Basic Mechanical Engineering

(Credit: 3.00)

EEE 1207

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Simplification of a Force and Couple System

Sometimes it is convenient to reduce a system of forces and couple moments acting on a body to a simpler form by replacing it with an *equivalent system*, consisting of a single resultant force acting at a specific point and a resultant couple moment.

A system is equivalent if the external effects it produces on a body are the same as those caused by the original force and couple moment system.

In this context, the external effects of a system refer to the *translating and rotating motion* of the body if the body is free to move, or it refers to the *reactive forces* at the supports if the body is held fixed.

For example, if the stick in fig.(a), is subjected to the force **F** at point A.

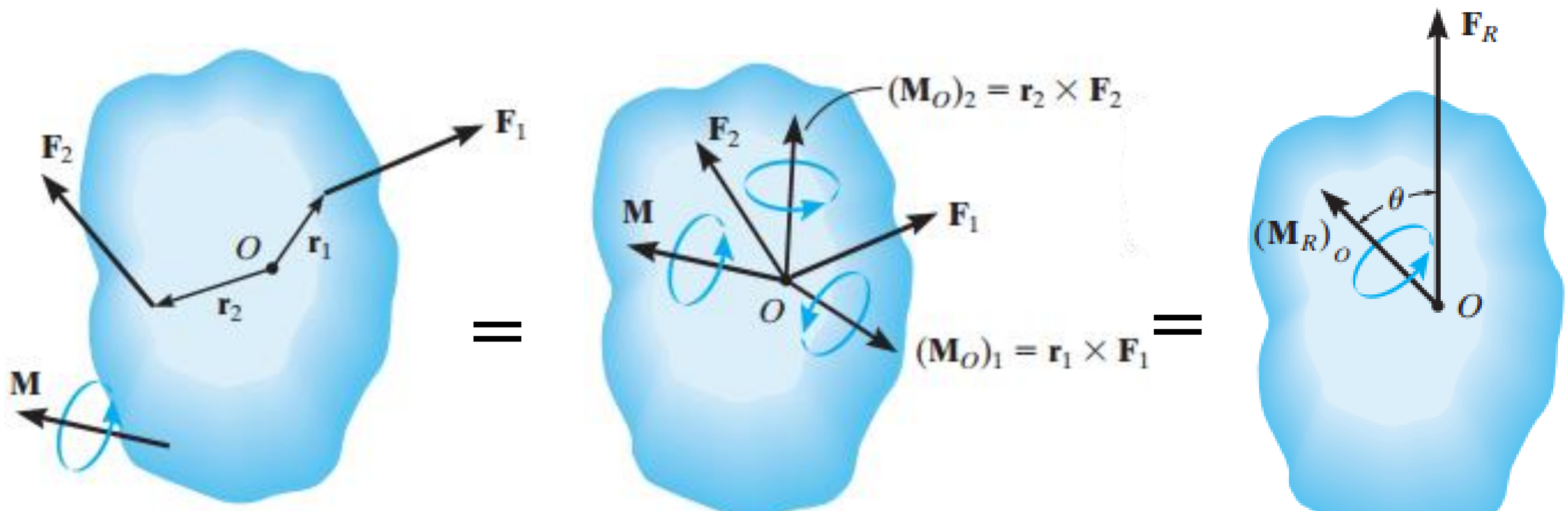
(a) (b) (c)

If **F** is applied perpendicular to the stick, as shown in fig. (*a*).

(a) (b) (c)

System of Forces and Couple Moments: Using the above method, a system of several forces and couple moments acting on a body can be reduced to an equivalent single resultant force acting at a point O and a resultant couple moment.

For example, in fig. (a), O is not on the line of action of $\mathbf{F}_1$, and so this force can be moved to point O provided a couple moment $(\mathbf{M}_O)_1 = \mathbf{r}_1 \times \mathbf{F}_1$ is added to the body.



(a) (b) (c)

Similarly, the couple moment $(\mathbf{M}_O)_2 = \mathbf{r}_2 \times \mathbf{F}_2$ should be added to the body when we move $\mathbf{F}_2$ to point O. Finally, since the couple moment $\mathbf{M}$ is a free vector, it can just be moved to point O.

By doing this, we obtain the equivalent system shown in fig. (*b*), which produces the same external effects (support reactions) on the body as that of the force and couple system shown in fig. (*a*).

If we sum the forces and couple moments, we obtain the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and the resultant couple moment $(\mathbf{M}_R)_O = \mathbf{M} + (\mathbf{M}_O)_1 + (\mathbf{M}_O)_2$, shown in fig. (*c*).

We can generalize the above method of reducing a force and couple system to an equivalent resultant force $\mathbf{F}_R$ acting at point O and a resultant couple moment $(\mathbf{M}_R)_O$ by using the following two equations.

$$\mathbf{F}_R = \Sigma\mathbf{F}$$

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O + \Sigma\mathbf{M}$$

The first equation states that the resultant force of the system is equivalent to the sum of all the forces; and the second equation states that the resultant couple moment of the system is equivalent to the sum of all the couple moments $\Sigma\mathbf{M}$ plus the moments of all the forces $\Sigma\mathbf{M}_O$ about point O.

