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\Downarrow \; S_{(s_x, s_y)}$$

Mirror Reflection





Reflection about X - axis

$$x' = x \quad y' = -y$$

$$M_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

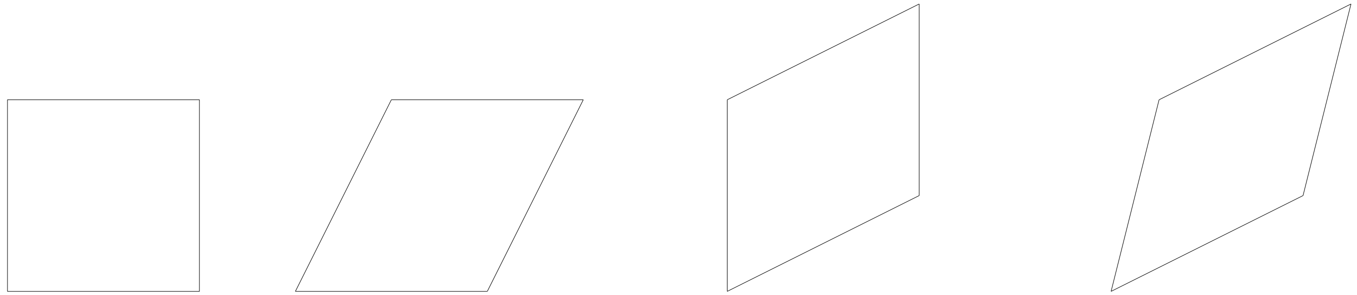
Reflection about Y - axis

$$x' = -x \quad y' = y$$

$$M_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Shearing Transformation

$$SH_x = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad SH_y = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad SH_{xy} = \begin{bmatrix} 1 & a & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



unit cube

Sheared in X direction

Sheared in Y direction

Sheared in both X and Y direction

Composite Transformation

- Composite transformation matrix based on the concatenation of transformation matrices
  - ✓ It leads to different results if the change in the order of transformation. Thus, the matrix, $[A] \cdot [B] \neq [B] \cdot [A]$

- For example, in order to rotate an object around an arbitrary point P(h,k):
  1) Translate P(h,k) to the origin.
  2) Rotate it around the origin.
  3) To finish, translate the center of rotation back as it was previously.

General Pivot point, P(h,k), Rotation of θ : $R_{\theta,P}$

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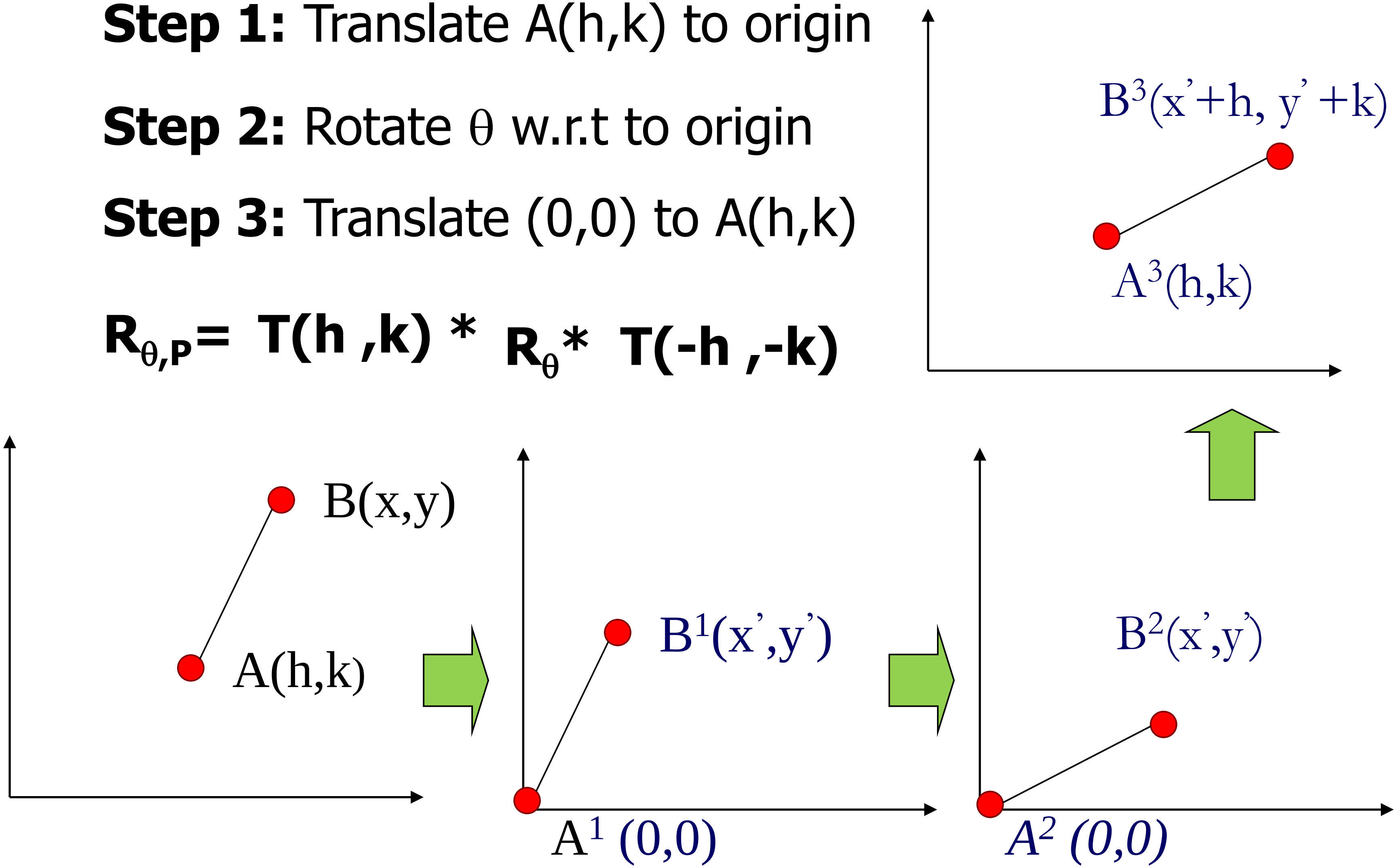
Step 1: Translate A(h,k) to origin

Step 2: Rotate θ w.r.t to origin

Step 3: Translate (0,0) to A(h,k)

$$R_{\theta,P} = T(h,k) * R_\theta * T(-h,-k)$$



Fixed point P(h,k) Scaling : $S_{sx,sy,p}$

Step 1: Translate P(h,k) to origin

Step 2: Scale $S(s_x,s_y)$ w.r.t origin

Step 3: Translate (0,0) to P(h,k)

$$S_{sx,sy,P} = T(h,k) * S(s_x,s_y) * T(-h,-k)$$