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Computer Graphics

3D Transformations

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Chapter Description

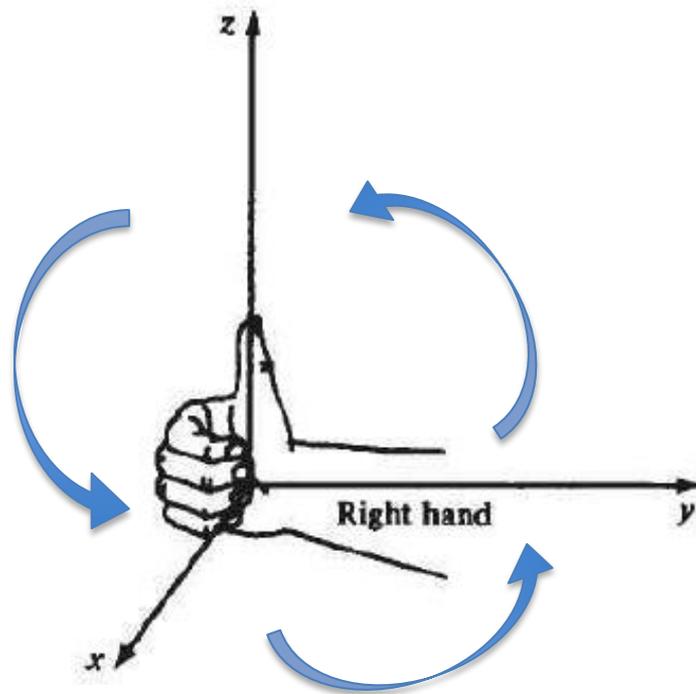
- **Aims**
 - Basic of Computer Graphics.
- **Expected Outcomes**
 - Understand the basic concept of computer graphics. (CO1: Knowledge)
 - Ability to use the computer graphics technology. (CO1: Knowledge)
- **References**
 - Computer Graphics by Zhigang Xiang, Schaum's Outlines.
 - Donald Hearn & M. Pauline Baker, Computer Graphics with OpenGL, 4th Edition, Boston : Addison Wesley, 2011.



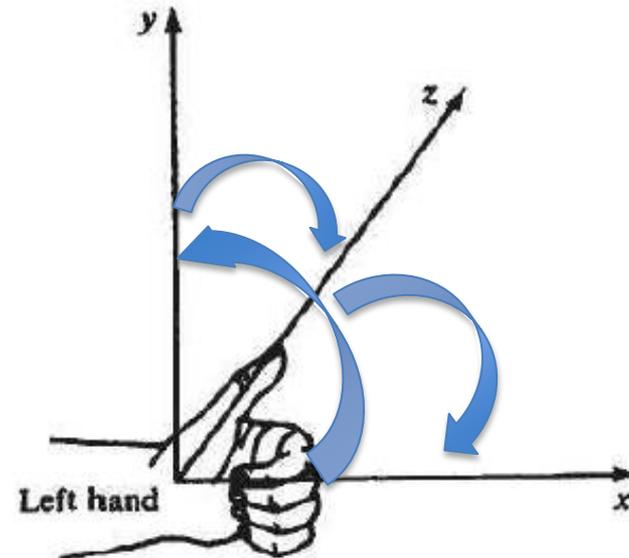
3D Transformations

- In this Chapter, we will learn the basics of
 - 3D geometry
 - And its 3D Transformations
 - Composite Transformations for 3D geometry.

Orientation



Right-handed Orientation



Left-handed Orientation

Vectors in 3D

$$V = [a, b, c]$$

Length using Euclidean Norm

$$|V| = \sqrt{a^2 + b^2 + c^2}$$

Dot Product: $V \cdot U$

$$\begin{aligned} & [x_v, y_v, z_v] \cdot [x_u, y_u, z_u] \\ & = x_v * x_u + y_v * y_u + z_v * z_u \end{aligned}$$