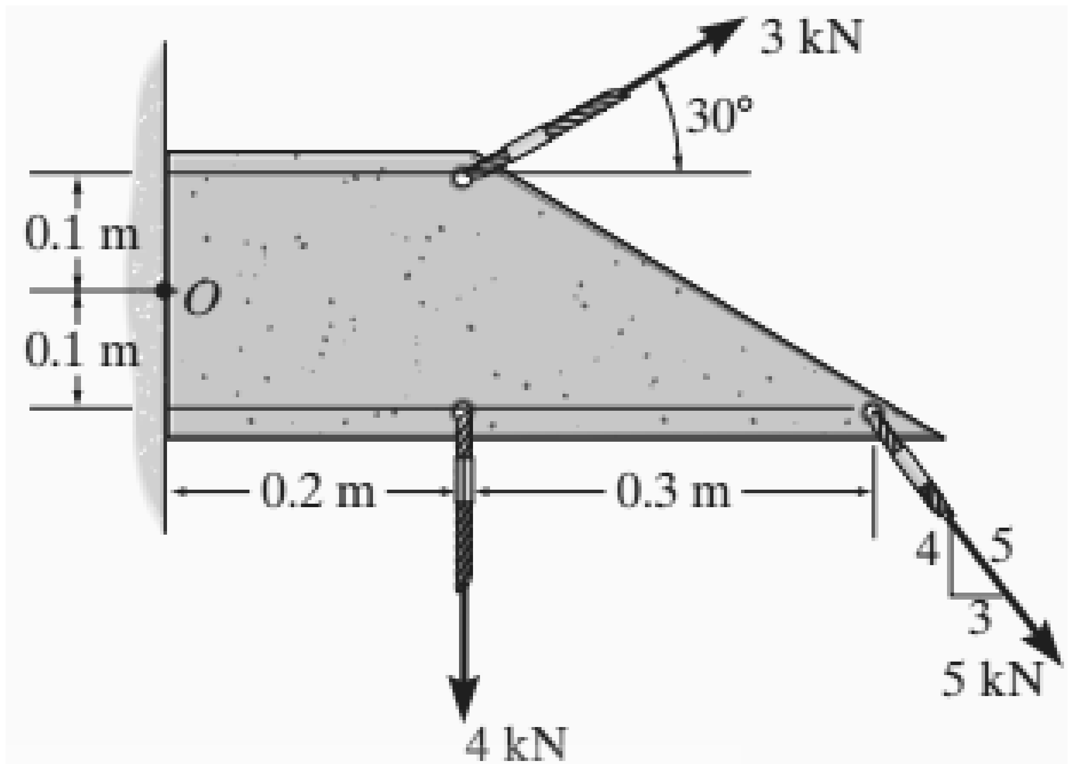
If the force system lies in the *x–y* plane and any couple moments are perpendicular to this plane, then the above equations reduce to the following three scalar equations.

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$

Here the resultant force is determined from the vector sum of its two components $(F_R)_x$ and $(F_R)_y$

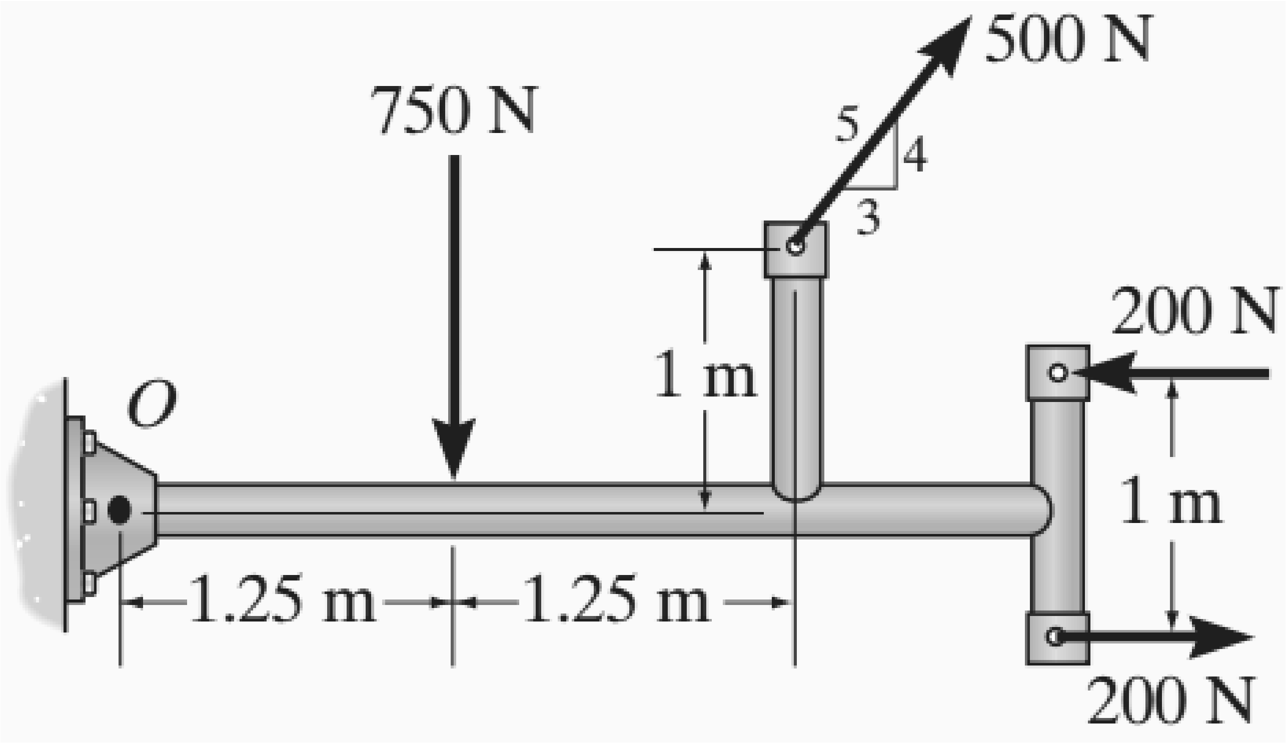
EXAMPLE: 4.14

Replace the force and couple system shown in fig. (*a*) by an equivalent resultant force and couple moment acting at point *O*.



