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Computer Graphics

Composite Transformation 3D

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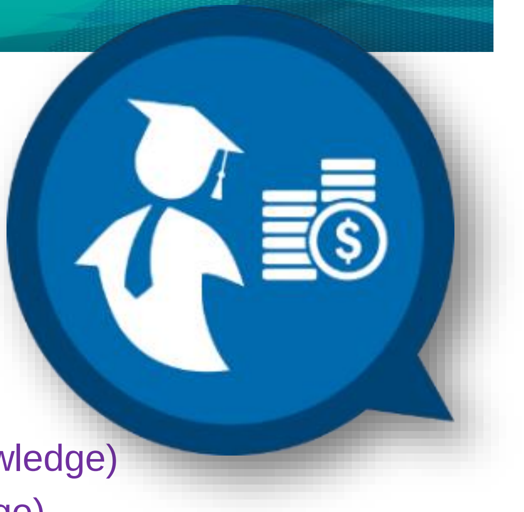
Chapter Description



- Aims
  - Basic of Computer Graphics.

- Expected Outcomes
  - Understand the basic concept of computer graphics. (CO1: Knowledge)
  - Ability to use the computer graphics technology. (CO1: Knowledge)

- References
  - Computer Graphics by Zhigang Xiang, Schaum's Outlines.
  - Donald Hearn & M. Pauline Baker, Computer Graphics with OpenGL, 4th Edition, Boston : Addison Wesley, 2011.

Composite Transformations – 3D

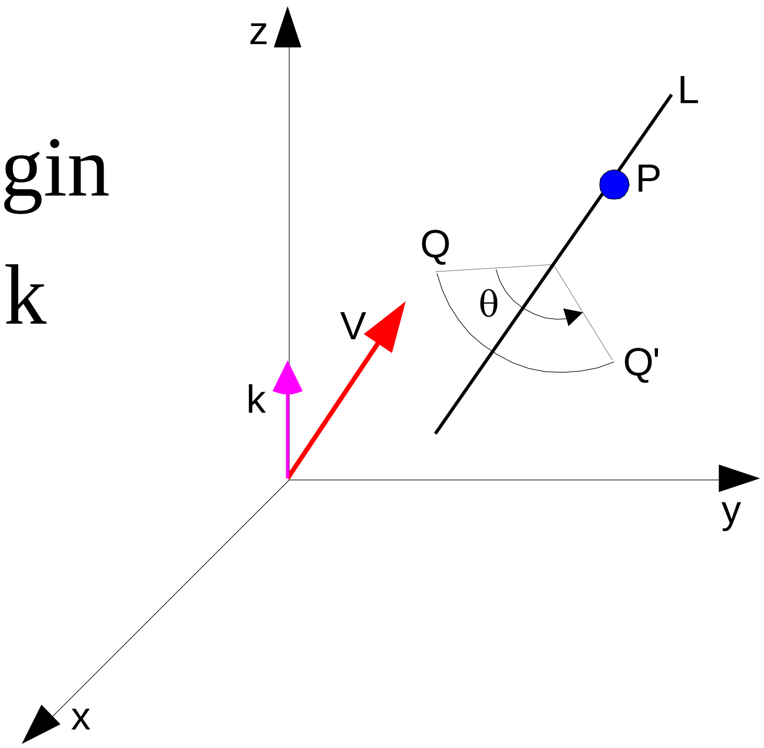
Basic composite transformations :

- $R_{\theta,L}$ = rotation about an axis L(**V**, P)
- $S_{sx,sy,P}$= scaling w.r.t. point P

$R_{\theta,L}$: rotation about an axis L

Let the axis L be represented by vector V and passing through point P

1. Translate P to the origin
2. Align V with vector k
3. Rotate θ° about k
4. Reverse step 2
5. Reverse step 1



$$T_{-P}^{-1} * A_V^{-1} * R_{\theta,k} * A_V * T_{-P}$$

Composition: Translate points

- Figure 1 to Figure 2:
  - P_1 is at Origin
  - P_1P_2 is along positive z-axis
  - P_1P_3 lies in positive y-axis half of yz plane