And, $V \cdot U = |V| |U| \cos \beta$

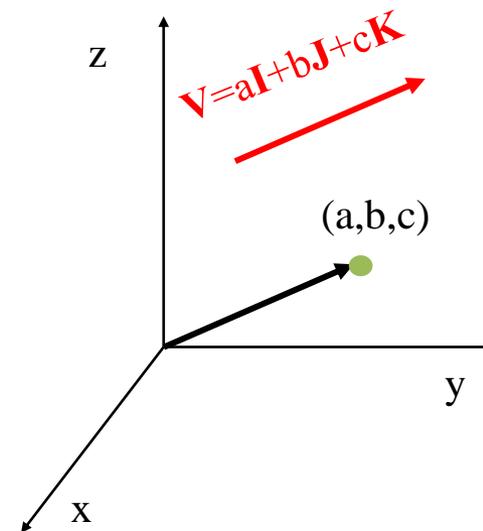
Cross Product : $V \times U$

$$[y_v * z_u - z_v * y_u, -x_v * z_u + z_v * x_u, x_v * y_u - y_v * x_u]$$

And, $V \times U = \eta |V| |U| \sin \beta$

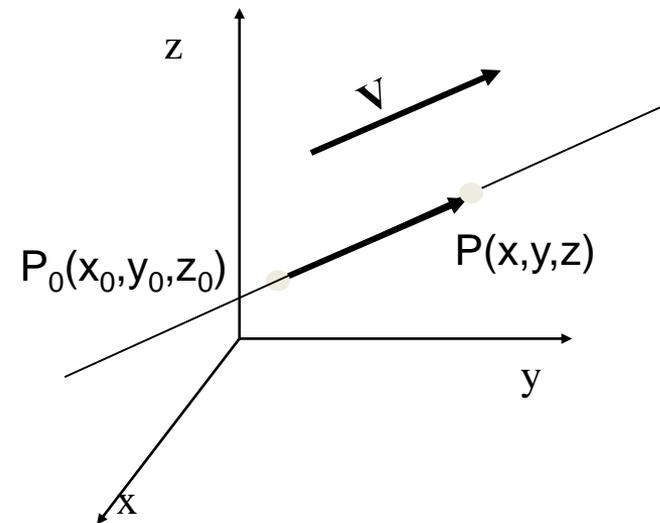
Moreover,

$$V \times U = - (U \times V)$$

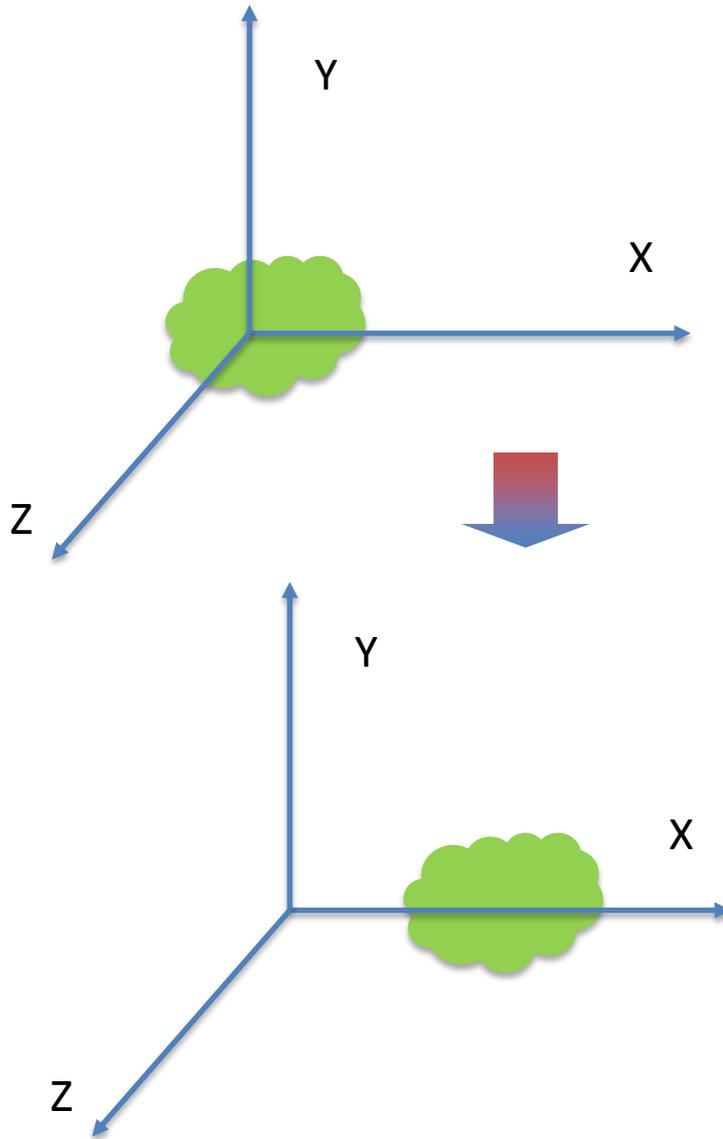


Vector Equation

- **Line** is defined by direction and point
 - direction vector, \vec{V}
 - A point, $P_0(x_0, y_0, z_0)$