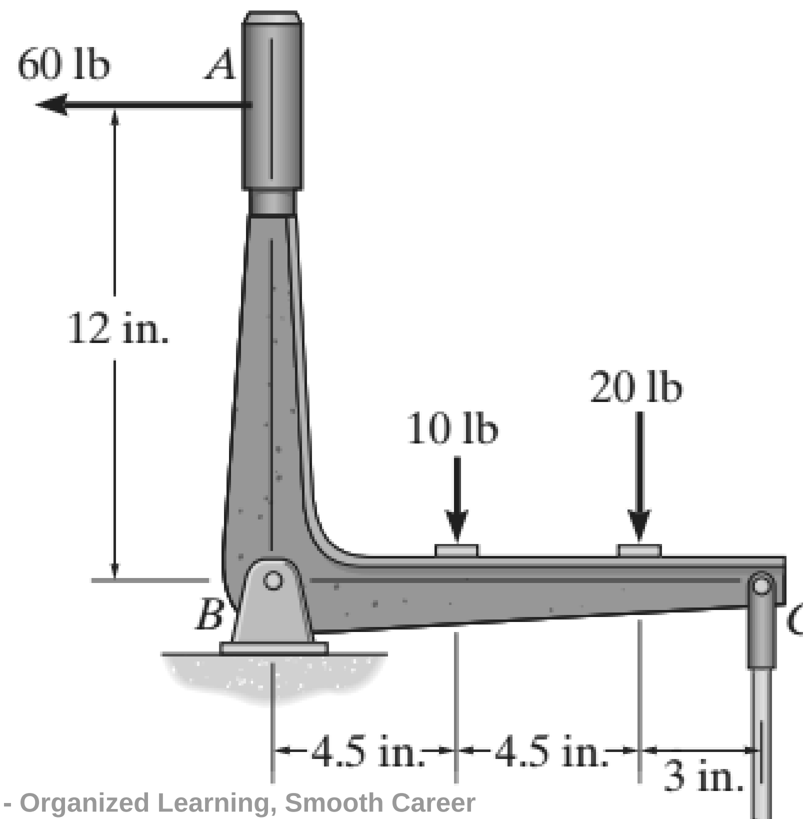
EXAMPLE: 4.15

Replace the force and couple system acting on the member in fig. (*a*) by an equivalent resultant force and couple moment acting at point *O*.



Problem No.: 4.104

Replace the force system acting on the crank by a resultant force, and specify where its line of action intersects *BA* measured from the pin at *B*.



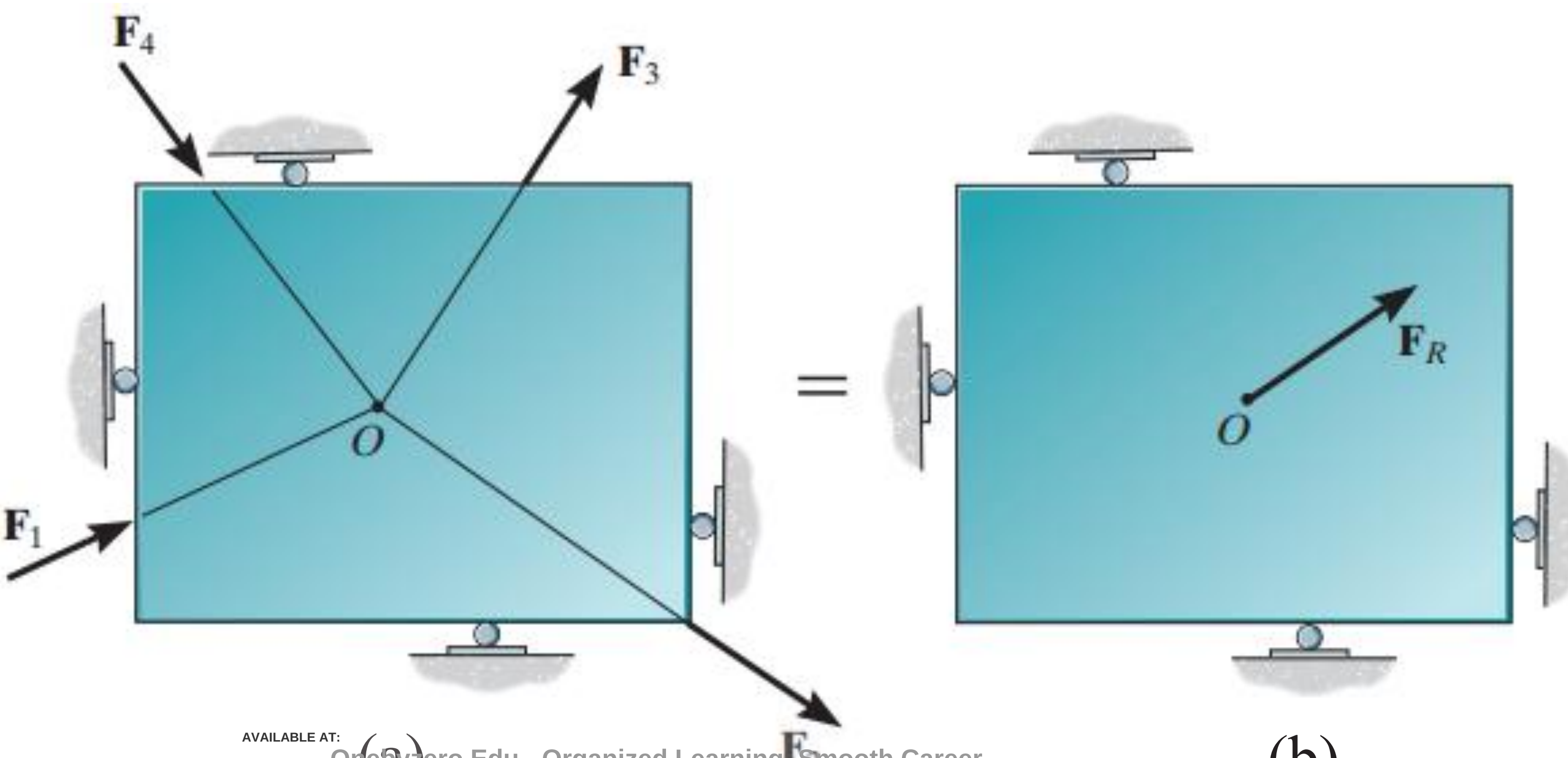
Further Simplification of a Force and Couple System

Previously, we developed a way to reduce a force and couple moment system acting on a rigid body into an equivalent resultant force $\mathbf{F}_R$ acting at a specific point O and a resultant couple moment $(\mathbf{M}_R)_O$.

The force system can be further reduced to an equivalent single resultant force provided the lines of action of $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are *perpendicular* to each other.

Because of this condition, only concurrent, coplanar, and parallel force systems can be further simplified.