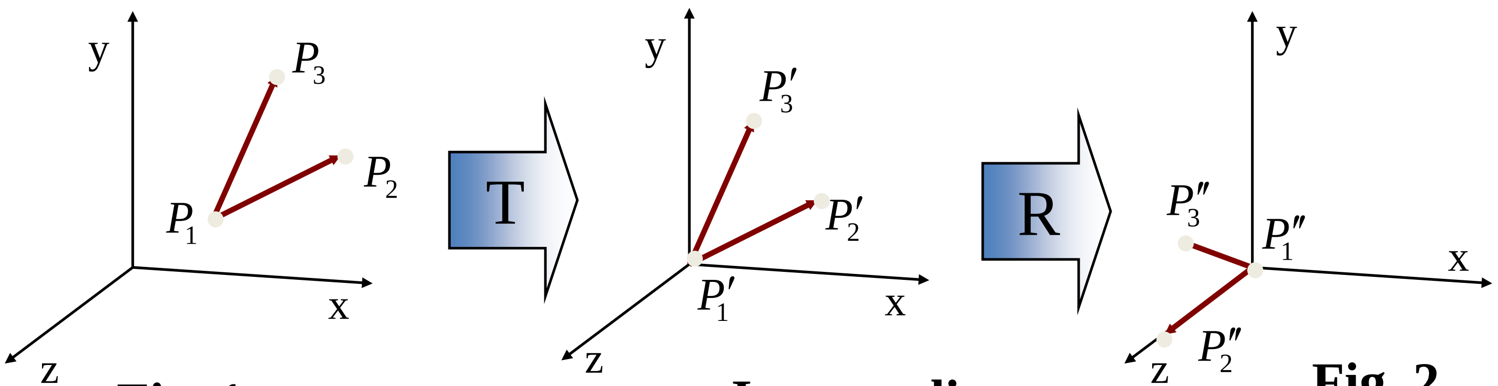


Fig. 1 **Intermediate** **Fig. 2**

Composition: Translate points (cont.)

- The Composite Transformation:

  Translation of P_1 to Origin $\Rightarrow$ T

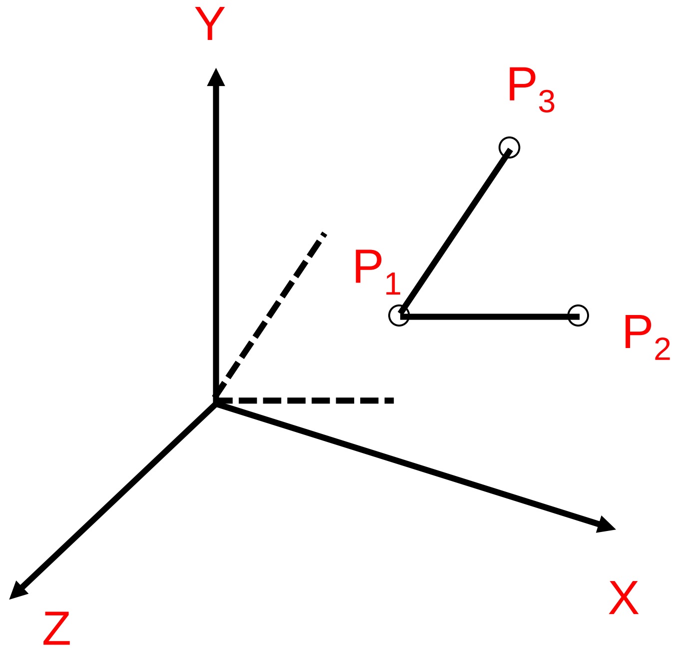
  Some Combination of Rotations $\Rightarrow$ **R**

Composition: Translate points (cont.)

1. Translate P_1 to Origin

$$T_{(-p_1)} = \begin{pmatrix} 1 & 0 & 0 & -x_1 \\ 0 & 1 & 0 & -y_1 \\ 0 & 0 & 1 & -z_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad P_1' = T_{(-p_1)}.P_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$P_2' = T_{(-p_1)}.P_2 = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \\ 1 \end{pmatrix} \qquad P_3' = T_{(-p_1)}.P_3 = \begin{pmatrix} x_3 - x_1 \\ y_3 - y_1 \\ z_3 - z_1 \\ 1 \end{pmatrix}$$



Composition: Translate points (cont.)