Translation – 3D



$$x' = x + d_x$$

$$y' = y + d_y$$

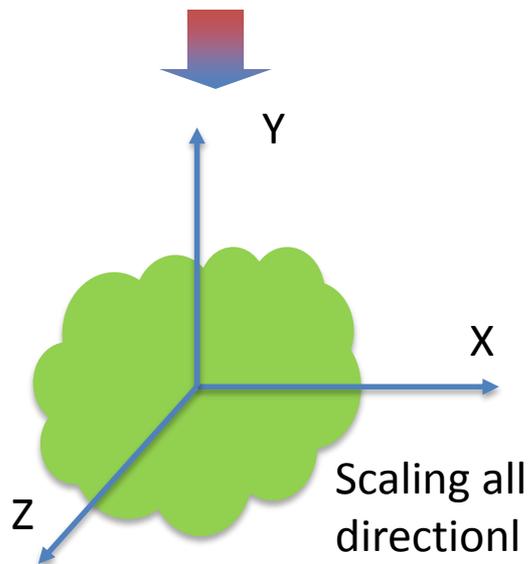
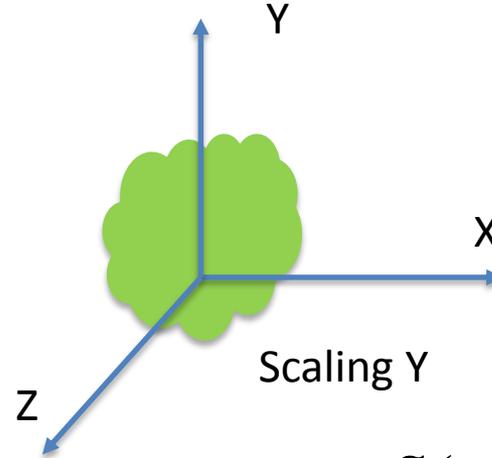
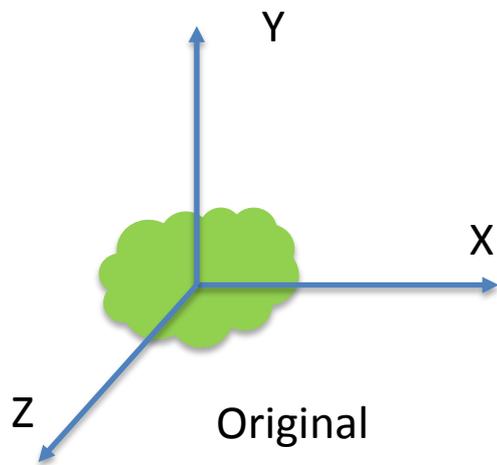
$$z' = z + d_z$$

$$\begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + d_x \\ y + d_y \\ z + d_z \\ 1 \end{bmatrix}$$