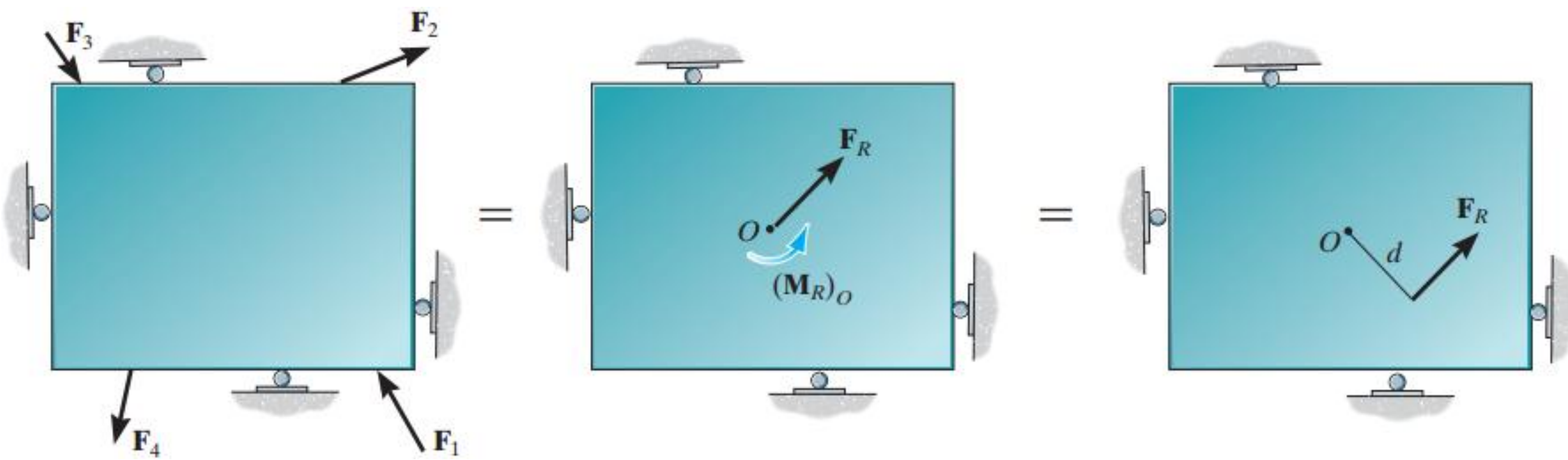
Concurrent Force System: Since a *concurrent force system* is one in which the lines of action of all the forces intersect at a common point *O*, fig. (a), then the force system produces no moment about this point. As a result, the equivalent system can be represented by a single resultant force $\mathbf{F}_R = \mathbf{F}$ acting at *O*, fig. (b).



(a) (b)

Coplanar Force System: In the case of a *coplanar force system*, the lines of action of all the forces lie in the same plane, fig. (a), and so the resultant force $\mathbf{F}_R = \mathbf{F}$ of this system also lies in this plane.

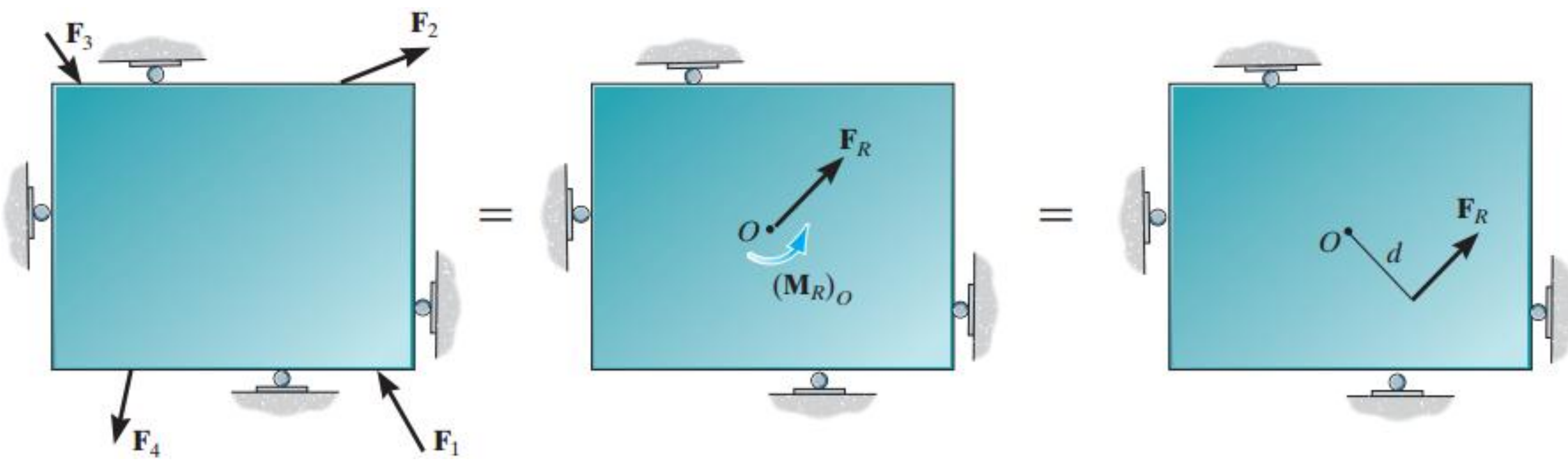
Furthermore, the moment of each of the forces about any point O is directed perpendicular to this plane. Thus, the resultant moment $(\mathbf{M}_R)_O$ and resultant force $\mathbf{F}_R$ will be *mutually perpendicular*, fig. (b).



(a) (b) (c)

The resultant moment can be replaced by moving the resultant force $\mathbf{F}_R$ a perpendicular or moment arm distance d away from point O such that $\mathbf{F}_R$ produces the *same moment* $(\mathbf{M}_R)_O$ about point O, fig. (c).

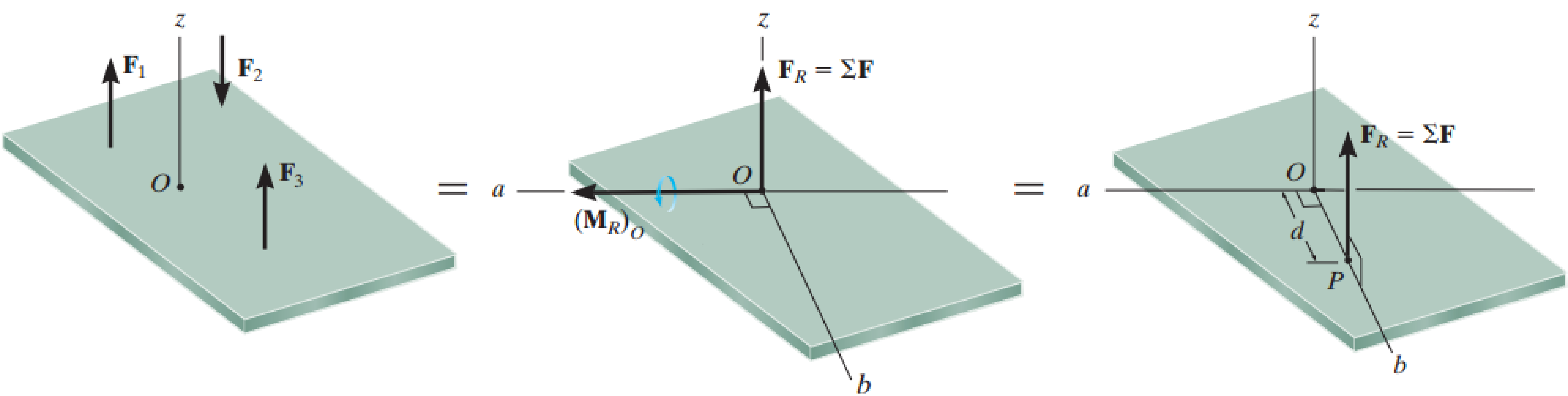
This distance d can be determined from the scalar equation $(M_R)_O = F_R d = M_O$ or $d = (M_R)_O / F_R$.