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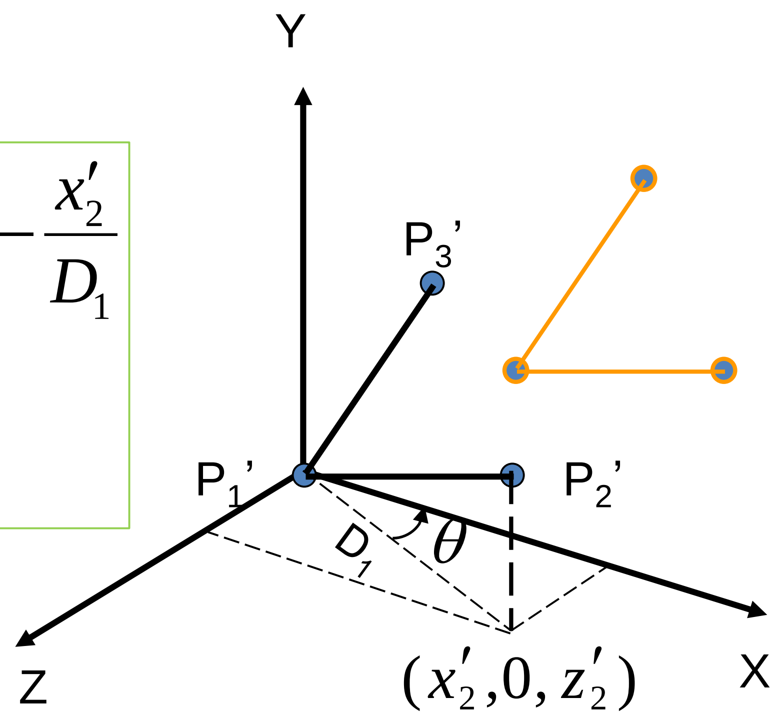
2. Rotate such that P_1P_2 lies on yz palne, Rotate about Y axis

- Angle = $-(90^o - \theta)$

$$D_1 = \sqrt{x_2'^2 + z_2'^2}$$

$$\sin(-(90^\circ - \theta)) = -\sin(90^\circ - \theta) = -\cos\theta = -\frac{x_2'}{D_1}$$

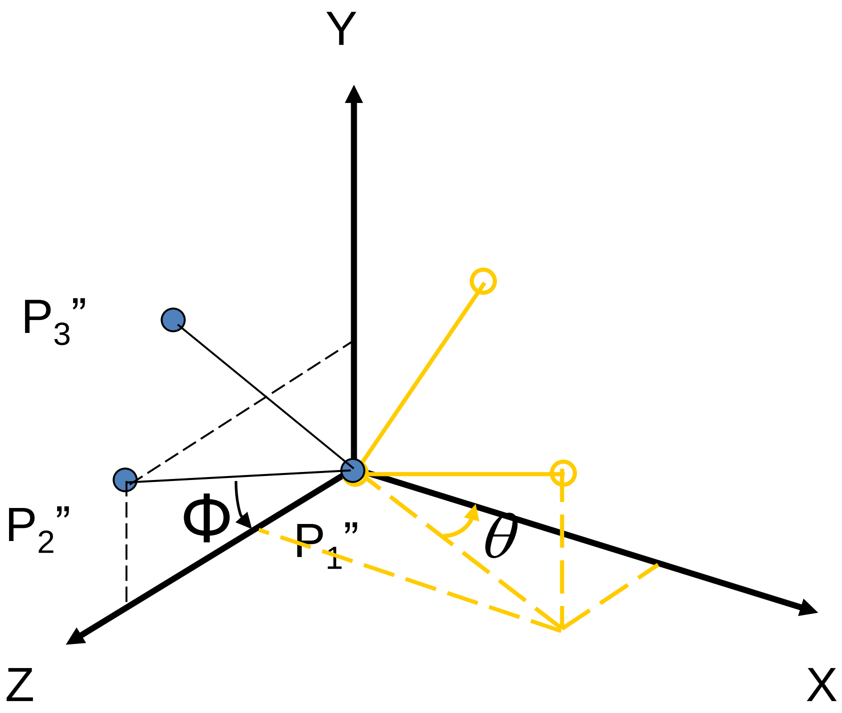
$$\cos(-(90^\circ - \theta)) = \cos(90^\circ - \theta) = \sin\theta = \frac{z_2'}{D_1}$$