\Downarrow \Downarrow \Downarrow

$$T(d_x, d_y, d_z) * P = P'$$

Scaling – 3D



$$x' = s_x * x$$

$$y' = s_y * y$$

$$z' = s_z * z$$

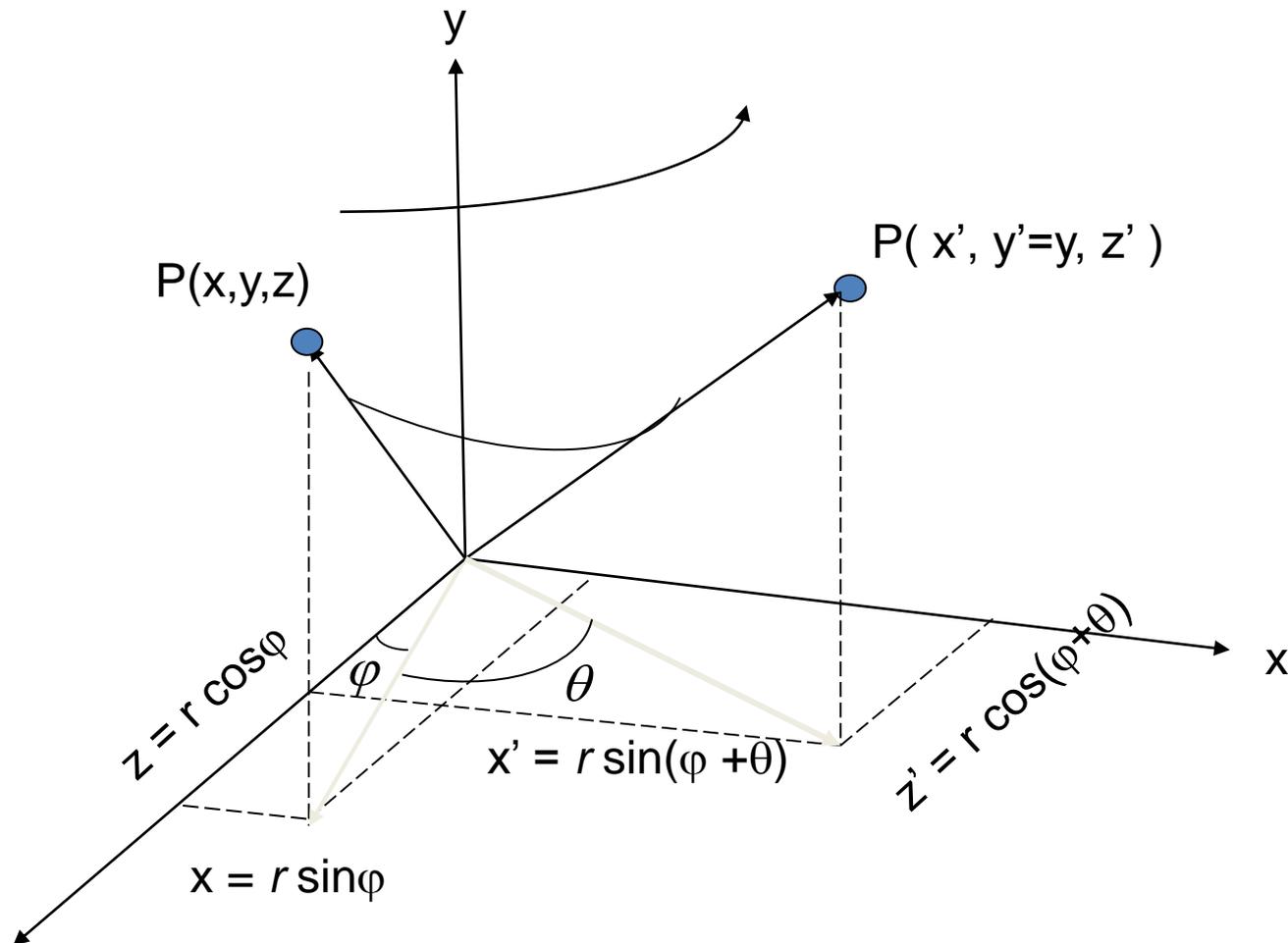


$$S(s_x, s_y, s_z) * P = P'$$



$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x * s_x \\ y * s_y \\ z * s_z \\ 1 \end{bmatrix}$$

Rotation in 3D



Rotation in 3D

$$x = r \sin \varphi$$

$$z = r \cos \varphi$$

$$\begin{aligned} x' &= r \sin(\varphi + \theta) = r \sin \varphi \cos \theta + r \cos \varphi \sin \theta \\ &= x \cos \theta + z \sin \theta \end{aligned}$$

$$y' = y$$

$$\begin{aligned} z' &= r \cos(\varphi + \theta) = r \cos \varphi \cos \theta - r \sin \varphi \sin \theta \\ &= z \cos \theta - x \sin \theta \\ &= -x \sin \theta + z \cos \theta \end{aligned}$$

Rotation in 3D

About y-axis

$$\begin{array}{ccc}
 R_{\theta, j} & * & P = & P' \\
 \Downarrow & & \Downarrow & \Downarrow \\
 \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & * & \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} & = & \begin{bmatrix} x * \cos \theta + z * \sin \theta \\ y \\ -x * \sin \theta + z * \cos \theta \\ 1 \end{bmatrix}
 \end{array}$$

Rotation – 3D

Rotation about z-axis:

$$\begin{array}{ccc}
 R_{\theta,k} & * & P = P' \\
 \Downarrow & & \Downarrow \\
 \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & * & \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x * \cos \theta - y * \sin \theta \\ x * \sin \theta + y * \cos \theta \\ z \\ 1 \end{bmatrix}
 \end{array}$$

Rotation about Y-axis & X-axis

About x-axis

$$\begin{array}{ccccc}
 R_{\theta,i} & * & P & = & P' \\
 \Downarrow & & \Downarrow & & \Downarrow \\
 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & * & \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} & = & \begin{bmatrix} x \\ y * \cos \theta - z * \sin \theta \\ y * \sin \theta + z * \cos \theta \\ 1 \end{bmatrix}
 \end{array}$$