(a) (b) (c)

Parallel Force System: The *parallel force system* shown in fig. (a) consists of forces that are all parallel to the *z* axis. Thus, the resultant force $\mathbf{F}_R = \mathbf{F}$ at point *O* must also be parallel to this axis, fig. (b).

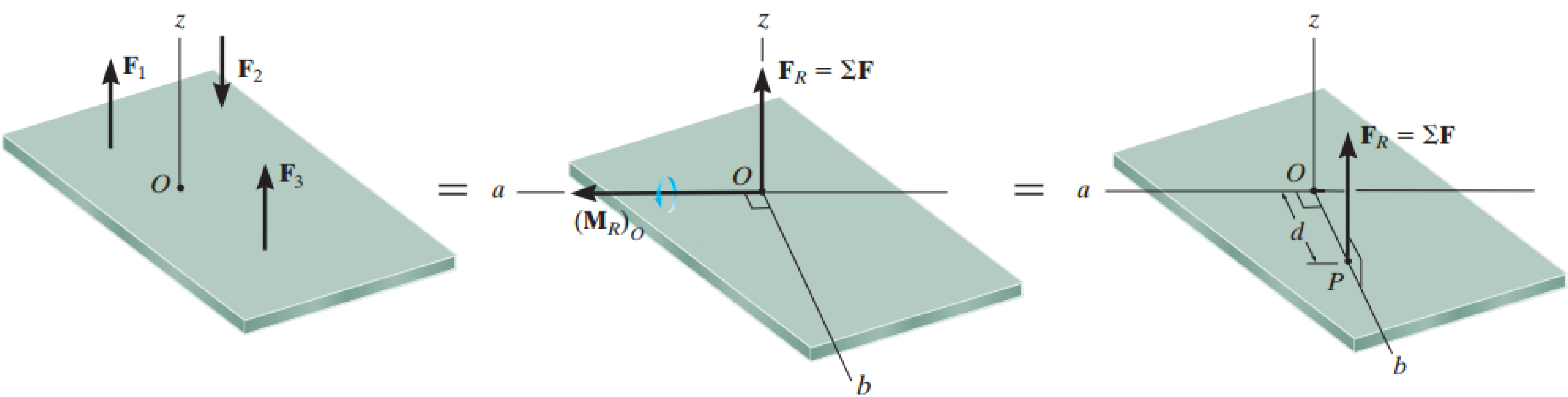
The moment produced by each force lies in the plane of the plate, and so the resultant couple moment, $(\mathbf{M}_R)_O$, will also lie in this plane, along the moment axis *a* since $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are mutually perpendicular.



(a) (b) (c)

As a result, the force system can be further reduced to an equivalent single resultant force $\mathbf{F}_R$, acting through point P located on the perpendicular b axis, fig. (c).

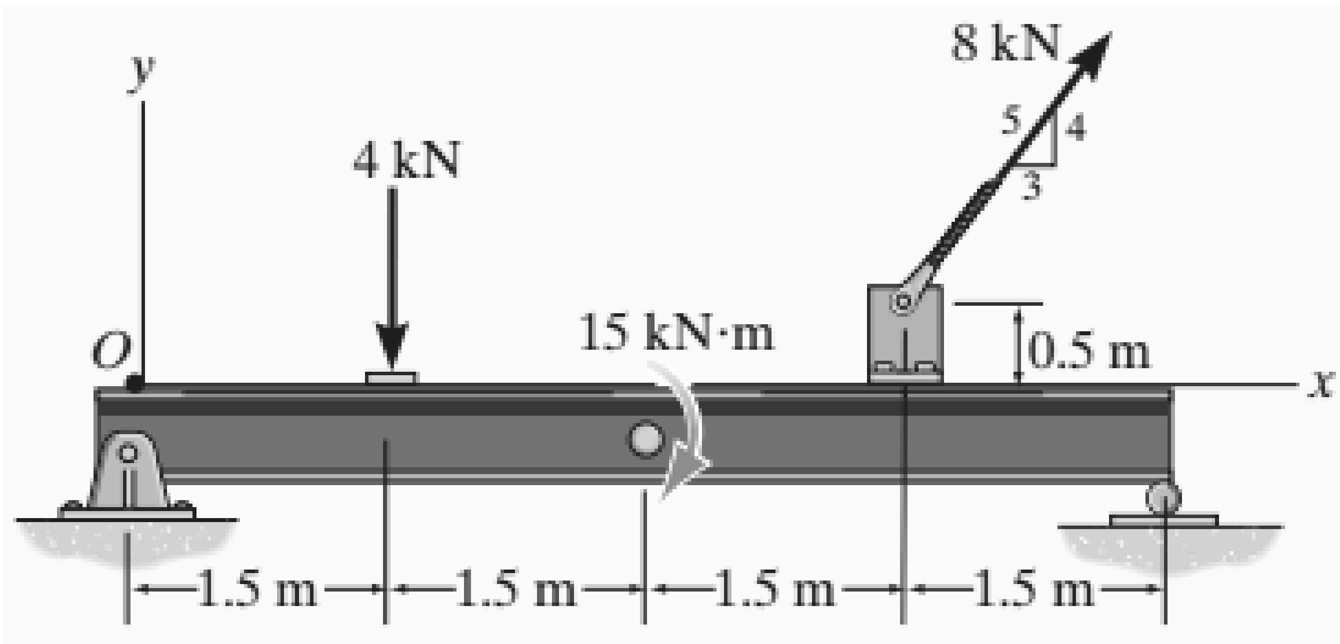
The distance d along this axis from point O requires $(M_R)_O = F_R d = M_O$ or $d = M_O > F_R$.