Composition: Translate points (cont.)

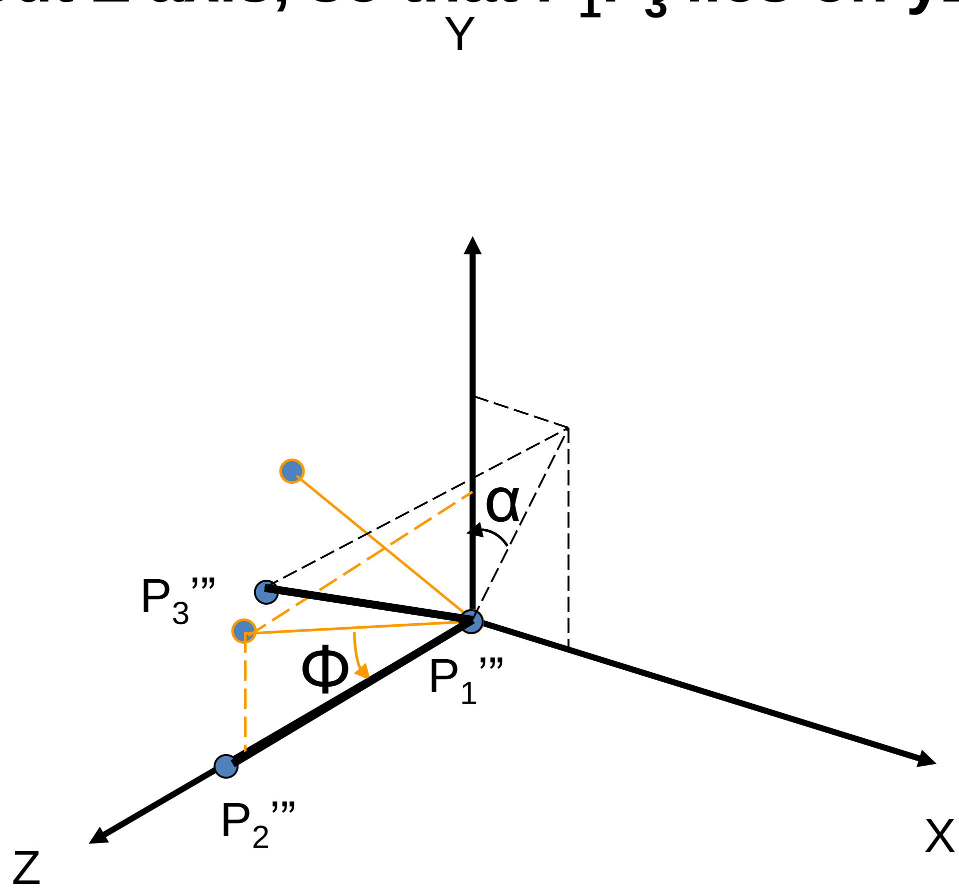
3. Rotate about X axis, so that P_1P_2 lies on z axis

$$P_2'' = R_{-(90^\circ - \theta)} \bullet P_1' = \begin{pmatrix} 0 \\ y_2' \\ D_1 \\ 1 \end{pmatrix}$$



Composition: Translate points (cont.)

4. Rotate about Z axis, so that P_1P_3 lies on yz palne



Composition: Translate points (cont.)

Finally,

$$C = R_{\alpha,k} \cdot R_{\phi,i} \cdot R_{(\theta-90),j} \cdot T_{(-p_1)}$$