(a) (b) (c)

EXAMPLE: 4.17

Replace the force and couple moment system acting on the beam in figure by an equivalent resultant force, and find where its line of action intersects the beam, measured from point *O*.



Force Summation. Summing the force components,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN} \rightarrow$$

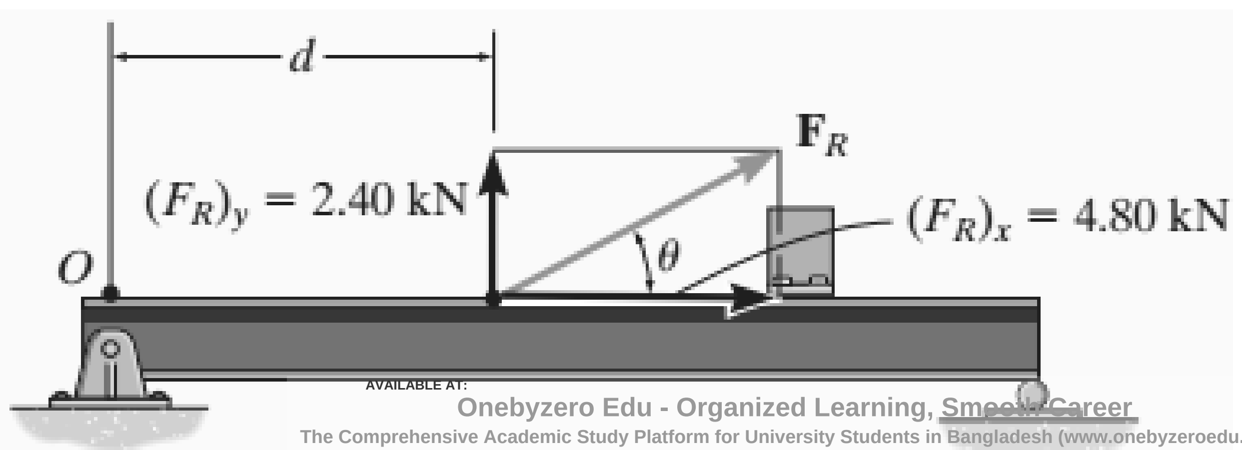
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN} \qquad \textit{Ans.}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ \qquad \textit{Ans.}$$

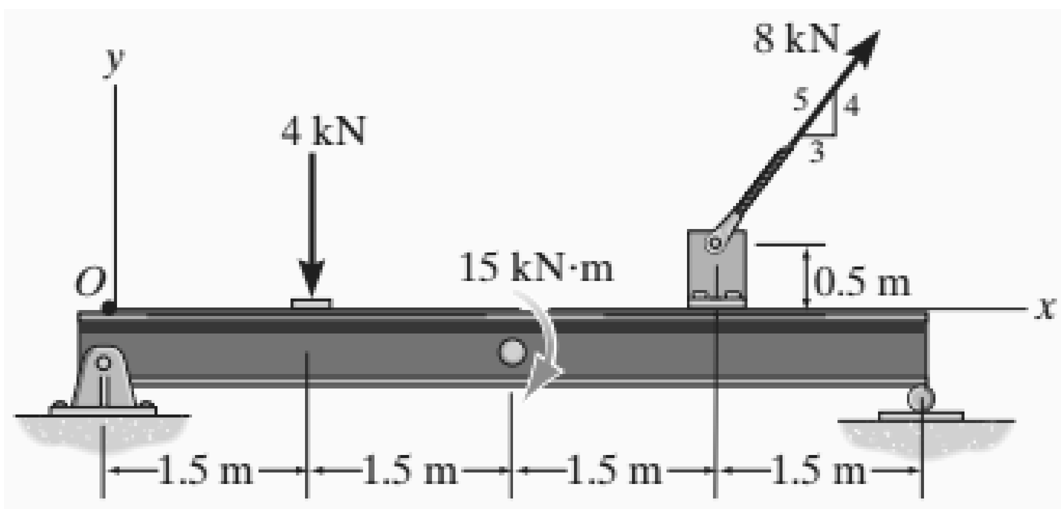


Moment Summation. We must equate the moment of $\mathbf{F}_R$ about point O in Fig. 4–44b to the sum of the moments of the force and couple moment system about point O in Fig. 4–44a. Since the line of action of $(\mathbf{F}_R)_x$ acts through point O, *only* $(\mathbf{F}_R)_y$ *produces a moment about this point*. Thus,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

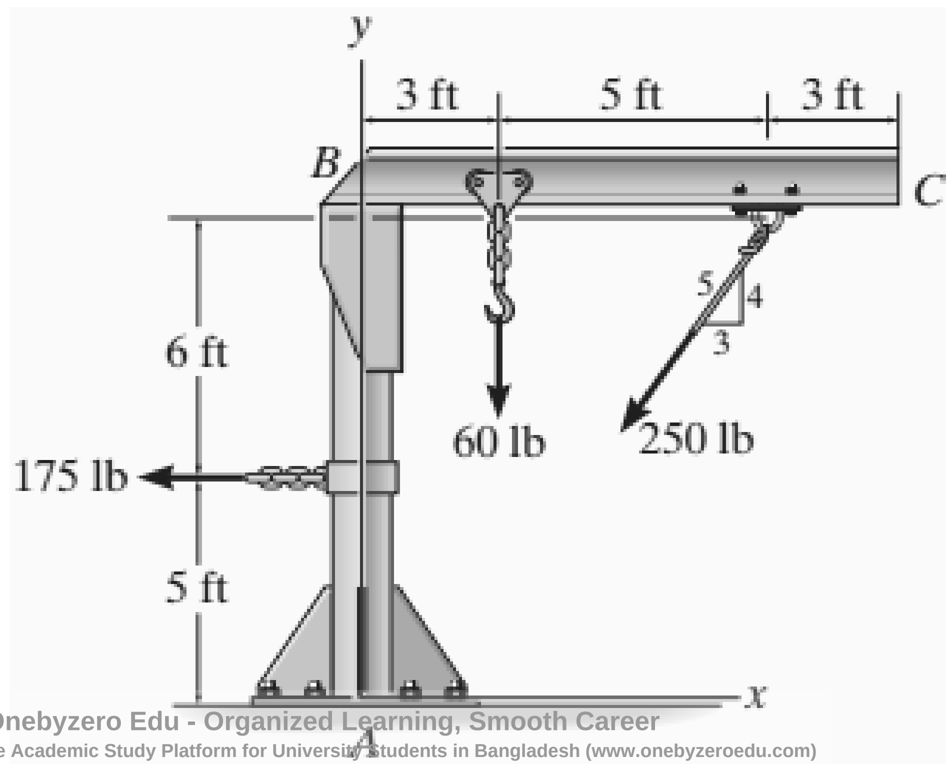
$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m} \qquad \textit{Ans.}$$



EXAMPLE: 4.18

The jib crane shown in figure is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column *AB* and boom *BC*.



SOLUTION

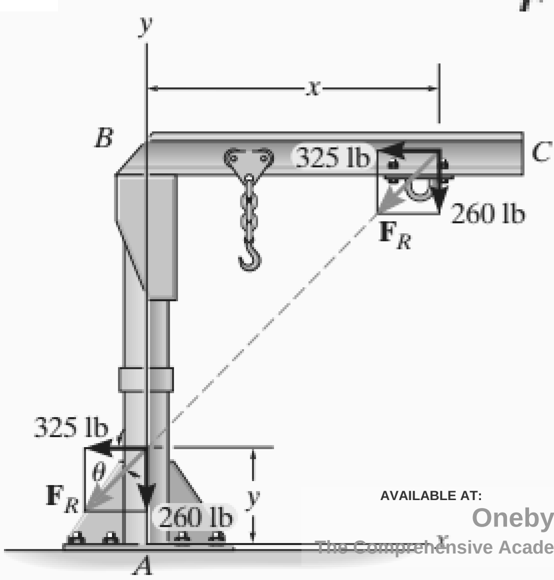
Force Summation. Resolving the 250-lb force into x and y components and summing the force components yields

$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x; \quad (F_R)_x = -250\text{ lb}\left(\tfrac{3}{5}\right) - 175\text{ lb} = -325\text{ lb} = 325\text{ lb} \leftarrow$$

$$+\uparrow(F_R)_y = \Sigma F_y; \quad (F_R)_y = -250\text{ lb}\left(\tfrac{4}{5}\right) - 60\text{ lb} = -260\text{ lb} = 260\text{ lb}\downarrow$$

As shown by the vector addition in Fig. 4–45*b*,

$$F_R = \sqrt{(325\text{ lb})^2 + (260\text{ lb})^2} = 416\text{ lb} \qquad \textit{Ans.}$$



$$\theta = \tan^{-1}\left(\frac{260\text{ lb}}{325\text{ lb}}\right) = 38.7° \qquad \textit{Ans.}$$

Moment Summation. Moments will be summed about point A. Assuming the line of action of $\mathbf{F}_R$ *intersects* AB at a distance y from A, Fig. 4–45b, we have

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325 \text{ lb } (y) + 260 \text{ lb } (0)$$

$$= 175 \text{ lb } (5 \text{ ft}) - 60 \text{ lb } (3 \text{ ft}) + 250 \text{ lb}\left(\tfrac{3}{5}\right)(11 \text{ ft}) - 250 \text{ lb}\left(\tfrac{4}{5}\right)(8 \text{ ft})$$

$$y = 2.29 \text{ ft} \qquad \textit{Ans.}$$

By the principle of transmissibility, $\mathbf{F}_R$ can be placed at a distance x where it intersects BC, Fig. 4–45b. In this case we have

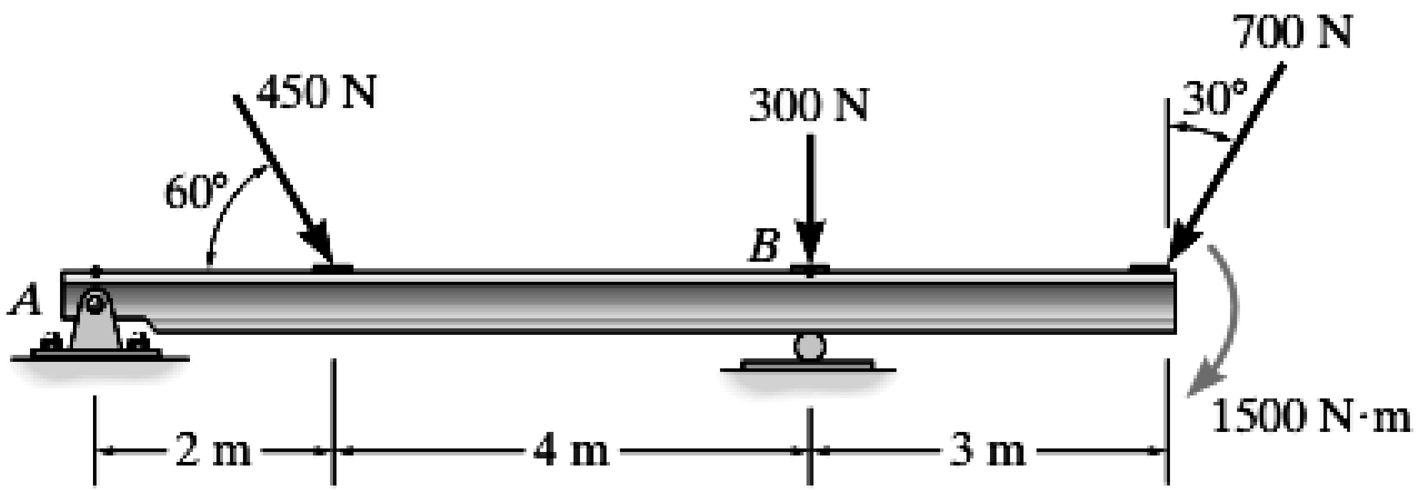
$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 325 \text{ lb } (11 \text{ ft}) - 260 \text{ lb } (x)$$

$$= 175 \text{ lb } (5 \text{ ft}) - 60 \text{ lb } (3 \text{ ft}) + 250 \text{ lb}\left(\tfrac{3}{5}\right)(11 \text{ ft}) - 250 \text{ lb}\left(\tfrac{4}{5}\right)(8 \text{ ft})$$

$$x = 10.9 \text{ ft} \qquad \textit{Ans.}$$

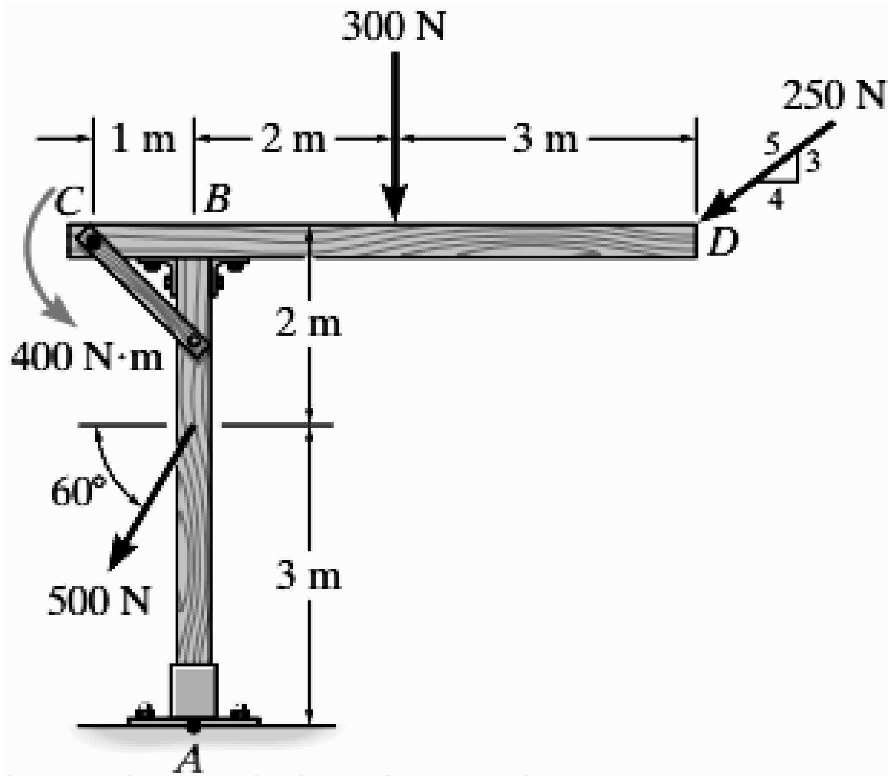
Problem No.: 4.117 and 4.118

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A and from *B*.



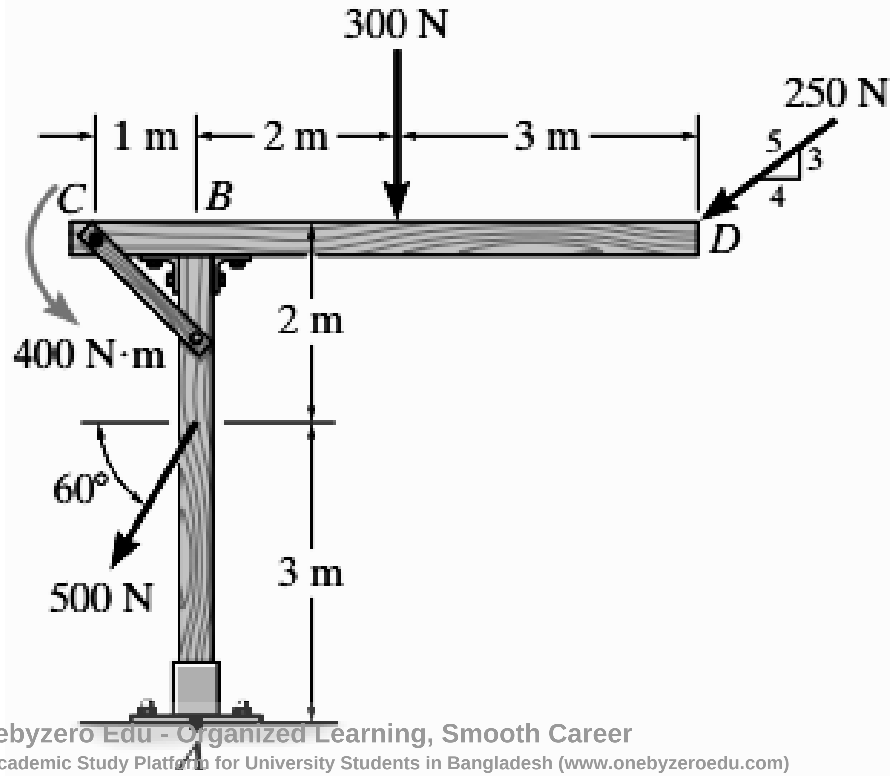
Problem No.: 4.120

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A



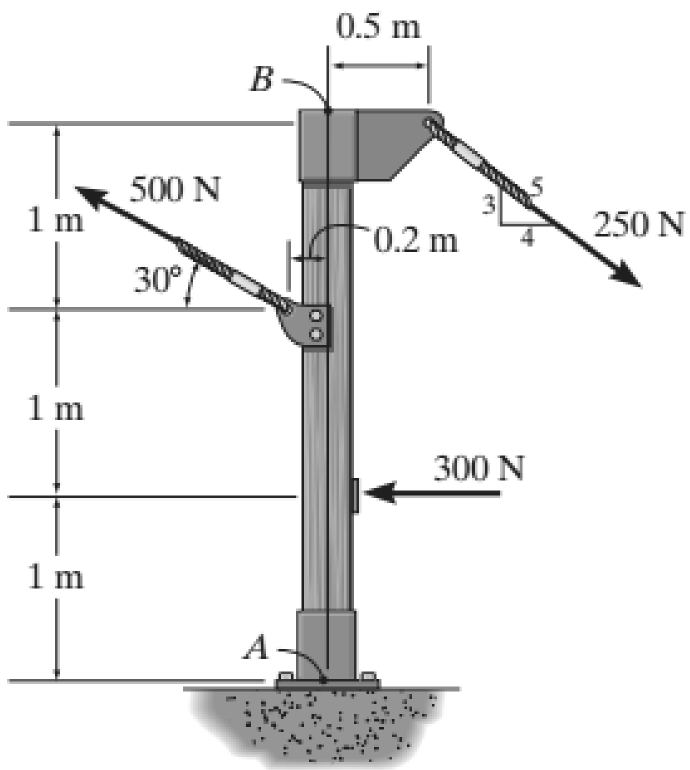
Problem No.: 4.121

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member *CD*, measured from end *C*.



Problem No.: 4.124

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post *AB* measured from point *A*.



Problem No.: 4.134

Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point *O*.